

---

# Contents

<b>1</b>	<b>Introduction</b> .....	1
1.1	Objectives of Analyzing Multiple Time Series .....	1
1.2	Some Basics .....	2
1.3	Vector Autoregressive Processes .....	4
1.4	Outline of the Following Chapters .....	5

---

## Part I Finite Order Vector Autoregressive Processes

---

<b>2</b>	<b>Stable Vector Autoregressive Processes</b> .....	13
2.1	Basic Assumptions and Properties of VAR Processes .....	13
2.1.1	Stable VAR( $p$ ) Processes .....	13
2.1.2	The Moving Average Representation of a VAR Process .....	18
2.1.3	Stationary Processes .....	24
2.1.4	Computation of Autocovariances and Autocorrelations of Stable VAR Processes .....	26
2.2	Forecasting .....	31
2.2.1	The Loss Function .....	32
2.2.2	Point Forecasts .....	33
2.2.3	Interval Forecasts and Forecast Regions .....	39
2.3	Structural Analysis with VAR Models .....	41
2.3.1	Granger-Causality, Instantaneous Causality, and Multi-Step Causality .....	41
2.3.2	Impulse Response Analysis .....	51
2.3.3	Forecast Error Variance Decomposition .....	63
2.3.4	Remarks on the Interpretation of VAR Models .....	66
2.4	Exercises .....	66
<b>3</b>	<b>Estimation of Vector Autoregressive Processes</b> .....	69
3.1	Introduction .....	69
3.2	Multivariate Least Squares Estimation .....	69

3.2.1	The Estimator .....	70
3.2.2	Asymptotic Properties of the Least Squares Estimator .	73
3.2.3	An Example .....	77
3.2.4	Small Sample Properties of the LS Estimator .....	80
3.3	Least Squares Estimation with Mean-Adjusted Data and Yule-Walker Estimation .....	82
3.3.1	Estimation when the Process Mean Is Known .....	82
3.3.2	Estimation of the Process Mean .....	83
3.3.3	Estimation with Unknown Process Mean .....	85
3.3.4	The Yule-Walker Estimator .....	85
3.3.5	An Example .....	87
3.4	Maximum Likelihood Estimation .....	87
3.4.1	The Likelihood Function .....	87
3.4.2	The ML Estimators.....	89
3.4.3	Properties of the ML Estimators .....	90
3.5	Forecasting with Estimated Processes .....	94
3.5.1	General Assumptions and Results .....	94
3.5.2	The Approximate MSE Matrix.....	96
3.5.3	An Example .....	98
3.5.4	A Small Sample Investigation .....	100
3.6	Testing for Causality .....	102
3.6.1	A Wald Test for Granger-Causality .....	102
3.6.2	An Example .....	103
3.6.3	Testing for Instantaneous Causality .....	104
3.6.4	Testing for Multi-Step Causality .....	106
3.7	The Asymptotic Distributions of Impulse Responses and Forecast Error Variance Decompositions .....	109
3.7.1	The Main Results .....	109
3.7.2	Proof of Proposition 3.6 .....	116
3.7.3	An Example .....	118
3.7.4	Investigating the Distributions of the Impulse Responses by Simulation Techniques .....	126
3.8	Exercises .....	130
3.8.1	Algebraic Problems .....	130
3.8.2	Numerical Problems .....	132
<b>4</b>	<b>VAR Order Selection and Checking the Model Adequacy ..</b>	<b>135</b>
4.1	Introduction .....	135
4.2	A Sequence of Tests for Determining the VAR Order.....	136
4.2.1	The Impact of the Fitted VAR Order on the Forecast MSE.....	136
4.2.2	The Likelihood Ratio Test Statistic .....	138
4.2.3	A Testing Scheme for VAR Order Determination .....	143
4.2.4	An Example .....	145
4.3	Criteria for VAR Order Selection .....	146

