The Inversion Test of the Investment Funds Efficiency Measures

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Abstract. The purpose of this article is to present the use of the inverse test in investment funds based on historical data. Kendall's coefficient is the known factor used to test rank correlations. As a measure of dependency is used at any sample size. Its distribution (except asymptotic distribution) is rarely used because of the rather difficult analytical form of the statistics used to test the hypotheses. This work will use the inversion test, which is a variant of the test based on correlation Kendall rank. In the case of a moderate sample, it is more convenient to consider the amount of inversion. It is equal to the number of incompatible pairs (in the sense described below) for variables with a continuous distribution (binding pairs are not possible). It turns out that the language of inversion is often more comfortable. This is particularly noticeable in case of second type error analysis. In the paper are presented the results of the test of the Sharpe and Treynor measures ability for investment rate of return prediction of Polish investment funds.

Keywords: Investment funds · Inversion test · Efficiency · Predictability

1 Introduction

Investing in the capital market allows individual and institutional investors to make money without contributing the work by investing their previously earned cash surplus. However, the risk is an inherent part of investing. According to the definition of investment, today are incurred expenditures, for future benefits. The future in a changing environment creates uncertainty for future benefits, and uncertainty creates risk. Investor makes an investment decision and expects that future cash flows generated by the investment will earn money. All investors like the idea of achieving high returns on the investment, most tend to dislike the high risks that are associated with anticipated high returns. The investor in the decision-making process must constantly make choices (trade off) between the rate of return and risk. Understanding the trade-off that have to be made between investment risk and expected rate of return is a base to investment decision making. Uncertainty and risk which are associated with capital investment require a special instrument supporting process of the investor's decision making.

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Risk understood as uncertainty, possibility that expected benefits will not be achieved, that benefits deviate from expected benefits follows the decision making process. Managers recognize that the expected return from risky activity tends to be higher than the expected return from less risky activities.

Investment funds allow reduce the risk of investment in the financial market by risk diversification building the portfolio of financial instruments. The individual investor must devote a lot of effort, and when he has small amount of capital simply is not able to effectively diversify his portfolio. The investment fund deposit gathering many participants may choose securities such a way that potential large drops, or even bankruptcy of one of the issuers, it was compensated by increases in the prices of shares in other companies, thus reducing the investment risks and giving it greater stability.

Evaluation of the efficiency of investment is always carried out in relation to the accepted reference point (benchmark). As a criterion for assessing the effectiveness of the funds shall be the rate of return, which is determined on the basis of changes in the value of shares, the level of risk incurred and the additional profits that it compensate. The most commonly used indicators to evaluate investment funds are indicators: Sharpe (Sp), Treynor (Tr) and Jensen (Je) [10]. These measures are risk-adjusted capital, as their design takes into account both the rate of return reached by the investment fund as well as the accompanying investment risk. These indicators are calculated based on the results of the estimation model (CML) Capital Market Line and the Capital Asset Pricing Model (CAPM).

Sharpe ratio is the ratio of the average additional rate of return, which is the surplus profit that comes from the fund over a risk-free rate to the standard deviation of the additional rate of return which is a derivative of total risk. A positive index value indicates the profit worked out by the fund is higher than the benchmark, which lets you choose a fund with the highest rate of return with minimal risk. If the index value is negative, it means that the profit of the fund is lower than the market risk-free rate for which is usually assumed profitability of T-bills.

$$Sp = \frac{R_p - R_f}{\sigma_p}$$

where:

R_p - the average rate of return of the investment fund at time t
R_f - the average rate of return on risk-free instruments at time t
σ_p - standard deviation of the returns of the investment fund at time t

The counter of this expression is the so-called risk premium, a kind of reward for the investor, that is additional income above the risk-free rate. The higher the Sharpe ratio, the higher is the efficiency of the tested fund.

Construction of Treynor's Ratio is similar to Sharpe ratio, Treynor ratio, however, takes into account two types of risks. One result of the general situation on the whole market and is called systematic risk (coefficient β), and the second, the specific risk is characteristic of the assets in the portfolio. Through appropriate diversification of assets portfolio, reducing the risk unsystematic it manages to reduce the total risk to the level of systematic risk.

$$Tp = \frac{R_p - R_f}{\beta_p}$$

where:

Rp - the average rate of return of the investment fund at time t
Rf - the average rate of return on risk-free instruments at time t
βp - systematic risk (coefficient β) of investment fund at time t

This indicator reflects the sensitivity to changes in the value of the instrument to changes in benchmark.

