

Contents

1	Edgeworth-Pareto Principle	1
1.1	Multicriteria Choice Problem	1
1.1.1	Set of Feasible Alternatives and Set of Selectable Alternatives	1
1.1.2	Decision-Maker	3
1.1.3	Vector Criterion	3
1.1.4	Multicriteria Problem	4
1.1.5	Preference Relation	5
1.1.6	Multicriteria Choice Problem	6
1.2	Binary Relations	6
1.2.1	Definition of Binary Relation	6
1.2.2	Types of Binary Relations	8
1.2.3	Ordering Relations	8
1.3	Exclusion Axiom and Set of Nondominated Alternatives	9
1.3.1	Asymmetry Requirement for Preference Relation	9
1.3.2	Exclusion Axiom	10
1.3.3	Set of Nondominated Alternatives	11
1.4	Edgeworth-Pareto Principle	12
1.4.1	Pareto Axiom	12
1.4.2	Pareto Set and Pareto Principle	12
1.4.3	Minimality of Exclusion Axiom and Pareto Axiom	13
1.5	Axioms of Transitivity and Compatibility	14
1.5.1	Transitivity Axiom	14
1.5.2	Compatibility Axiom	15
1.6	Finding of Pareto Set	18
1.6.1	Sets of Pareto Optimal Alternatives and Vectors	18
1.6.2	Calculation of Pareto Set	18
1.6.3	Design Algorithm for Pareto Set	20
1.6.4	Geometry of 2D Pareto Set	21

2	Pareto Set Reduction Based on Elementary Information Quantum	23
2.1	Invariance Requirement for Preference Relation	23
2.1.1	Relations Invariant with Respect to Linear Positive Transformation	23
2.1.2	Cone Relations	24
2.2	Definition of Elementary Information Quantum	29
2.2.1	Original Multicriteria Choice Problem.	29
2.2.2	Elementary Information Quantum: Motivation	30
2.2.3	Definition of Elementary Information Quantum	31
2.2.4	Properties of Elementary Information Quantum	32
2.2.5	Connection to Lexicographic Relation.	34
2.3	Pareto Set Reduction Using Elementary Information Quantum.	35
2.3.1	Simplification of Basic Definition	35
2.3.2	Pareto Set Reduction Based on Elementary Information Quantum	37
2.3.3	Geometrical Aspects	43
2.4	Scales of Criteria and Invariance of Measurements	46
2.4.1	Quantitative and Qualitative Scales	46
2.4.2	Pareto Set Invariance with Respect to Strictly Increasing Transformation of Criteria.	48
2.4.3	Invariance of Theorem 2.5 with Respect to Linear Positive Transformation.	48
3	Pareto Set Reduction Based on General Information Quantum	51
3.1	Definition and Properties of General Information Quantum	51
3.1.1	Basic Definitions.	51
3.1.2	Properties of Information Quantum.	52
3.2	Pareto Set Reduction Using Information Quantum	56
3.2.1	Simplified Definition of Information Quantum	56
3.2.2	Pareto Set Reduction Based on Information Quantum.	56
3.3	Geometrical Illustrations to the Problem with Three Criteria	63
3.3.1	Tricriteria Problem in General Form.	63
3.3.2	Case of Linear Criteria	65
4	Pareto Set Reduction Using Elementary Collections of Information Quanta	67
4.1	Consistent Collections of Information Quanta.	67
4.1.1	Preliminary Analysis	67
4.1.2	Definition of Consistent Collection of Vectors	68
4.1.3	Criteria of Consistency	69
4.1.4	Essential Information About Relative Importance of Criteria	74

