Almost any kind of human activity is associated with the following situations: There exist several alternatives, and a human being is free to choose any most suitable for him.

Best choice problems form the subject of decision theory. Using the latter, it is possible to perform choice in a more reasonable way, taking advantage of available information about preferences. This theory allows eliminating wittingly unsuitable alternatives, with thorough consideration of possible negative consequences caused by half-baked choice.

An extremely wide and practically important class of choice problems concerns multicriteria choice problems, where the quality of an accepted alternative is assessed by several criteria simultaneously. Given a set of criteria, a common approach here employs the Edgeworth–Pareto principle stating that the “best” alternatives are the Pareto optimal ones only. However, the Pareto set is often large, and the final choice within it seems difficult. This aspect leads to the so-called Pareto set reduction problem. The solution to this problem, i.e., the well-justified reduction of the Pareto set, is impossible without information about the decision-maker’s preferences. A major source of such information consists in the decision-maker’s preference relation. In the elementary case, this information indicates which of two Pareto optimal alternatives is preferable for the decision-maker (DM). Such kind of information (known as an information quantum) serves for eliminating one of the two alternatives, thereby slightly facilitating further choice of the “final” alternative. By adopting some rather natural constraints (axioms) that regulate the DM’s choice procedure, one can reduce much more Pareto optimal alternatives using a single quantum. At the same time, with a collection of information quanta available, one can expect to obtain a relatively narrow set appreciably simplifying the final choice.

Adhering to a rigorous form of exposition, the author still endeavored not to lose the connection between theory and practice, involving all available means for the informal discussion and visualization of all new notions and results.

This book is intended for the specialists in the field of decision-making, requiring a standard university course on mathematics from a reader. No doubt, this
book will be useful for those who solve multicriteria problems by occupation, namely researchers, design engineers, product engineers, developers, analysts. In addition, this book can be of certain value for the undergraduates and postgraduates specialized in mathematics, economics, and engineering.

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The formulas, figures, and assertions have the double numbering system, where the first number corresponds to the chapter.

The symbols □ and ■ indicate the beginning and end of a proof, respectively.

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