2 The Inversion Test

The coefficient τ-Kendall (Magiera R. 2002) is used to describe the correlation between order variables. In order to calculate τ-Kendall, the observations in the sample should be compiled into all possible pairs and classified into three categories.

Compatible pairs – either variable or in the first observation both are larger than the second or both smaller, the number of such pairs will be marked as P_z.

Incompatible pairs – the variables change in the opposite direction, one of them is greater for the observation in pair for which the second one is smaller, the number of such pairs is marked as P_n.

Bonded pair – in both observations one variable has the same value, the number of such pairs – P_w.

Estimator of τ-Kendalla can be calculated from the formula

$$\tau = \frac{P_z - P_n}{P_z + P_n + P_w}$$

This coefficient is contained in the interval (−1, 1).

Because

$$P_z + P_n + P_w = \binom{n}{2} = \frac{n(n-1)}{2}$$

then

$$\tau = 2\frac{P_z - P_n}{n(n-1)}$$

where:
n – sample size.
P_z – number of compatible pairs.
P_n – number of incompatible pairs.

A permutation tool is a convenient tool for analyzing variables in the order scale.

Permutation is a function that transforms the set of natural numbers {1, 2,…, n} into oneself. Observations of any real random variable can be ordered according to the natural order if there are no equal ones. This happens if the assumed random variable is assumed to be continuous.

Let

$$N_n = \frac{n(n-1)}{2}$$

will be the maximum number of inversions in permutation with n arguments

Let $\left\{ \begin{matrix} N_n \\ k \end{matrix} \right\}$ will be the number of permutations having exactly k inversion.

If $N_1 = 1$, then from definition $\left\{ \begin{matrix} N_1 \\ 0 \end{matrix} \right\} = 0$

For $N_2 = 2$, is $\left\{ \begin{matrix} N_2 \\ 0 \end{matrix} \right\} = 1$ i $\left\{ \begin{matrix} N_2 \\ 1 \end{matrix} \right\} = 1.$

For $N_3 = 3$ is $\left\{ \begin{matrix} N_3 \\ 0 \end{matrix} \right\} = \left\{ \begin{matrix} 3 \\ 0 \end{matrix} \right\} = 1, \left\{ \begin{matrix} 3 \\ 1 \end{matrix} \right\} = 2, \left\{ \begin{matrix} 3 \\ 2 \end{matrix} \right\} = 2, \left\{ \begin{matrix} 3 \\ 3 \end{matrix} \right\} = 1$

Similary for $N_4 = 6$:

$$\left\{ \begin{matrix} N_4 \\ 0 \end{matrix} \right\} = 1, \left\{ \begin{matrix} N_4 \\ 1 \end{matrix} \right\} = 3, \left\{ \begin{matrix} N_4 \\ 2 \end{matrix} \right\} = 5, \left\{ \begin{matrix} N_4 \\ 3 \end{matrix} \right\} = 6,$$

$$\left\{ \begin{matrix} N_4 \\ 4 \end{matrix} \right\} = 5, \left\{ \begin{matrix} N_4 \\ 5 \end{matrix} \right\} = 3, \left\{ \begin{matrix} N_4 \\ 6 \end{matrix} \right\} = 1$$

In the general case:

$$\left\{ \begin{matrix} N_n \\ k \end{matrix} \right\} = \sum\nolimits_{i=\max(0,k-n+1)}^{k} \left\{ \begin{matrix} N_{n-1} \\ i \end{matrix} \right\} \quad (1)$$

3 Inversion Test for Investment Funds

The study was conducted for 36 investment funds with legal form of mutual funds or expertly open, operating on the Polish market. These are different types of funds: Money Funds, Debt Funds, Mixed Funds and Stock Funds, Table 1. The study period covers the years 2012–2016.

The following theorem [1] will be used to test hypotheses about inversions.

Theorem

Let p be a probability of inversion and

$$P(I_n = k) = \left(\left\{ \begin{matrix} N_n \\ k \end{matrix} \right\} p^k q^{N_n - k}\right) / \sum\nolimits_{k=0}^{N_n} \left\{ \begin{matrix} N_n \\ k \end{matrix} \right\} p^k q^{N_n - k}$$
$$= \left(p_{n,k} \cdot p^k q^{N_n - k}\right) / \sum\nolimits_{k=0}^{N_n} p_{n,k} \cdot p^k q^{N_n - k} \quad (2)$$