- 4.2 Consideration of Two Elementary Information Quanta 75
 - 4.2.1 Case of Two Mutually Independent Quanta 75
 - 4.2.2 Case Where One of Two Criteria Is More Important Than the Two Others 76
 - 4.2.3 Pareto Set Reduction When Each of Two Criteria Is More Important Than the Third One 82
- 4.3 Pareto Set Reduction Based on a Finite Number of Some Information Quanta 86
 - 4.3.1 Usage of Information Quanta of Point-Set Type 86
 - 4.3.2 Usage of Information Quanta of Set-Point Type 91
- 5 Pareto Set Reduction Based on Collections of Information Quanta 97**
 - 5.1 Closed Collections of Information Quanta 97
 - 5.1.1 Closed Collection of Two Information Quanta and Its Consistency 97
 - 5.1.2 Reduction of the Pareto Set Using Closed Collection 100
 - 5.2 Cyclic Collections of Information Quanta 110
 - 5.2.1 Definition and Consistency of Cyclic Collection of Information Quanta 110
 - 5.2.2 Pareto Set Reduction Based on Cyclic Collections of Information Quanta 113
 - 5.3 Geometrical Algorithm of New Vector Criterion Design 120
 - 5.3.1 Preliminary Analysis 120
 - 5.3.2 Geometrical Algorithm and Its Justification 122
 - 5.3.3 Example 125
 - 5.4 Algebraic Algorithm of Vector Criterion Recalculation 126
 - 5.4.1 Statement of the Problem 126
 - 5.4.2 Construction of Dual Cone Generators 128
 - 5.4.3 Algorithm of Information Quanta Consideration 133
 - 5.5 Reduction of Finite Pareto Set 134
 - 5.5.1 Main Idea 134
 - 5.5.2 Majorant Relation 135
 - 5.5.3 Example 136
 - 5.5.4 Upper Estimation Algorithm 137
- 6 Completeness Property of Information Quanta 139**
 - 6.1 Preliminary Analysis 139
 - 6.1.1 Problem Statement 139
 - 6.1.2 Geometrical Aspects 140
 - 6.1.3 Distance Between Cones 141
 - 6.2 First Completeness Theorem 143
 - 6.2.1 Statement of Mathematical Problem 143
 - 6.2.2 First Completeness Theorem 144

6.3	Second Completeness Theorem	146
6.3.1	Example	146
6.3.2	Second Completeness Theorem.	147
6.3.3	Case of Finite Set of Feasible Vectors	149
7	Pareto Set Reduction Using Fuzzy Information	151
7.1	Statement of Fuzzy Multicriteria Choice Problem.	151
7.1.1	Basic Notions from Theory of Fuzzy Sets	151
7.1.2	Fuzzy Multicriteria Choice Problem	152
7.1.3	Axioms of Fuzzy Reasonable Choice.	154
7.1.4	Fuzzy Pareto Principle	155
7.2	Fuzzy Information About Preference Relation and Its Consistency	157
7.2.1	Definition and Some Properties of Information Quantum on Fuzzy Preference Relation	157
7.2.2	Consistent Collection of Fuzzy Information Quanta	159
7.3	Pareto Set Reduction Based on Fuzzy Information Quantum	162
7.3.1	Basic Result	162
7.3.2	Example	164
7.3.3	Case of a Fuzzy Set of Feasible Alternatives	165
7.4	Pareto Set Reduction Based on Collections of Fuzzy Information Quanta	166
7.4.1	Pareto Set Reduction Using Two Fuzzy Information Quanta	166
7.4.2	Example	168
7.4.3	Fuzzy Cyclic Information Quanta and Their Consistency.	170
7.4.4	Pareto Set Reduction Via Elementary Fuzzy Cyclic Information Quantum	172
8	Decision-Making Based on Information Quanta: Methodology and Practice	179
8.1	How Do Humans Make Their Decisions?	179
8.1.1	Mental Components of Decision-Making Process	179
8.1.2	Decision-Making Strategy of Humans in Multicriteria Environment	181
8.2	Methodology of Axiomatic Approach Application for Pareto Set Reduction	183
8.2.1	Mathematical Modeling.	183
8.2.2	Elicitation of Information About DM's Preference Relation.	186
8.2.3	Sequential Reduction of Pareto Set.	187

- 8.3 Linear Scalarization Approach 190
 - 8.3.1 Linear Combination of Criteria 190
 - 8.3.2 Linear Scalarization as a Choice Tool for Specific Pareto
Optimal Vector 192
 - 8.3.3 Normalization of Criteria and Assignment of
Scalarization Coefficients 194
 - 8.3.4 Using Linear Scalarization at the Final Stage of Pareto
Set Reduction 196
 - 8.3.5 Combined Approach with Multiplicative Combination
of Criteria 199
- 8.4 Combined Methods for Pareto Set Reduction 200
 - 8.4.1 Using Uniform Metric 200
 - 8.4.2 Using Euclidean Metric 204
- 8.5 Customs Duty Optimization 206
 - 8.5.1 Problem Statement 206
 - 8.5.2 Solution of Posed Problems 208
 - 8.5.3 Pareto Set Reduction 212
- 8.6 Production Output Increase Problem with Resource Costs 215
 - 8.6.1 Problem Statement 215
 - 8.6.2 Pareto Set Reduction 215
- 8.7 Weakening of Basic Axiomatics 219
 - 8.7.1 Weakening of Compatibility Axiom 219
 - 8.7.2 Weakening of Invariance Axiom 221
- Concluding Remarks 223**
- References 227**
- Index 231**



<http://www.springer.com/978-3-319-67872-6>

Reduction of the Pareto Set

An Axiomatic Approach

Noghin, V.D.

2018, XIX, 232 p. 24 illus., Hardcover

ISBN: 978-3-319-67872-6