Table of Contents

Preface .......................................................................................................................... 15
Acknowledgments ........................................................................................................ 17

1 ASYMPTOTIC APPROXIMATIONS ................................................................. 19
   1.1 Asymptotic series .................................................. 19
   1.2 Fundamental concepts of asymptotics [129] ............... 21
   1.3 Transformations of asymptotical series [129] ............... 23
   1.4 Nonuniform expansions [129] ................................ 25
   1.5 Non-dimensionalization .................................... 27
   1.6 Asymptotics of integrals [129] ............................... 29

2 REGULAR PERTURBATIONS OF PARAMETERS ................................. 47
   2.1 Eigenvalue problems ........................................ 47
   2.2 Stability of oval cylindrical shell uniformly loaded by external pressure ...................... 58
   2.3 Stability of the cantilever beam .............................. 60
   2.4 Adjoint operators method .................................. 62
   2.5 Transformation of coordinates and variables ............ 65
   2.6 Asymptotic and real error .................................. 71
   2.7 Numerical verification of asymptotic solution ............ 75
   2.8 Removal of nonuniformities ............................... 77
   2.9 Nonlinear vibrations of a stringer shell ...................... 81
   2.10 Non-quasilinear asymptotics of nonlinear system ..... 84
   2.11 Artificial small parameters ................................. 88
   2.12 Method of small $\delta$ ....................................... 91
   2.13 Method of large $\delta$ ....................................... 94
   2.14 Choice of zero approximation ............................ 98
   2.15 Lyapunov–Schmidt procedure ............................. 101
   2.16 Nonlinear periodical vibrations of continuous structures .. 103

3 SINGULAR PERTURBATION PROBLEMS ................................................. 117
   3.1 The method of Gol’denveizer-Vishik-Lyusternik [313, 645, 672, 673, 674] ............... 117
   3.2 Multiscale method ........................................... 122
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.3</td>
<td>Newton polygon and asymptotic integration parameters</td>
<td>125</td>
</tr>
<tr>
<td>3</td>
<td>3.4</td>
<td>Stretched plate bending</td>
<td>131</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
<td>Simplification of the static equations of a cylindrical shell</td>
<td>133</td>
</tr>
<tr>
<td>3</td>
<td>3.6</td>
<td>Boundary layer: Papkovitch approach</td>
<td>136</td>
</tr>
<tr>
<td>3</td>
<td>3.7</td>
<td>Edge boundary layer</td>
<td>138</td>
</tr>
<tr>
<td>3</td>
<td>3.8</td>
<td>Incorporating of the singular part of solution</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>3.9</td>
<td>Plane theory of elasticity</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>3.10</td>
<td>Asymptotic foundation model</td>
<td>144</td>
</tr>
<tr>
<td>3</td>
<td>3.11</td>
<td>Vibrations of reinforced conical shells</td>
<td>149</td>
</tr>
<tr>
<td>4</td>
<td>BOUNDARY VALUE PROBLEMS OF ISOTROPIC CYLINDRICAL SHELLS</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.1</td>
<td>Governing relations</td>
<td>157</td>
</tr>
<tr>
<td>4</td>
<td>4.2</td>
<td>Operator method</td>
<td>159</td>
</tr>
<tr>
<td>4</td>
<td>4.3</td>
<td>Simplified boundary value problems</td>
<td>160</td>
</tr>
<tr>
<td>5</td>
<td>BOUNDARY VALUE PROBLEMS – ORTHOTROPIC SHELLS</td>
<td>169</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5.1</td>
<td>Governing relations</td>
<td>169</td>
</tr>
<tr>
<td>5</td>
<td>5.2</td>
<td>Statical problems</td>
<td>173</td>
</tr>
<tr>
<td>5</td>
<td>5.3</td>
<td>Non-linear dynamical problems</td>
<td>182</td>
</tr>
<tr>
<td>5</td>
<td>5.4</td>
<td>Stability problems</td>
<td>189</td>
</tr>
<tr>
<td>5</td>
<td>5.5</td>
<td>Error estimation using Newton’s method</td>
<td>197</td>
</tr>
<tr>
<td>6</td>
<td>COMPOSITE BOUNDARY VALUE PROBLEMS – ISOTROPIC SHELLS</td>
<td>201</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6.1</td>
<td>Statical problems</td>
<td>202</td>
</tr>
<tr>
<td>6</td>
<td>6.2</td>
<td>Equations of higher order approximations</td>
<td>206</td>
</tr>
<tr>
<td>6</td>
<td>6.3</td>
<td>Error estimation</td>
<td>208</td>
</tr>
<tr>
<td>6</td>
<td>6.4</td>
<td>Dynamical problems</td>
<td>211</td>
</tr>
<tr>
<td>6</td>
<td>6.5</td>
<td>Non-linear dynamical problems</td>
<td>217</td>
</tr>
<tr>
<td>7</td>
<td>COMPOSITE BOUNDARY VALUE PROBLEMS – ORTHOTROPIC SHELLS</td>
<td>223</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7.1</td>
<td>Statical problems</td>
<td>223</td>
</tr>
<tr>
<td>7</td>
<td>7.2</td>
<td>Dynamical problems</td>
<td>234</td>
</tr>
<tr>
<td>7</td>
<td>7.3</td>
<td>Non-linear dynamical problems</td>
<td>235</td>
</tr>
<tr>
<td>7</td>
<td>7.4</td>
<td>Stability problems</td>
<td>239</td>
</tr>
<tr>
<td>8</td>
<td>AVERAGING</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8.1</td>
<td>Two-scales approach</td>
<td>241</td>
</tr>
<tr>
<td>8</td>
<td>8.2</td>
<td>Visco-elastic problems and ’freezing’ method</td>
<td>244</td>
</tr>
<tr>
<td>8</td>
<td>8.3</td>
<td>The successive change of variables</td>
<td>246</td>
</tr>
<tr>
<td>8</td>
<td>8.4</td>
<td>Application of the Lie groups</td>
<td>249</td>
</tr>
<tr>
<td>8</td>
<td>8.5</td>
<td>Whitham method (non-linear WKB approach)</td>
<td>256</td>
</tr>
</tbody>
</table>
9 CONTINUALIZATION ........................................ 259

