Spatial Problem Solving in Spatial Structures

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Abstract. The ability to solve spatial tasks is crucial for everyday life and therefore of great importance for cognitive agents. In artificial intelligence (AI) we model this ability by representing spatial configurations and spatial tasks in the form of knowledge *about* space and time. Augmented by appropriate algorithms, such representations enable the generation of knowledge-based solutions to spatial problems. In comparison, natural embodied and situated cognitive agents often solve spatial tasks without detailed knowledge about underlying geometric and mechanical laws and relationships. They directly relate actions and their effects through physical affordances inherent in their bodies and their environments. Examples are found in everyday reasoning and also in descriptive geometry. In an ongoing research effort we investigate how spatial and temporal structures *in the body and the environment* can support or even replace reasoning effort in computational processes. We call the direct use of spatial structure *Strong Spatial Cognition*. Our contribution describes cognitive principles of an extended paradigm of cognitive processing. The work aims (i) to *understand* the effectiveness and efficiency of natural problem solving approaches; (ii) to *overcome* the need for detailed representations required in the knowledge-based approach; and (iii) to *build* computational cognitive systems that make use of these principles.

Keywords: Cognitive systems · Spatial cognition · Spatial problem solving · Strong spatial cognition

1 Introduction: AI and Cognitive Systems

Cognitive agents – be they humans, animals, or autonomous robots – comprise brains or computers connected to sensors and actuators. These components are arranged in the agents' bodies in ways that allow them to interact with one another and with their spatial environments. In this paper, we consider the entire aggregate (cognitive agent including its body and the environment) as a *full cognitive system* (Fig. 1). We investigate how spatial processes performed by an agent in the environment can support computational processes.

Consider the *distribution*, *coordination*, and *execution* of spatial tasks among the system components of spatially situated cognitive agents. In a pure information processing/AI approach, the elements of the spatial problem outside the brain or computer would be considered "outside the system." Inside the system they are
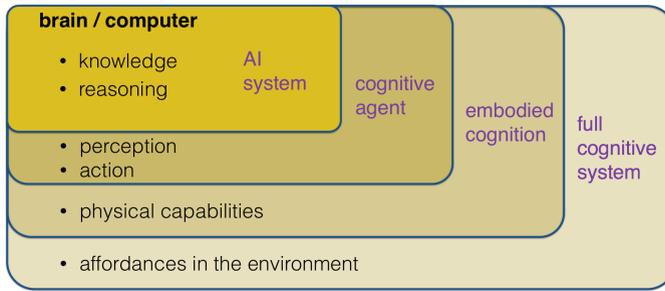
Fig. 1. Structure of a full cognitive system. (Adapted from Freksa 2015b, p. 11)

described in terms of some knowledge representation language or pattern. This allows the computer to perform formal reasoning (or other computational processing) on the knowledge representation. To obtain this knowledge representation, physical, topological, and geometric relations in the problem configuration must be transformed into abstract *information.* The tasks then can be performed entirely on the information processing level, where physical, topological, and geometric relations and physical affordances no longer persist.

However, the classical information-processing oriented division between (a) brain or computer and (b) perception, action, body, and environment, is only one way of distributing the activities involved in cognitive processing. As in natural problem solving approaches, we can include the spatial problem domain as part of the system and (1) maintain some of the spatial relations in their original form; (2) simulate spatial relations and interactions through motion models; or (3) use *mild abstraction* (Freksa et al. 2018) for their representation.

One of the pillars of knowledge representation research is that processing structures of problem solving processes differ within and across types of representation (cf. Marr 1982). Most importantly, certain processing structures facilitate certain forms of processing (Sloman 1985; Larkin and Simon 1987). In particular, certain *spatial* structures facilitate certain forms of *spatial* problem solving (Barkowsky et al. 1994; Freksa 2013, 2015a, b; Freksa and Schultheis 2014; Freksa et al. 2016; Furbach et al. 2016). *Spatial* problem solving is a particularly interesting and important class of problems that mobile cognitive agents, such as most animals and autonomous robots, must deal with all the time. Accordingly, we investigate structures that specifically facilitate solutions to spatial problems. However, it has been argued that spatial cognition can provide mechanisms for non-spatial problem solving, as well (e.g. Lakoff and Johnson 1980); thus, if successful for spatial problem solving, the importance of this research may extend into other domains of cognition.

From depictive (constructive) geometry we know that certain computation can be replaced by geometric construction (and vice versa). Often, constructive procedures appear simpler than the corresponding computations and they also lead more frequently to insights into the nature of the problem and the solution. Our approach aims at relating *spatial constructions* and the corresponding *computations* in their respective underlying structures (substrates) in order to assess and compare the spatial problem

solving processes in the framework of a full cognitive system that comprises both spatial and computational operations.

