Contents

Volume I Basic Theory

1 Generalized Differentiation in Banach Spaces .................. 3
  1.1 Generalized Normals to Nonconvex Sets ....................... 4
    1.1.1 Basic Definitions and Some Properties .............. 4
    1.1.2 Tangential Approximations .......................... 12
    1.1.3 Calculus of Generalized Normals ..................... 18
    1.1.4 Sequential Normal Compactness of Sets .............. 27
    1.1.5 Variational Descriptions and Minimality .......... 33
  1.2 Coderivatives of Set-Valued Mappings ...................... 39
    1.2.1 Basic Definitions and Representations ........... 40
    1.2.2 Lipschitzian Properties .......................... 47
    1.2.3 Metric Regularity and Covering ..................... 56
    1.2.4 Calculus of Coderivatives in Banach Spaces ...... 70
    1.2.5 Sequential Normal Compactness of Mappings ....... 75
  1.3 Subdifferentials of Nonsmooth Functions ................... 81
    1.3.1 Basic Definitions and Relationships .............. 82
    1.3.2 Fréchet-Like \( \varepsilon \)-Subgradients
        and Limiting Representations ....................... 87
    1.3.3 Subdifferentiation of Distance Functions .......... 97
    1.3.4 Subdifferential Calculus in Banach Spaces .......... 112
    1.3.5 Second-Order Subdifferentials ..................... 121
  1.4 Commentary to Chap. 1 .................................. 132

2 Extremal Principle in Variational Analysis ...................... 171
  2.1 Set Extremality and Nonconvex Separation .................... 172
    2.1.1 Extremal Systems of Sets ......................... 172
    2.1.2 Versions of the Extremal Principle
        and Supporting Properties ........................ 174
    2.1.3 Extremal Principle in Finite Dimensions .......... 178
  2.2 Extremal Principle in Asplund Spaces ....................... 180
2.2.1 Approximate Extremal Principle .......................... 180
 in Smooth Banach Spaces .......................... 180
2.2.2 Separable Reduction ............................... 183
2.2.3 Extremal Characterizations of Asplund Spaces ........ 195
2.3 Relations with Variational Principles ....................... 203
 2.3.1 Ekeland Variational Principle ....................... 204
 2.3.2 Subdifferential Variational Principles ................. 206
 2.3.3 Smooth Variational Principles ....................... 210
2.4 Representations and Characterizations in Asplund Spaces ... 214
 2.4.1 Subgradients, Normals, and Coderivatives .......................... 214
   in Asplund Spaces ............................... 214
 2.4.2 Representations of Singular Subgradients .......................... 223
 and Horizontal Normals to Graphs and Epigraphs ............... 223
2.5 Versions of Extremal Principle in Banach Spaces ............ 230
  2.5.1 Axiomatic Normal and Subdifferential Structures .... 231
  2.5.2 Specific Normal and Subdifferential Structures ........... 235
  2.5.3 Abstract Versions of Extremal Principle .............. 245
2.6 Commentary to Chap. 2 .................................. 249

3 Full Calculus in Asplund Spaces ........................... 261
 3.1 Calculus Rules for Normals and Coderivatives ............... 261
   3.1.1 Calculus of Normal Cones .......................... 262
   3.1.2 Calculus of Coderivatives ........................... 274
 3.1.3 Strictly Lipschitzian Behavior .......................... 287
      and Coderivative Scalarization .......................... 287
 3.2 Subdifferential Calculus and Related Topics ............... 296
   3.2.1 Calculus Rules for Basic and Singular Subgradients .... 296
   3.2.2 Approximate Mean Value Theorem .......................... 308
      with Some Applications .............................. 308
   3.2.3 Connections with Other Subdifferentials ................. 317
   3.2.4 Graphical Regularity of Lipschitzian Mappings ........... 327
   3.2.5 Second-Order Subdifferential Calculus .................. 335
 3.3 SNC Calculus for Sets and Mappings ....................... 341
   3.3.1 Sequential Normal Compactness of Set Intersections 341
      and Inverse Images .............................. 341
   3.3.2 Sequential Normal Compactness for Sums .......................... 349
      and Related Operations with Maps ................... 349
   3.3.3 Sequential Normal Compactness for Compositions 354
      of Maps ........................................ 354
 3.4 Commentary to Chap. 3 .................................. 361