Table 1. Tested investment funds

Type	Investment fund (TFI)
Money funds	INVESTOR Gotówkowy SFIO (Investors TFI)
	KBC Pieniężny (KBC TFI)
	MetLife Pieniężny (MetLife TFI)
	NN Lokacyjny Plus FIO (NN Investment Partners TFI)
	UniKorona Pieniężny FIO (Union Investment TFI)
Debt funds	Aviva Investors Obligacji Dynamiczny FIO (Aviva Investors Poland TFI)
	ALLIANZ Obligacji Plus FIO (Allianz Polska S.A. TFI)
	KBC Papierów Dłużnych FIO (KBC TFI)
	NN Obligacji FIO (NN Investment Partners TFI)
	PZU Ochrony Majątku FIO (PZU S.A. TFI)
	PZU Papierów Dłużnych POLONEZ FIO (PZU S.A. TFI)
	Skarbiec Depozytowy DPW FIO (Skarbiec TFI)
Mixed funds	ALLIANZ Aktywnej Alokacji FIO (Allianz Polska S.A.)
	Investor Zabezpieczenia Emerytalnego FIO (Investors TFI)
	Investor Zrównoważony FIO (Investors TFI)
	KBC Stabilny FIO (KBC TFI)
	MetLife Ochrony Wzrostu SFIO (MetLife TFI)
	MILLENNIUM Cyklu Koniunkturalnego FIO (Millennium TFI)
	NN Zrównoważony FIO (NN Investment Partners TFI)
	Noble Fund Mieszany FIO (Noble Funds TFI)
	Noble Fund Timingowy FIO (Noble Funds TFI)
	PKO Stabilnego Wzrostu FIO (PKO TFI)
	UniKorona Zrównoważony FIO (Union Investment TFI)
Stock funds	AVIVA Nowoczesnych Technologii FIO (Aviva Investors Poland TFI)
	BPH Akcji FIO (BPH TFI)
	KBC Akcji Małych i Średnich Spółek FIO (KBC TFI)
	KBC Akcyjny FIO {KBC TFI)
	Millennium Dynamicznych Spółek FIO (Millennium TFI)
	NN Akcji FIO (NN Investment Partners TFI)
	NN Średnich i Małych Spółek FIO (NN Investment Partners TFI)
	Noble Fund Akcji FIO (Noble Funds TFI)
	NOBLE FUND Akcji Małych i Średnich Spółek FIO (Noble Funds TFI)
	Novo Akcji FIO (Opera TFI)
	PKO Akcji Małych i Średnich Spółek FIO (PKO TFI)
	PZU Akcji Małych i Średnich Spółek FIO (PZU S.A. TFI)

Source: [11]

For selected funds were calculated: the expected rate of return, standard deviation, coefficient of variation, coefficient β and the efficiency measures of Sharpe and Treynor. In Table 2 there are shown expected value of rate of return, standard deviation, β coefficient, Sharpe and Traynor's coefficients for selected investment funds.

Table 2. Statistic for selected funds

Fund type	Selected fund	Date	2012	2013	2014	2015	2016
Money	MetLife Pieniężny	R*	0.093	0.023	0.038	0.018	0.012
		σ	0.048	0.057	0.015	0.010	0.018
		β	0.009	0.042	0.015	0.001	0.002
		Sharpe	1.936	0.395	2.454	1.685	0.508
		Treynor	10.490	0.539	2.545	11.622	3.923
Debt	Aviva Investors Obligacji Dynamiczny	R*	0.151	0.053	0.101	0.022	0.025
		σ	0.098	0.221	0.103	0.115	0.087
		β	0.080	0.105	0.127	0.045	0.002
		Sharpe	1.531	0.236	0.974	0.190	0.261
		Treynor	1.889	0.495	0.794	0.482	9.302
Mixed	MetLife Ochrony Wzrostu	R*	0.144	−0.070	−0.044	−0.076	−0.027
		σ	0.224	0.287	0.193	0.140	0.076
		β	0.325	0.462	0.467	0.318	0.113
		Sharpe	0.641	−0.246	−0.231	−0.542	−0.393
		Treynor	0.441	−0.153	−0.095	−0.239	−0.264
Stock	AVIVA Nowoczesnych Technologii	R*	0.112	0.247	−0.026	0.029	0.103
		σ	0.442	0.415	0.222	0.292	0.339
		β	0.569	0.523	0.416	0.272	0.285
		Sharpe	0.252	0.595	−0.120	0.100	0.297
		Treynor	0.196	0.471	−0.064	0.107	0.353

Source: own work

The basis for the fund's ranking is the Sharpe and Traynor's measure [10]. This is a commonly used methods for evaluating the quality of investment for investment funds, Table 3.

Calculating the number of inversions requires several comparisons of rankings from two consecutive years. In the penultimate line of the No. 4 table, the number of inversions was calculated, and in the last line the probability of inversion was estimated by frequency (Table 4).

