One of the interesting questions in the study of links between mathematics, art and creativity is whether a mathematician’s creativity leads him to invent a new world, or rather makes him discover one that already exists in its own right. It could seem like a superfluous question of very little interest. Though it might seem so to non mathematicians, it certainly isn’t so for many mathematicians like Roger Penrose, who has dedicated a part of the book *The Emperor’s New Mind* [1] to this subject. “In mathematics, should one talk of invention or of discovery?” asks Roger Penrose. There are two possible answers to the question: when the mathematician obtains new results, he creates only some elaborate mental constructions which, although have no relation to physical reality whatsoever, nevertheless possess such power and elegance that they are able to make the researcher believe that these “mere mental constructions” have their very own reality. Alternatively, do mathematicians discover that “these mere mental constructions” are already there, a truth whose existence is completely independent of their workings out? Penrose is inclined toward the second point of view, even though he adds that the problem is not as simple as it seems. His opinion is that in mathematics one determines situations for which the term discovery is certainly more appropriate than the term invention. There are cases in which the results essentially derive from the structure itself, more than from the input of mathematicians. Penrose cites the example of complex numbers: “Later we find many other magical properties that these complex numbers possess, properties that we had no inkling about at first. These properties are just there. They were not put there by Cardano, nor by Bombelli, nor Wallis, nor Coates, nor Euler, nor Wessel, nor Gauss, despite the undoubted farsightedness of these, and other, great mathematicians; such magic was inherent in the very structure that they gradually uncovered.”

When mathematicians discover a structure of this kind, it means that they stumbled upon that which Penrose calls “works of God”. So, are mathematicians mere explorers? Fortunately not all mathematical structures are so strictly predetermined. There are cases in which “the results are obtained equally by merit of the structure and of the mathematicians’ calculations”; in this case, Penrose says, it is more appropriate to use the word invention than the word discovery. Hence there is room for what he calls works of man, though he notes that the discoveries of structures that are works of God are of vastly greater importance than the ‘mere’ inventions that are the works of man.
One can make analogous distinctions in the arts and in engineering. “Great works of art are indeed ‘closer to God’ than are lesser ones.”

Among artists, Penrose claims, the idea that their most important works reveal eternal truths is not uncommon, that they have “some kind of prior ethereal existence”, while the lesser important works have a more personal character, and are arbitrary, mortal constructions. In mathematics, this need to believe in an immaterial and eternal existence, at least of the most profound mathematical concepts (the works of God), is felt even more strongly. Penrose observes that “there is a compelling uniqueness and universality in such mathematical ideas which seems to be of quite a different order from that which one could expect in the arts or or engineering.” A work of art can be appreciated or brought into question in different epochs, but no-one can put in doubt a correct proof of a mathematical result.

Penrose explains very explicitly that mathematicians think of their discipline as a highly creative activity that has nothing to envy of the creativity of artists and indeed, because of the uniqueness and universality of mathematical creation, one that should be regarded as superior to the artistic discipline; Penrose doesn’t write it explicitly, but many mathematicians think that mathematics is the true Art; a difficult, laborious art, with it’s very own language and symbolism, that produces universally accepted results.

Penrose takes up the question of the existence in its own right of the world of mathematical ideas, a question that has dogged the science of mathematics since its inception. It comes from the book Matière à pensée, co-authored by a mathematician, Alain Connes, winner of the Fields medal, and a neurobiologist, Jean-Pierre Changeux [2].

In one of the chapters in the book, entitled Invention ou découverte the neurobiologist, speaking of the nature of mathematical objects, recalls that there is a realist attitude directly inspired by Plato, an attitude that can be summarised in the phrase: the world is populated by ideas that have a reality separate from physical reality. Supporting his observation, Changeux quotes a claim of Dieudonné, according to whom “mathematicians accept that mathematical objects possess a reality distinct from physical reality”, a reality that can be compared to that which Plato accords to his Ideas. From this point of view it is of secondary importance whether or not the mathematical world is a divine creation, as the mathematician Cantor (1874–1918) believed: “The highest perfection of God is the possibility of creating an infinite set, and his immense bounty leads him to do so.”

We are in complete divine mathesis, in complete metaphysics. This is surprising to serious scientists, comments Changeux. Connes, the mathematician, not at all bothered by the biologist’s arguments, replies very clearly that he identifies strongly with the realist point of view. After emphasising that the sequence of prime numbers has a more stable reality than the material reality surrounding us, he notes that “one can compare the work of a mathematician to that of an explorer discovering the world.” If practical experience leads us to discover pure and simple facts such as, for example, that the sequence of prime numbers appear to have no end, the work of a mathematician consists of proving that there exists an infinite number of primes. Once this property has been proved, no-one can ever claim to having found the largest prime of all. It would be easy to show them
that they are wrong. Connes concludes “so we clash with a reality just as incontestable as the physical one”. Indeed, it is arguably more real than any physical reality.

Still in the aforementioned book, we read that mathematics is a universal language, but that is not all, for the generative nature of mathematics has a crucial role. Otherwise one could not explain the incredible and unforeseeable usefulness of mathematics, from meteorology to the structure of DNA, to the simulation of boats for the America’s Cup.

Imagination is not enough to do mathematics; you need knowledge of the language and techniques; imagination, creativity, the ability to make problems explicit, to formalise and solve them, whenever possible.

In recent years, thanks to the advent of computer graphics, the role of visualisation in some sectors of mathematics has increased significantly. It was inevitable that this sort of new Visual Mathematics would open new ways for links between creativity in the arts and in mathematics. We wish to deal with some of these aspects in this new book of the series called significantly “Mathematics and Culture”, which was born of an idea of Valeria Marchiafava and Michele Emmer in September 1997, in Turin.

References