4 Characterizations of Well-Posedness ........................ 377
 and Sensitivity Analysis ................................. 378
 4.1 Neighborhood Criteria and Exact Bounds ...................... 378
   4.1.1 Neighborhood Characterizations of Covering .......... 378
### Volume II Applications

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Constrained Optimization and Equilibria</td>
<td>3</td>
</tr>
<tr>
<td>5.1</td>
<td>Necessary Conditions in Mathematical Programming</td>
<td>3</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Minimization Problems with Geometric Constraints</td>
<td>4</td>
</tr>
<tr>
<td>5.1.2</td>
<td>Necessary Conditions under Operator Constraints</td>
<td>9</td>
</tr>
<tr>
<td>5.1.3</td>
<td>Necessary Conditions under Functional Constraints</td>
<td>22</td>
</tr>
<tr>
<td>5.1.4</td>
<td>Suboptimality Conditions for Constrained Problems</td>
<td>41</td>
</tr>
<tr>
<td>5.2</td>
<td>Mathematical Programs with Equilibrium Constraints</td>
<td>46</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Necessary Conditions for Abstract MPECs</td>
<td>47</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Variational Systems as Equilibrium Constraints</td>
<td>51</td>
</tr>
<tr>
<td>5.2.3</td>
<td>Refined Lower Subdifferential Conditions for MPECs via Exact Penalization</td>
<td>61</td>
</tr>
<tr>
<td>5.3</td>
<td>Multiobjective Optimization</td>
<td>69</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Optimal Solutions to Multiobjective Problems</td>
<td>70</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Generalized Order Optimality</td>
<td>73</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Extremal Principle for Set-Valued Mappings</td>
<td>83</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Optimality Conditions with Respect to Closed Preferences</td>
<td>92</td>
</tr>
<tr>
<td>5.3.5</td>
<td>Multiobjective Optimization with Equilibrium Constraints</td>
<td>99</td>
</tr>
<tr>
<td>5.4</td>
<td>Subextremality and Suboptimality at Linear Rate</td>
<td>109</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Linear Subextremality of Set Systems</td>
<td>110</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Linear Suboptimality in Multiobjective Optimization</td>
<td>115</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Linear Suboptimality for Minimization Problems</td>
<td>125</td>
</tr>
<tr>
<td>5.5</td>
<td>Commentary to Chap. 5</td>
<td>131</td>
</tr>
</tbody>
</table>
6 Optimal Control of Evolution Systems in Banach Spaces . . 159

6.1 Optimal Control of Discrete-Time and Continuous-
time Evolution Inclusions ........................................... 160

6.1.1 Differential Inclusions and Their Discrete
Approximations ...................................................... 160

6.1.2 Bolza Problem for Differential Inclusions
and Relaxation Stability ......................................... 168

6.1.3 Well-Posed Discrete Approximations
of the Bolza Problem ............................................ 175

6.1.4 Necessary Optimality Conditions for Discrete-
Time Inclusions .................................................. 184

6.1.5 Euler-Lagrange Conditions for Relaxed Minimizers . . 198

6.2 Necessary Optimality Conditions for Differential Inclusions
without Relaxation ................................................. 210

6.2.1 Euler-Lagrange and Maximum Conditions
for Intermediate Local Minimizers ............................ 211

6.2.2 Discussion and Examples ................................... 219

6.3 Maximum Principle for Continuous-Time Systems
with Smooth Dynamics ............................................. 227

6.3.1 Formulation and Discussion of Main Results ............ 228

6.3.2 Maximum Principle for Free-Endpoint Problems ....... 234

6.3.3 Transversality Conditions for Problems
with Inequality Constraints ..................................... 239

6.3.4 Transversality Conditions for Problems
with Equality Constraints ....................................... 244

6.4 Approximate Maximum Principle in Optimal Control .... 248

6.4.1 Exact and Approximate Maximum Principles
for Discrete-Time Control Systems ............................ 248

6.4.2 Uniformly Upper Subdifferentiable Functions .......... 254

6.4.3 Approximate Maximum Principle
for Free-Endpoint Control Systems .......................... 258

6.4.4 Approximate Maximum Principle under Endpoint
Constraints: Positive and Negative Statements ............. 268

6.4.5 Approximate Maximum Principle
under Endpoint Constraints: Proofs and
Applications ......................................................... 276