4.3.1	Minimizing the Forecast MSE	146
4.3.2	Consistent Order Selection	148
4.3.3	Comparison of Order Selection Criteria	151
4.3.4	Some Small Sample Simulation Results	153
4.4	Checking the Whiteness of the Residuals	157
4.4.1	The Asymptotic Distributions of the Autocovariances and Autocorrelations of a White Noise Process	157
4.4.2	The Asymptotic Distributions of the Residual Autocovariances and Autocorrelations of an Estimated VAR Process	161
4.4.3	Portmanteau Tests	169
4.4.4	Lagrange Multiplier Tests	171
4.5	Testing for Nonnormality	174
4.5.1	Tests for Nonnormality of a Vector White Noise Process	174
4.5.2	Tests for Nonnormality of a VAR Process	177
4.6	Tests for Structural Change	181
4.6.1	Chow Tests	182
4.6.2	Forecast Tests for Structural Change	184
4.7	Exercises	189
4.7.1	Algebraic Problems	189
4.7.2	Numerical Problems	191
<b>5</b>	<b>VAR Processes with Parameter Constraints</b>	<b>193</b>
5.1	Introduction	193
5.2	Linear Constraints	194
5.2.1	The Model and the Constraints	194
5.2.2	LS, GLS, and EGLS Estimation	195
5.2.3	Maximum Likelihood Estimation	200
5.2.4	Constraints for Individual Equations	201
5.2.5	Restrictions for the White Noise Covariance Matrix	202
5.2.6	Forecasting	204
5.2.7	Impulse Response Analysis and Forecast Error Variance Decomposition	205
5.2.8	Specification of Subset VAR Models	206
5.2.9	Model Checking	212
5.2.10	An Example	217
5.3	VAR Processes with Nonlinear Parameter Restrictions	221
5.4	Bayesian Estimation	222
5.4.1	Basic Terms and Notation	222
5.4.2	Normal Priors for the Parameters of a Gaussian VAR Process	223
5.4.3	The Minnesota or Litterman Prior	225
5.4.4	Practical Considerations	227
5.4.5	An Example	227

5.4.6	Classical versus Bayesian Interpretation of $\bar{\alpha}$ in Forecasting and Structural Analysis .....	228
5.5	Exercises .....	230
5.5.1	Algebraic Exercises .....	230
5.5.2	Numerical Problems .....	231

**Part II Cointegrated Processes**

<b>6</b>	<b>Vector Error Correction Models</b> .....	237
6.1	Integrated Processes .....	238
6.2	VAR Processes with Integrated Variables .....	243
6.3	Cointegrated Processes, Common Stochastic Trends, and Vector Error Correction Models .....	244
6.4	Deterministic Terms in Cointegrated Processes .....	256
6.5	Forecasting Integrated and Cointegrated Variables .....	258
6.6	Causality Analysis .....	261
6.7	Impulse Response Analysis .....	262
6.8	Exercises .....	265
<b>7</b>	<b>Estimation of Vector Error Correction Models</b> .....	269
7.1	Estimation of a Simple Special Case VECM .....	269
7.2	Estimation of General VECMs .....	286
7.2.1	LS Estimation .....	287
7.2.2	EGLS Estimation of the Cointegration Parameters .....	291
7.2.3	ML Estimation .....	294
7.2.4	Including Deterministic Terms .....	299
7.2.5	Other Estimation Methods for Cointegrated Systems .....	300
7.2.6	An Example .....	302
7.3	Estimating VECMs with Parameter Restrictions .....	305
7.3.1	Linear Restrictions for the Cointegration Matrix .....	305
7.3.2	Linear Restrictions for the Short-Run and Loading Parameters .....	307
7.3.3	An Example .....	309
7.4	Bayesian Estimation of Integrated Systems .....	309
7.4.1	The Model Setup .....	310
7.4.2	The Minnesota or Litterman Prior .....	310
7.4.3	An Example .....	312
7.5	Forecasting Estimated Integrated and Cointegrated Systems .....	315
7.6	Testing for Granger-Causality .....	316
7.6.1	The Noncausality Restrictions .....	316
7.6.2	Problems Related to Standard Wald Tests .....	317
7.6.3	A Wald Test Based on a Lag Augmented VAR .....	318
7.6.4	An Example .....	320
7.7	Impulse Response Analysis .....	321

7.8	Exercises .....	323
7.8.1	Algebraic Exercises .....	323
7.8.2	Numerical Exercises .....	324
<b>8</b>	<b>Specification of VECMs .....</b>	<b>325</b>
8.1	Lag Order Selection .....	325
8.2	Testing for the Rank of Cointegration .....	327
8.2.1	A VECM without Deterministic Terms .....	328
8.2.2	A Nonzero Mean Term .....	330
8.2.3	A Linear Trend .....	331
8.2.4	A Linear Trend in the Variables and Not in the Cointegration Relations .....	331
8.2.5	Summary of Results and Other Deterministic Terms ..	332
8.2.6	An Example .....	335
8.2.7	Prior Adjustment for Deterministic Terms .....	337
8.2.8	Choice of Deterministic Terms .....	341
8.2.9	Other Approaches to Testing for the Cointegrating Rank	342
8.3	Subset VECMs .....	343
8.4	Model Diagnostics .....	345
8.4.1	Checking for Residual Autocorrelation .....	345
8.4.2	Testing for Nonnormality .....	348
8.4.3	Tests for Structural Change .....	348
8.5	Exercises .....	351
8.5.1	Algebraic Exercises .....	351
8.5.2	Numerical Exercises .....	352

