# Contents

1 **Prologue** ................................................................. 1  
1.1 Precise Position Measurements ................................ 4  
1.2 Outline of this Thesis .............................................. 8  
  1.2.1 Organization of Thesis ....................................... 9  
References ................................................................. 9  

2 **Quantum Fluctuations in Linear Systems** ................. 13  
  2.1 Kinematics of Fluctuations in Quantum Mechanics ............ 14  
    2.1.1 Operational Description of Fluctuations in Time .......... 16  
    2.1.2 Spectral Densities and Uncertainty Relations .......... 18  
  2.2 Dynamics Due to a Thermal Environment ....................... 23  
    2.2.1 Effect of Fluctuations from a Thermal Environment ..... 26  
  2.3 Dynamics Due to a Meter ......................................... 29  
    2.3.1 Effect of Fluctuations from a Meter ..................... 30  
References ................................................................. 32  

3 **Phonons and Photons** ........................................... 35  
  3.1 Phonons: Quantised Linear Elastodynamics ................. 35  
    3.1.1 Classical Description of Navier-Euler-Bernoulli Elastic  
      Field ....................................................... 37  
    3.1.2 Quantised Modes of the Elastic Field .................. 41  
    3.1.3 Mechanical Oscillator in Thermal Equilibrium .......... 43  
  3.2 Photons: Description and Detection .......................... 46  
    3.2.1 Quadrature, Number, and Phase Operators .............. 49  
    3.2.2 Quantum and Classical Fluctuations  
      in the Optical Field ..................................... 50  
    3.2.3 Detection of Optical Fluctuations ....................... 54  
    3.2.4 From Propagating Modes to Standing Waves:  
      Optical Cavity Coupled to a Waveguide .................... 69  
References ................................................................. 79  

4 Photon-Phonon Coupling: Cavity Optomechanics

4.1 Perturbing an Optical Cavity

4.2 Effective Description: Single-Mode Cavity Optomechanics

4.2.1 Steady-State Shifts

4.2.2 Dynamical Back-Action

4.3 Continuous Linear Measurement Using Cavity Optomechanics

References

5 Experimental Platform: Cryogenic Near-Field Cavity Optomechanics

5.1 Stressed Nanostring Coupled to an Optical Microcavity

5.1.1 Near-Field Coupling

5.1.2 Mechanical Properties of Stressed Radio-Frequency Beams

5.2 Measurement and Calibration of Thermomechanical Motion

5.3 Cryogenic Operation

5.3.1 Nature of Elastic Force: Radiation Pressure Versus Thermoelasticity

5.4 Experimental Schematic

References

6 Observation and Feedback-Suppression of Measurement Back-Action

6.1 Quantum-Noise-Limited Position Measurement

6.1.1 Measurement Imprecision and Back-Action in a Split-Mode Cavity

6.1.2 Measurement Imprecision

6.1.3 Measurement Back-Action

6.2 Feedback Suppression of Back-Action

6.2.1 Synthesis of a Linear Quadratic Gaussian Controller

6.2.2 Feedback by Cold Damping

6.2.3 Implementation of Feedback

6.2.4 Feedback Cooling to Near the Ground State

6.3 Conclusion

References

7 Observation of Quantum Correlations Using Feedback

7.1 Quantum Correlations Due to Light-Motion Interaction

7.1.1 Manifestation as Ponderomotive Squeezing

7.1.2 Manifestation as Sideband Asymmetry

7.2 Observation of Quantum Correlations

7.2.1 Observation of Ponderomotive Squeezing

7.2.2 Observation of Sideband Asymmetry Using Feedback
<table>
<thead>
<tr>
<th>Chapter/Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3</td>
<td>Conclusion</td>
<td>188</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>189</td>
</tr>
<tr>
<td>8</td>
<td>Epilogue</td>
<td>191</td>
</tr>
<tr>
<td>8.1</td>
<td>Quantum Correlations for Metrology and Control</td>
<td>192</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>195</td>
</tr>
<tr>
<td></td>
<td>Appendix A: Uncertainty Inequalities</td>
<td>197</td>
</tr>
<tr>
<td></td>
<td>Appendix B: Miscellanea on Elastodynamics</td>
<td>201</td>
</tr>
<tr>
<td></td>
<td>Appendix C: Response of an Imbalanced Interferometer</td>
<td>211</td>
</tr>
</tbody>
</table>
Quantum Limits on Measurement and Control of a Mechanical Oscillator
Sudhir, V.
2018, XIX, 214 p. 46 illus., 43 illus. in color., Hardcover
ISBN: 978-3-319-69430-6