6.4.6 Control Systems with Delays and of Neutral Type ..... 290

6.5 Commentary to Chap. 6 ....................................... 297

7 Optimal Control of Distributed Systems ..................... 335

7.1 Optimization of Differential-Algebraic Inclusions with Delays.. 336

7.1.1 Discrete Approximations of Differential-Algebraic
Inclusions .......................................................... 338

7.1.2 Strong Convergence of Discrete Approximations ....... 346
7.1.3 Necessary Optimality Conditions for Difference-Algebraic Systems ........................................ 352
7.1.4 Euler-Lagrange and Hamiltonian Conditions for Differential-Algebraic Systems .................. 357

7.2 Neumann Boundary Control of Semilinear Constrained Hyperbolic Equations .................. 364
7.2.1 Problem Formulation and Necessary Optimality Conditions for Neumann Boundary Controls .......... 365
7.2.2 Analysis of State and Adjoint Systems in the Neumann Problem .................................. 369
7.2.3 Needle-Type Variations and Increment Formula ...................................................... 376
7.2.4 Proof of Necessary Optimality Conditions ............................................................ 380

7.3 Dirichlet Boundary Control of Linear Constrained Hyperbolic Equations ...................... 386
7.3.1 Problem Formulation and Necessary Optimality Conditions for Dirichlet Controls .............. 387
7.3.2 Existence of Dirichlet Optimal Controls .................................................................... 390
7.3.3 Adjoint System in the Dirichlet Problem ................................................................. 391
7.3.4 Proof of Optimality Conditions ............................................................................... 395

7.4 Minimax Control of Parabolic Systems with Pointwise State Constraints ....................... 398
7.4.1 Problem Formulation and Splitting ............................................................................ 400
7.4.2 Properties of Mild Solutions and Minimax Existence Theorem ................................ 404
7.4.3 Suboptimality Conditions for Worst Perturbations ................................................. 410
7.4.4 Suboptimal Controls under Worst Perturbations ..................................................... 422
7.4.5 Necessary Optimality Conditions under State Constraints ..................................... 427

7.5 Commentary to Chap. 7 .................................................. 439

8 Applications to Economics ................................................. 461
8.1 Models of Welfare Economics .................................................................................. 461
8.1.1 Basic Concepts and Model Description ................................................................. 462
8.1.2 Net Demand Qualification Conditions for Pareto and Weak Pareto Optimal Allocations ...... 465
8.2 Second Welfare Theorem for Nonconvex Economies ................................................ 468
8.2.1 Approximate Versions of Second Welfare Theorem ............................................. 469
8.2.2 Exact Versions of Second Welfare Theorem ......................................................... 474
8.3 Nonconvex Economies with Ordered Commodity Spaces ........................................... 477
8.3.1 Positive Marginal Prices ....................................................................................... 477
8.3.2 Enhanced Results for Strong Pareto Optimality ..................................................... 479
8.4 Abstract Versions and Further Extensions .................................................................... 484
8.4.1 Abstract Versions of Second Welfare Theorem .................................................... 484
8.4.2 Public Goods and Restriction on Exchange ......................................................... 490
8.5 Commentary to Chap. 8 ................................................................................. 492
XXII Contents

References .......................................................... 477
List of Statements .................................................. 543
Glossary of Notation ............................................... 565
Subject Index ....................................................... 569
Variational Analysis and Generalized Differentiation I
Basic Theory
Mordukhovich, B.S.
2006, XXII, 579 p., Hardcover
ISBN: 978-3-540-25437-9