Our work studies spatial problems, identifies principles of solving spatial problems inside the spatial domain, and compares spatial approaches with purely computational approaches. We also investigate how to determine a suitable approach for solving a spatial problem from a problem specification and how to control spatial actions to solve spatial problems in a goal-oriented manner.

2   State of the Art in Spatial Problem Solving

Spatial problem solving has been a fundamental research topic in AI from the very beginning. Initially, spatial relations were treated like other features: task-relevant aspects of the domain were formalized and represented in some kind of data structure; general computation and reasoning methods were applied; and the result of the computation was interpreted in terms of the target domain. Taking into account the ubiquity of space and time in real environments, approaches have been developed that give spatial and temporal relations a special status and that are specifically tailored towards specific aspects of space (such as topology, orientation, and distance). In the following overview we consider five perspectives that have been taken for solving spatial problems. Most approaches take into account several of these perspectives.

2.1   Knowledge-Based Perspectives (K)

Knowledge-based approaches have dominated AI for most of the past 60 years. In such approaches, facts and relations about space in general and about specific problem domains are encoded as knowledge that describes the domain. Problem solving is then performed by computation that operates on the description level. Ontologies have become a much-used approach to formally describe properties of domains (e.g., Bateman et al. 2010 for the spatial domain). Commonsense reasoning, being one of the oldest research areas of AI, makes extensive use of formalized spatial knowledge. Yet, as Davis and Marcus (2015) point out, progress in the field has been slow. Qualitative spatial reasoning (Egenhofer and Franzosa 1991; Freksa 1991b; Cohn and Renz 2008; Dylla et al. 2017) has been an active research area since the late 1980s; specific knowledge about spatial relations and spatial operations defines spatial structures and makes up spatial calculi for reasoning on the basis of human-understandable spatial concepts. Some cognitive robotics approaches include qualitative spatial calculi (Mansouri and Pecora 2013; Wolter and Wallgrün 2012) to provide knowledge-based support for object identification, spatial orientation, and robot actions in space.

2.2   Computational Adaptation and Learning (L)

As manual encoding of extensive knowledge is cumbersome, learning algorithms have been developed that generate spatial information from sensor data (e.g. SLAM - Thrun et al. 2005; Frese et al. 2005) or generate new knowledge about spatial actions from rudimentary knowledge (Wörgötter et al. 2015). Deep Learning (Goodfellow et al.

2016) and 'cognitive computing' (Modha et al. 2011; Kelly 2015) approaches combine a multitude of methods in order to derive new knowledge from vast amounts of data. Whereas SLAM specifically exploits spatial structure in the sensor data, the latter two approaches do not necessarily require perceptual or other spatial input; they are mainly used for processing knowledge in an abstract form. Adaptation and learning approaches typically make use of large amounts of behavioral correlations rather than relying on the internal domain structure.

2.3   Analogical Representation and Analogical Reasoning (A)

Analogical reasoning pays particular attention to the structure of the represented domain and to the processes operating on them. Sloman (1971, 1975) analyzed structural characteristics in comparison to descriptive ('Fregean') representations as well as effects of representational structures on the processing characteristics. In the spirit of analogical representation and reasoning and in recognition of the power of two-dimensional visualization and perception, the field of *Diagrammatic Reasoning* evolved (Glasgow et al. 1995; Goel et al. 2010). This research area was motivated by (i) papers by mathematicians and other theoreticians who confessed that they obtain their insights and understanding of problems not by looking at formulas but by drawing and studying diagrams; and by (ii) influences of cognitive psychology that acknowledge essential differences between processing 2D layouts and processing their linearized descriptions. In his famous book *How to solve it,* Polya (1956) analyzed cognitive processes that lead from problem statements to their solutions. Diagrams visualize spatial and spatialize non-spatial situations to make them accessible to visual perception and spatial analysis. In AI, Funt (1980) and Chandrasekaran (2006) have proposed retina-like and more general perceptual representational structures that make certain aspects of spatial configurations – such as spatial neighborhood, shape, or size – directly accessible to computational processes.