---

**Part III Structural and Conditional Models**

---

<b>9</b>	<b>Structural VARs and VECMs .....</b>	<b>357</b>
9.1	Structural Vector Autoregressions .....	358
9.1.1	The A-Model .....	358
9.1.2	The B-Model .....	362
9.1.3	The AB-Model .....	364
9.1.4	Long-Run Restrictions à la Blanchard-Quah .....	367
9.2	Structural Vector Error Correction Models .....	368
9.3	Estimation of Structural Parameters .....	372
9.3.1	Estimating SVAR Models .....	372
9.3.2	Estimating Structural VECMs .....	376
9.4	Impulse Response Analysis and Forecast Error Variance Decomposition .....	377
9.5	Further Issues .....	383
9.6	Exercises .....	384
9.6.1	Algebraic Problems .....	384
9.6.2	Numerical Problems .....	385

<b>10</b>	<b>Systems of Dynamic Simultaneous Equations</b>	387
10.1	Background	387
10.2	Systems with Unmodelled Variables	388
10.2.1	Types of Variables	388
10.2.2	Structural Form, Reduced Form, Final Form	390
10.2.3	Models with Rational Expectations	393
10.2.4	Cointegrated Variables	394
10.3	Estimation	395
10.3.1	Stationary Variables	396
10.3.2	Estimation of Models with $I(1)$ Variables	398
10.4	Remarks on Model Specification and Model Checking	400
10.5	Forecasting	401
10.5.1	Unconditional and Conditional Forecasts	401
10.5.2	Forecasting Estimated Dynamic SEMs	405
10.6	Multiplier Analysis	406
10.7	Optimal Control	408
10.8	Concluding Remarks on Dynamic SEMs	411
10.9	Exercises	412

---

## Part IV Infinite Order Vector Autoregressive Processes

---

<b>11</b>	<b>Vector Autoregressive Moving Average Processes</b>	419
11.1	Introduction	419
11.2	Finite Order Moving Average Processes	420
11.3	VARMA Processes	423
11.3.1	The Pure MA and Pure VAR Representations of a VARMA Process	423
11.3.2	A VAR(1) Representation of a VARMA Process	426
11.4	The Autocovariances and Autocorrelations of a VARMA( $p, q$ ) Process	429
11.5	Forecasting VARMA Processes	432
11.6	Transforming and Aggregating VARMA Processes	434
11.6.1	Linear Transformations of VARMA Processes	435
11.6.2	Aggregation of VARMA Processes	440
11.7	Interpretation of VARMA Models	442
11.7.1	Granger-Causality	442
11.7.2	Impulse Response Analysis	444
11.8	Exercises	444
<b>12</b>	<b>Estimation of VARMA Models</b>	447
12.1	The Identification Problem	447
12.1.1	Nonuniqueness of VARMA Representations	447
12.1.2	Final Equations Form and Echelon Form	452
12.1.3	Illustrations	455