10 HOMOGENIZATION ........................................ 267
10.1 ODEs with rapidly oscillating coefficients ................. 267
10.2 Axisymmetric bending of corrugated circle plate .......... 272
10.3 Deformation of reinforced membrane ....................... 276
10.4 Ribbed strip – two-scale and Fourier homogenization .... 280
10.5 Ribbed plate – direct homogenization ....................... 284
10.6 Perforated membrane .................................... 292
10.7 Composite with periodic cubic inclusions ................. 302
10.8 Torsion of bar with periodic parallelepiped inclusions ... 307
10.9 Solution of cell problem: perturbation of boundary form 313
10.10 Linear vibrations of a beam with concentrated masses and
discrete supports ........................................ 319

11 INTERMEDIATE ASYMPTOTICS – DYNAMICAL EDGE
EFFECT METHOD .......................................... 333
11.1 Linear preliminaries .................................... 333
11.2 Nonlinear beam vibrations ................................ 336
11.3 Nonlinear rectangular plate vibrations ..................... 340
11.4 Nonlinear shallow shell vibrations ........ .................. 345
11.5 Rayleigh-Ritz-Bolotin approach .......................... 353
11.6 Parallelogram plate vibrations ............................ 357
11.7 Sectorial plate nonlinear vibrations ........................ 360

12 LOCALIZATION .......................................... 363
12.1 Localization in linear chains .............................. 363
12.2 Localization in nonlinear chain ............................ 370
12.3 Localization of shell buckling ............................. 372
12.4 Localization of vibration in plates and shells ............. 373

13 IMPROVEMENT OF PERTURBATION SERIES ................. 377
13.1 Padé approximants (PA) .................................. 377
13.2 The effect of autocorrection .............................. 379
13.3 Extending of perturbation series .......................... 381
13.4 Improvement iterative procedures convergence .......... 384
13.5 Nonuniformities elimination ............................. 387
13.6 Error estimation of asymptotic approaches ................. 388
13.7 Localized solutions and blow-up phenomenon ............ 388
13.8 Gibbs phenomena ....................................... 389
13.9 Boundary conditions perturbation method ................. 392
13.10 Bifurcation problem .................................... 401
13.11 Borel summation and superasymptotics .................. 402
13.12 Domb–Sykes plot [340, 659] .............................. 404
# Table of Contents

13.14 Analytical continuation [407] ................................. 409

**14 MATCHING OF LIMITING ASYMPOTOTIC EXPANSIONS .... 417**

14.1 Two-point Padé approximants .......................... 417
14.2 Quasifractional approximants .......................... 424
14.3 Post-buckling behaviour of shallow convex shell .......... 429

**15 COMPLEX VARIABLES IN NONLINEAR DYNAMICS .......... 435**

15.1 Nonlinear oscillator with cubic anharmonicity ........... 436
15.2 System of two weakly coupled nonlinear oscillators ....... 447
15.3 Nonlinear dynamics of an infinite chain of coupled oscillators . 453
15.4 Nonlinear dynamics of an infinite chain of coupled particles .... 460

**16 OTHER ASYMPTOTICAL APPROACHES ...................... 463**

16.1 Matched asymptotic procedure .......................... 463
16.2 Hilbert transform ....................................... 465
16.3 Normal forms in non-linear problems ...................... 467
16.4 WKB - approach ....................................... 470
16.5 The WKB method and turning points ..................... 473
16.6 A distributional approach ................................ 478

**AFTERWORD ................................................. 483**

Asymptotics and Computers .................................. 483
Are Asymptotic Methods a Panacea? ....................... 485

**References .................................................. 489**

**Subject index ................................................ 529**