2.4   Biology-Inspired Approaches (B)

Alternative autonomous systems inspired by biological role models have been proposed in biocybernetics and AI research. Such systems perceive their environment and act in a goal-oriented manner. For example, Braitenberg's (1984) vehicles demonstrate smart spatial behavior without requiring explicit symbolic representation of spatial information; these vehicles directly replicate specific aspects of neural sensory-motor connectivity that implicitly responds to spatial arrangements and inherent spatial structures. From an engineering perspective, Brooks (1991) proposed the *subsumption architecture* to implement intelligent reactive systems without representing knowledge about the domain. Like Braitenberg, Brooks emphasizes physical interaction with the environment as a primary source of constraints on the design of intelligent systems. He argues for focusing on the interface to the real world, in order to avoid the need for reliance on a representation. In their elaborate book, Pfeifer and Scheier (1999) describe this class of approaches as a new way of understanding intelligence *(Nouvelle AI)*. Goel et al. (2012) use biological role models for conceiving design systems that manifest cognitive, collaborative, conceptual, and creative characteristics.

2.5   Cognition-Based Approaches (C)

In *The design of everyday things*, Norman (2013) distinguishes 'knowledge in the world' from 'knowledge in the head'. Maintaining features and relations in their original form and context corresponds to what Norman calls *knowledge in the world.* Use of knowledge in the world involves the use of perception in order to solve problems. He explains why people need both types of knowledge to manage everyday tasks. Gibson (1979) introduced the notion of *affordance* to characterize conditions that permit actions in physical environments. As Gibson developed his theory in the context of visual perception, the notion was understood by various authors (including initially by Norman) to refer exclusively to conditions that can be perceptually identified; different uses of the notion *affordance* have caused considerable confusion in the cognitive science community that seems to have scared some researchers away from what is a highly beneficial notion, if used in a well-defined manner.

Qualitative spatial relations have provided a conceptual framework to comprehend space-specific structures and processes underlying topological and geometric affordances (Freksa 1991a; Gooday and Cohn 1994; Egenhofer and Mark 1995). Wintermute and Laird (2008) proposed to augment qualitative representation and reasoning in cognitive architectures by quantitative simulations of spatial relations and interactions, in order to make physical affordances accessible to computational approaches. A Dagstuhl Seminar (Rome et al. 2008) approached the topic of affordance-based robot control as a perspective on directly coupling perception, action, and reasoning in real-time. Raubal and Moratz (2008) present an extended theory of affordances that differentiates between different kinds of affordances in order to characterize functional models of affordance-based agents. Kirsh (2013) discusses human imagination and the role of (i) physical interaction; (ii) thinking with brain and body; (iii) physically performing vs. watching; and (iv) thinking with things for effective cognition and for finding answers to sometimes long-standing questions. In our work we address these issues with a constructive approach and theoretical analysis.

3   Spatial Solutions to Spatial Problems

The Strong Spatial Cognition team at the Bremen Spatial Cognition Center[1] has studied example problems from the literature such as the shortest route problem (Dreyfus and Haugeland 1974), Archimedes' volume comparison problem (Vitruvius 2007), and classical geometric construction problems. We demonstrated or outlined spatial procedures to solving these problems (Freksa 2013, 2015a, b; Freksa and Schultheis 2014; Freksa et al. 2016, 2018). In collaboration with other universities we started to investigate approaches to compare formal and spatial solutions to solving spatial problems (Furbach et al. 2016).

Previously, in the framework of the CRC/TR 8 Spatial Cognition, the team had investigated spatial relations in geographic maps and varieties of formal representations. *Mild abstraction* (Freksa et al. 2018) was identified as a form of analogical

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[1] http://bscc.spatial-cognition.de.

representation employed in geographic paper maps to facilitate physical operations such as perception, route-following with a finger, and manipulation in similar ways as in the represented real-world domain. Mild abstraction may abstract only from few aspects, while preserving structural spatial properties Perception is required to use mildly abstracted representations – but the perception task typically is easier than the same task under real-world conditions, for example due to the modified scale.

4 The Strong Spatial Cognition Paradigm

Our work largely builds on the perspectives **A**, **B**, and **C** outlined in the state-of-the-art section. Also, **K** is important for the meta-level of planning and organizing sub-tasks of spatial problem solving. **L** only peripherally plays a role, as we prescribe spatial structure that learning approaches would derive. In our approach, we specifically target the direct use of spatial structure. For example, we study the concept of a 'string' as a deformable 1D spatial entity whose length is invariant under shape transformations (Freksa et al. 2016). A certain class of spatial problems requires length comparison, while absolute length is irrelevant (e.g., the shortest path problem). Arbitrarily shaped strings are difficult to compare with regards to their length. Simple spatial *pull* and *align* operations, however, can transform arbitrarily shaped strings into straight and aligned strings that are easily compared through perceptual operations. Similar operations can be found for other aspects of space, such as angles.