12.2	The Gaussian Likelihood Function . . . . .	459
12.2.1	The Likelihood Function of an MA(1) Process . . . . .	459
12.2.2	The MA( $q$ ) Case . . . . .	461
12.2.3	The VARMA(1, 1) Case . . . . .	463
12.2.4	The General VARMA( $p, q$ ) Case . . . . .	464
12.3	Computation of the ML Estimates . . . . .	467
12.3.1	The Normal Equations . . . . .	468
12.3.2	Optimization Algorithms . . . . .	470
12.3.3	The Information Matrix . . . . .	473
12.3.4	Preliminary Estimation . . . . .	474
12.3.5	An Illustration . . . . .	477
12.4	Asymptotic Properties of the ML Estimators . . . . .	479
12.4.1	Theoretical Results . . . . .	479
12.4.2	A Real Data Example . . . . .	486
12.5	Forecasting Estimated VARMA Processes . . . . .	487
12.6	Estimated Impulse Responses . . . . .	490
12.7	Exercises . . . . .	491
<b>13</b>	<b>Specification and Checking the Adequacy of VARMA Models</b> . . . . .	<b>493</b>
13.1	Introduction . . . . .	493
13.2	Specification of the Final Equations Form . . . . .	494
13.2.1	A Specification Procedure . . . . .	494
13.2.2	An Example . . . . .	497
13.3	Specification of Echelon Forms . . . . .	498
13.3.1	A Procedure for Small Systems . . . . .	499
13.3.2	A Full Search Procedure Based on Linear Least Squares Computations . . . . .	501
13.3.3	Hannan-Kavalieris Procedure . . . . .	503
13.3.4	Poskitt's Procedure . . . . .	505
13.4	Remarks on Other Specification Strategies for VARMA Models . . . . .	507
13.5	Model Checking . . . . .	508
13.5.1	LM Tests . . . . .	508
13.5.2	Residual Autocorrelations and Portmanteau Tests . . . . .	510
13.5.3	Prediction Tests for Structural Change . . . . .	511
13.6	Critique of VARMA Model Fitting . . . . .	511
13.7	Exercises . . . . .	512
<b>14</b>	<b>Cointegrated VARMA Processes</b> . . . . .	<b>515</b>
14.1	Introduction . . . . .	515
14.2	The VARMA Framework for $I(1)$ Variables . . . . .	516
14.2.1	Levels VARMA Models . . . . .	516
14.2.2	The Reverse Echelon Form . . . . .	518
14.2.3	The Error Correction Echelon Form . . . . .	519
14.3	Estimation . . . . .	521

14.3.1	Estimation of $ARMA_{RE}$ Models	521
14.3.2	Estimation of $EC-ARMA_{RE}$ Models	522
14.4	Specification of $EC-ARMA_{RE}$ Models	523
14.4.1	Specification of Kronecker Indices	523
14.4.2	Specification of the Cointegrating Rank	525
14.5	Forecasting Cointegrated VARMA Processes	526
14.6	An Example	526
14.7	Exercises	528
14.7.1	Algebraic Exercises	528
14.7.2	Numerical Exercises	529
<b>15</b>	<b>Fitting Finite Order VAR Models to Infinite Order Processes</b>	<b>531</b>
15.1	Background	531
15.2	Multivariate Least Squares Estimation	532
15.3	Forecasting	536
15.3.1	Theoretical Results	536
15.3.2	An Example	538
15.4	Impulse Response Analysis and Forecast Error Variance Decompositions	540
15.4.1	Asymptotic Theory	540
15.4.2	An Example	543
15.5	Cointegrated Infinite Order VARs	545
15.5.1	The Model Setup	546
15.5.2	Estimation	549
15.5.3	Testing for the Cointegrating Rank	551
15.6	Exercises	552
<hr/>		
<b>Part V Time Series Topics</b>		
<hr/>		
<b>16</b>	<b>Multivariate ARCH and GARCH Models</b>	<b>557</b>
16.1	Background	557
16.2	Univariate GARCH Models	559
16.2.1	Definitions	559
16.2.2	Forecasting	561
16.3	Multivariate GARCH Models	562
16.3.1	Multivariate ARCH	563
16.3.2	MGARCH	564
16.3.3	Other Multivariate ARCH and GARCH Models	567
16.4	Estimation	569
16.4.1	Theory	569
16.4.2	An Example	571
16.5	Checking MGARCH Models	576
16.5.1	ARCH-LM and ARCH-Portmanteau Tests	576



