## Contents

Preface ........................................................................... V

Part I The curve smoothing problem

1 Curve evolution and image processing ......................... 3
  1.1 Shape recognition ............................................ 5
    1.1.1 Axioms for shape recognition......................... 7
    1.1.2 ... and their consequences .......................... 8
  1.2 Curve smoothing ............................................ 8
    1.2.1 The linear curve scale space ......................... 9
    1.2.2 Towards an intrinsic heat equation ................. 10
  1.3 An axiomatic approach of curve evolution ................. 12
    1.3.1 Basic requirements .................................... 12
    1.3.2 First conclusions and first models ................. 13
  1.4 Image and contour smoothing ................................ 15
  1.5 Applications .................................................. 15
    1.5.1 Active contours ........................................ 15
    1.5.2 Principles of a shape recognition algorithm ...... 17
    1.5.3 Optical character recognition ...................... 18
  1.6 Organization of the volume ................................ 19
  1.7 Bibliographical notes ....................................... 20

2 Rudimentary bases of curve geometry .......................... 23
  2.1 Jordan curves ............................................... 23
  2.2 Length of a curve .......................................... 24
  2.3 Euclidean parameterization ................................ 25
  2.4 Motion of graphs ............................................ 27
Part II Theoretical curve evolution

3 Geometric curve shortening flow ........................................ 31
   3.1 What kind of equations for curve smoothing? ................. 32
      3.1.1 Invariant flows .................................. 32
      3.1.2 Symmetry group of flow .......................... 32
   3.2 Differential invariants ....................................... 40
      3.2.1 General form of invariant flows ..................... 40
      3.2.2 The mean curvature flow is the Euclidean intrinsic heat flow 42
      3.2.3 The affine invariant flow: the simplest affine invariant curve flow .................................. 42
   3.3 General properties of second order parabolic flows: a digest .... 46
      3.3.1 Existence and uniqueness for mean curvature and affine flows .................................. 46
      3.3.2 Short-time existence in the general case ............... 48
      3.3.3 Evolution of convex curves ........................ 48
      3.3.4 Evolution of the length ........................... 49
      3.3.5 Evolution of the area ............................. 49
   3.4 Smoothing staircases ........................................ 50
   3.5 Bibliographical notes ....................................... 53

4 Curve evolution and level sets .................................... 55
   4.1 From curve operators to function operators and vice versa .... 57
      4.1.1 Signed distance function and supporting function ..... 57
      4.1.2 Monotone and translation invariant operators ....... 57
      4.1.3 Level sets and their properties ...................... 58
      4.1.4 Extension of sets operators to functions operators .... 60
      4.1.5 Characterization of monotone, contrast invariant operator 62
      4.1.6 Asymptotic behavior of morphological operators ....... 64
      4.1.7 Morphological operators yield PDEs .................. 69
   4.2 Curve evolution and Scale Space theory ...................... 70
      4.2.1 Multiscale analysis are given by PDEs ............... 70
      4.2.2 Morphological scale space ........................... 73
   4.3 Viscosity solutions ........................................ 74
      4.3.1 Definition of viscosity solution ...................... 76
      4.3.2 Proof of uniqueness: the maximum principle .......... 80
      4.3.3 Existence of solution by Perron’s Method ............ 85
      4.3.4 Contrast invariance of level sets flow .............. 88
      4.3.5 Viscosity solutions shorten level lines .......... 89
   4.4 Morphological operators and viscosity solution ............ 90
      4.4.1 Median filter and mean curvature motion ............... 90
      4.4.2 Affine invariant schemes ............................ 92
   4.5 Conclusions ............................................... 93
   4.6 Curvature thresholding .................................... 93
### Part III Numerical curve evolution

#### 5 Classical numerical methods for curve evolution

- 5.1 Parametric methods
  - 5.1.1 Finite difference methods
  - 5.1.2 Finite element schemes
- 5.2 Non parametric methods
  - 5.2.1 Sethian’s level sets methods
  - 5.2.2 Alvarez and Guichard’s finite differences scheme
  - 5.2.3 A monotone and convergent finite difference schemes
  - 5.2.4 Bence, Merriman and Osher scheme for mean curvature motion
  - 5.2.5 Elliptic regularization

#### 6 A geometrical scheme for curve evolution

- 6.1 Preliminary definitions
- 6.2 Erosion
- 6.3 Properties of the erosion
- 6.4 Erosion and level sets
  - 6.4.1 Consistency
  - 6.4.2 Convergence
- 6.5 Evolution by a power of the curvature
  - 6.5.1 Consistency
  - 6.5.2 Convergence
- 6.6 A numerical implementation of the erosion
  - 6.6.1 Scale covariance
  - 6.6.2 General algorithm
  - 6.6.3 Eroded set and envelope
  - 6.6.4 Swallow tails
- 6.7 Numerical experiments
  - 6.7.1 Evolving circles
  - 6.7.2 Convex polygons
  - 6.7.3 Unclosed curve
  - 6.7.4 Evolving non convex curves
  - 6.7.5 Invariance
  - 6.7.6 Numerical maximum principle
<table>
<thead>
<tr>
<th>Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.7.7 Image filtering</td>
<td>156</td>
</tr>
<tr>
<td>6.8 Bibliographical notes</td>
<td>166</td>
</tr>
<tr>
<td>Conclusion and perspectives</td>
<td>167</td>
</tr>
<tr>
<td>Discussion</td>
<td>167</td>
</tr>
<tr>
<td>Open problems</td>
<td>169</td>
</tr>
<tr>
<td>A Proof of Thm. 4.34</td>
<td>171</td>
</tr>
<tr>
<td>References</td>
<td>177</td>
</tr>
<tr>
<td>Index</td>
<td>185</td>
</tr>
</tbody>
</table>
Geometric Curve Evolution and Image Processing
Cao, F.
2003, X, 194 p., Softcover
ISBN: 978-3-540-00402-8