With our work, we take an important step beyond the state of the art and introduce a paradigm shift: we aim at preserving spatial structure and directly exploit features of simultaneous spatial transformations. Initially we represent spatial objects and configurations using the objects and configurations themselves or their *physical* models, rather than via abstract representations. The core advantages of this approach are: information loss due to early representational commitments is avoided; and no decision needs to be made beforehand about which aspects of the world to represent in a certain way, which aspects to abstract away, and which spatial reference frame to use. This can be decided partly during the problem solving process. Then, additional contextual information may become available that can guide the choice of the specific abstraction to be used.

Even more important: objects and configurations frequently are aggregated in a natural and meaningful way; for example, a chair may consist of a seat, several legs, and a back; if I move or deform one component of a chair, I automatically (and simultaneously!) move or deform other components and the entire chair, and vice versa (cf. the frame problem, McCarthy and Hayes 1969). This property is not intrinsically given in abstract representations of physical objects; but it is an extremely important property from a cognitive point of view, as no computational processing cycles are required for simulating the physical effects or for reasoning about them. Thus, manipulability of physical structures may become an important feature of cognitive processing, and not merely a property of physical objects.

Our approach is to *isolate* and *simplify* the specific spatial problem to be solved, e.g. by removing task-irrelevant entities and features from the spatial configuration or by *reconstructing the essence* of the spatial configuration through mild abstraction. In

general, it will be difficult to prescribe the precise preprocessing steps for solving a problem; but for the special case of spatial problems it is feasible to provide useful heuristics. These can serve as meta-knowledge which can be used to control actions on the physical level. After successful preprocessing, it will be possible in certain cases to 'read' an answer to the problem through perception directly off the resulting configuration; in other cases, the resulting spatial configuration may be a more suitable starting point for a knowledge-based approach to solving the problem.

A main hypothesis of our approach is that the 'intelligence' of cognitive systems is located not only in specific abstract problem-solving approaches, but also – and perhaps more importantly – in the capability of recognizing characteristic problem structures and of selecting particularly suitable problem-solving approaches for given tasks. Formal representations may not facilitate the recognition of such structures, due to a bias inherent in the abstraction. This is where *mild abstraction* can help.

The insight that spatial relations and physical operations are strongly connected to cognitive processing may lead to a different division of labor between the perceptual, the representational, the computational, and the locomotive parts of cognitive interaction than the one we currently pursue in AI systems: rather than putting all the 'intelligence' of the system into computing, our approach aims at putting more intelligence into the interactions between components and structures of the full cognitive system. More specifically, we aim at exploiting intrinsic structures of space and time in order to simplify the tasks to be solved.

We hypothesize that the flexible assignment of physical and computational resources for cognitive problem solving may be closer to the workings of natural cognitive systems than an almost exclusively computational approach. For example, when we as cognitive agents search for a certain object in our environment, we have at least two strategies at our disposal: we can represent the object in our mind and try to imagine and mentally reconstruct where it could or should be – the classical AI approach; or we can visually search for the object in our physical environment. Which is better (or more promising) depends on a variety of factors including memory, physical effort, the size of the physical environment, etc.; frequently a clever combination of both approaches will be best.

Strong Spatial Cognition research is primarily carried out as basic cognitive systems research: we identify and relate a set of cognitive principles and ways of combining them to obtain cognitive performance in spatio-temporal domains. We bring together three areas of expertise: (1) **cognitive systems research** – to investigate cognitive architectures and trade-offs between explicit and implicit representations; (2) **theory** – to characterize and analyze the resulting structures and operations; and (3) **implementation** – to construct and explore various cognitive system configurations.. The Strong Spatial Cognition approach aims at developing and exploring a novel paradigm for cognitive processing based on the integration of results obtained in various disciplines of cognitive science.

5   Example of Strong Spatial Cognition Problem Solving[2]

Suppose an agent's task is to identify the shortest route that connects a location A with a location B given several possible paths in a route network that can be chosen. A classical knowledge-based approach would (i) represent the lengths of the route sections, (ii) compute various alternatives of configuring these sections to connect A and B and (iii) determine the option with the smallest overall length. Note that the lengths of the route sections need to be known to use this approach although the absolute length of the resulting route is not of interest. Also note that several alternatives have to be computed and compared before the one route of interest can be identified.