16.5.2	LM and Portmanteau Tests for Remaining ARCH . . . . .	577
16.5.3	Other Diagnostic Tests . . . . .	578
16.5.4	An Example . . . . .	578
16.6	Interpreting GARCH Models . . . . .	579
16.6.1	Causality in Variance . . . . .	579
16.6.2	Conditional Moment Profiles and Generalized Impulse Responses . . . . .	580
16.7	Problems and Extensions . . . . .	582
16.8	Exercises . . . . .	584
<b>17</b>	<b>Periodic VAR Processes and Intervention Models . . . . .</b>	<b>585</b>
17.1	Introduction . . . . .	585
17.2	The VAR( $p$ ) Model with Time Varying Coefficients . . . . .	587
17.2.1	General Properties . . . . .	587
17.2.2	ML Estimation . . . . .	589
17.3	Periodic Processes . . . . .	591
17.3.1	A VAR Representation with Time Invariant Coefficients . . . . .	592
17.3.2	ML Estimation and Testing for Time Varying Coefficients . . . . .	595
17.3.3	An Example . . . . .	602
17.3.4	Bibliographical Notes and Extensions . . . . .	604
17.4	Intervention Models . . . . .	604
17.4.1	Interventions in the Intercept Model . . . . .	605
17.4.2	A Discrete Change in the Mean . . . . .	606
17.4.3	An Illustrative Example . . . . .	608
17.4.4	Extensions and References . . . . .	609
17.5	Exercises . . . . .	609
<b>18</b>	<b>State Space Models . . . . .</b>	<b>611</b>
18.1	Background . . . . .	611
18.2	State Space Models . . . . .	613
18.2.1	The Model Setup . . . . .	613
18.2.2	More General State Space Models . . . . .	624
18.3	The Kalman Filter . . . . .	625
18.3.1	The Kalman Filter Recursions . . . . .	626
18.3.2	Proof of the Kalman Filter Recursions . . . . .	630
18.4	Maximum Likelihood Estimation of State Space Models . . . . .	631
18.4.1	The Log-Likelihood Function . . . . .	632
18.4.2	The Identification Problem . . . . .	633
18.4.3	Maximization of the Log-Likelihood Function . . . . .	634
18.4.4	Asymptotic Properties of the ML Estimator . . . . .	636
18.5	A Real Data Example . . . . .	637
18.6	Exercises . . . . .	641

---

**Appendix**

---

<b>A</b>	<b>Vectors and Matrices</b> .....	645
A.1	Basic Definitions .....	645
A.2	Basic Matrix Operations .....	646
A.3	The Determinant .....	647
A.4	The Inverse, the Adjoint, and Generalized Inverses .....	649
A.4.1	Inverse and Adjoint of a Square Matrix .....	649
A.4.2	Generalized Inverses .....	650
A.5	The Rank .....	651
A.6	Eigenvalues and -vectors – Characteristic Values and Vectors ..	652
A.7	The Trace .....	653
A.8	Some Special Matrices and Vectors .....	653
A.8.1	Idempotent and Nilpotent Matrices .....	653
A.8.2	Orthogonal Matrices and Vectors and Orthogonal Complements .....	654
A.8.3	Definite Matrices and Quadratic Forms .....	655
A.9	Decomposition and Diagonalization of Matrices .....	656
A.9.1	The Jordan Canonical Form .....	656
A.9.2	Decomposition of Symmetric Matrices .....	658
A.9.3	The Choleski Decomposition of a Positive Definite Matrix .....	658
A.10	Partitioned Matrices .....	659
A.11	The Kronecker Product .....	660
A.12	The vec and vech Operators and Related Matrices .....	661
A.12.1	The Operators .....	661
A.12.2	Elimination, Duplication, and Commutation Matrices ..	662
A.13	Vector and Matrix Differentiation .....	664
A.14	Optimization of Vector Functions .....	671
A.15	Problems .....	675
<b>B</b>	<b>Multivariate Normal and Related Distributions</b> .....	677
B.1	Multivariate Normal Distributions .....	677
B.2	Related Distributions .....	678
<b>C</b>	<b>Stochastic Convergence and Asymptotic Distributions</b> .....	681
C.1	Concepts of Stochastic Convergence .....	681
C.2	Order in Probability .....	684
C.3	Infinite Sums of Random Variables .....	685
C.4	Laws of Large Numbers and Central Limit Theorems .....	689
C.5	Standard Asymptotic Properties of Estimators and Test Statistics .....	692
C.6	Maximum Likelihood Estimation .....	693
C.7	Likelihood Ratio, Lagrange Multiplier, and Wald Tests .....	694

C.8 Unit Root Asymptotics.....	698
C.8.1 Univariate Processes .....	698
C.8.2 Multivariate Processes .....	703
<b>D Evaluating Properties of Estimators and Test Statistics by Simulation and Resampling Techniques.....</b>	<b>707</b>
D.1 Simulating a Multiple Time Series with VAR Generation Process .....	707
D.2 Evaluating Distributions of Functions of Multiple Time Series by Simulation .....	708
D.3 Resampling Methods.....	709
<b>References.....</b>	<b>713</b>
<b>Index of Notation .....</b>	<b>733</b>
<b>Author Index.....</b>	<b>741</b>
<b>Subject Index .....</b>	<b>747</b>



<http://www.springer.com/978-3-540-40172-8>

New Introduction to Multiple Time Series Analysis

Lütkepohl, H.

2005, XXI, 764 p., Hardcover

ISBN: 978-3-540-40172-8