Dreyfus and Haugeland ([1974](#)) describe a spatial approach to this task. Here we present a mildly abstracted version of a route network: a map in which all regions that do not correspond to routes are missing; the routes are represented here by colored
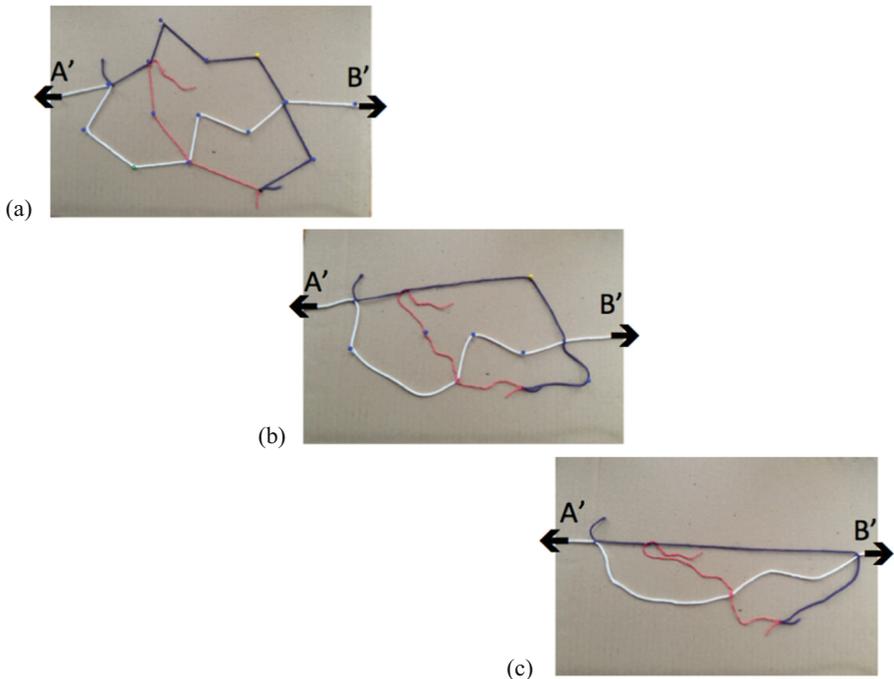

Fig. 2.   Determining the shortest route from point A to point B by physical manipulation of a mildly abstracted representation of a route network. (a) The (non-elastic) strings corresponding to route segments preserve the relative distance relations of the original route segments. The distance relations are invariant with respect to physical manipulations (pulling apart strings at A' and B') which distort angles and shapes of the route network (b) and (c). The shortest route is identified as the route corresponding to the straight connection between A' and B' in (c).

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[2] This example is adapted from Freksa ([2015b](#)) pp. 81–82.

strings. We obtain a deformable map consisting only of route representations that preserve the relative lengths of the original route sections (Fig. 2a).

The map permits certain spatial reconfigurations of the network through deformation, while preserving topology and important geometric constraints. In particular, an agent can (carefully) pull apart the positions A' and B' on the string map (Fig. 2b) that correspond to locations A and B until a string of route sections forms a straight line between these positions (Fig. 2c); due to the *geometric properties of the representation*, the route sections corresponding to the sections on the straight line represent the shortest route between A and B.

This approach avoids computation by reducing the problem to the relevant single dimension of length on which a basic geometric principle *straight line is shortest connection* can be directly applied. In this example, computational problem solving operations have been replaced by spatial operations.

6   Conclusions and Outlook

Strong Spatial Cognition sets out to provide answers to research questions of the following kind:

- How can physical operations replace computation in spatial problem solving?
- How can we characterize the trade-off between computation and physical action?
- What is the scope of application for cognitive operations in the spatial and temporal domain?
- What are the relations between computational constraints and spatial affordances?
- Which meta-knowledge is needed to control spatial actions for targeted problem solving?
- How general is the proposed paradigm?

The goal of this research is to develop an implementable theory of spatial problem solving in the framework of a full cognitive system. The theory will relate and compare concrete spatial actions and perceptions with abstract operations. It will provide a control structure to adequately allocate resources in specific problem contexts.

Ultimately, we envision technical applications in cyber-physical systems of physically supported cognitive configurations, for example in the development of future *intelligent materials* ('smart skin'), where distributed spatio-temporal computation is required but needs to be minimized with respect to computation cycles and energy consumption.

Our approach builds on research on spatial and temporal relations, their representation in memory, and qualitative spatial and temporal reasoning. We pursue broadly applicable cognitive principles, which can be configured to help design tomorrow's intelligent assistants. Our philosophy is to understand and exploit pertinent features of space and time as modality-specific properties of cognitive systems. Such features will enable powerful specialized approaches in the domain of space and time, as space and time are most basic for perception and action and are ubiquitous in cognitive processing. Furthermore, there are strong arguments that space and time-based approaches will not be limited to the spatial and temporal domains, as most

of human cognition is rooted in the interaction in space and time (e.g. Lakoff and Johnson 1980). The understanding and use of spatial and temporal structures will be beneficial for both cognitive science and cognitively inspired systems and AI approaches.

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