The size of sample is 36. Value of p is the frequency of inversion. Maximal value of inversions equals $630 = (36 \cdot 35)/2$. NI- is the number of inversions. Thus $p = NI/630$.

In Table 5 chosen values of distribution function are presented. They are calculated using formulas (1) and (2). In Table 5, the values used for testing are bolded.

Table 3. Ranking by Sharpe and Treynor's measure

Ranking by Sharpe's measure						Ranking by Treynor's measure					
Rank	2012	2013	2014	2015	2016	Rank	2012	2013	2014	2015	2016
1	31	10	1	1	16	1	1	31	1	30	4
2	34	31	16	16	5	2	35	10	35	1	1
3	1	25	35	31	35	3	16	16	16	18	5
4	10	28	10	35	9	4	31	35	10	31	16
5	13	22	31	10	1	5	10	25	31	16	35
6	4	35	4	26	8	6	34	22	4	10	9
7	18	20	34	25	15	7	30	28	30	4	8
8	30	32	30	22	14	8	18	1	18	35	15
9	16	3	18	18	32	9	4	4	15	34	34
10	5	14	15	23	3	10	13	20	34	25	10
11	17	9	5	15	10	11	5	3	13	23	14
12	26	8	13	30	22	12	15	32	5	22	22
13	27	23	27	8	36	13	17	14	27	26	13
14	9	16	9	28	28	14	26	15	9	8	3
15	23	15	12	9	4	15	23	9	12	9	25
16	2	1	17	4	12	16	27	8	17	15	32
17	12	4	8	32	34	17	9	23	8	28	20
18	15	27	36	34	25	18	2	34	36	3	28
19	36	19	33	14	26	19	20	5	33	32	26
20	35	24	11	20	27	20	12	27	11	14	36
21	11	17	14	3	11	21	36	13	14	20	12
22	24	26	19	13	29	22	25	24	19	13	17
23	8	29	21	17	20	23	24	19	21	17	27
24	20	7	28	5	19	24	32	17	7	24	29
25	19	11	7	24	17	25	8	26	28	5	19
26	21	12	20	27	13	26	11	29	20	27	11
27	32	36	3	12	24	27	28	7	3	11	24
28	25	34	6	19	7	28	22	11	6	12	7
29	29	5	2	11	21	29	19	12	2	19	18
30	22	13	24	7	18	30	21	36	24	7	21
31	28	21	22	29	33	31	29	30	29	6	33
32	7	30	23	6	6	32	7	21	23	29	6
33	3	18	26	21	31	33	3	18	25	21	23
34	33	33	29	36	23	34	14	33	26	36	31
35	14	6	25	33	30	35	33	6	22	33	2
36	6	2	32	2	2	36	6	2	32	2	30

Source: own work

Table 4. Comparisons of ranking

Rank 2012	2013	Rank 2013	2014	Rank 2014	2015	Rank 2015	2016
1	2	1	4	1	1	1	5
2	28	2	5	2	2	2	1
3	16	3	35	3	4	3	33
4	1	4	24	4	5	4	3
5	30	5	31	5	3	5	11
6	17	6	3	6	16	6	19
7	33	7	26	7	18	7	18
8	32	8	36	8	12	8	12
9	14	9	27	9	9	9	30
10	29	10	21	10	11	10	34
11	21	11	14	11	24	11	7
12	22	12	17	12	22	12	35
13	18	13	32	13	26	13	6
14	11	14	2	14	15	14	14
15	13	15	10	15	27	15	4
16	36	16	1	16	23	16	15
17	26	17	6	17	13	17	9
18	15	18	13	18	34	18	17
19	27	19	22	19	35	19	8
20	6	20	30	20	29	20	23
21	25	21	16	21	19	21	10
22	20	22	33	22	28	22	26
23	12	23	34	23	33	23	25
24	7	24	25	24	14	24	2
25	19	25	20	25	30	25	27
26	31	26	15	26	20	26	20
27	8	27	18	27	21	27	16
28	3	28	7	28	32	28	24
29	23	29	11	29	36	29	21
30	5	30	12	30	25	30	28
31	4	31	23	31	8	31	22
32	24	32	8	32	10	32	32
33	9	33	9	33	6	33	29
34	34	34	19	34	31	34	13
35	10	35	28	35	7	35	31
36	35	36	29	36	17	36	36
NI	339		320		222		213
p	0.538		0.508		0.352		0.338

Source: own work

Table 5. Distribution of inversions. Chosen values.

NI (p = 0.05)	213	214	215	216	217
Distr function	**0.0026**	0.0028	0.0031	0.0033	0.0036
NI (p = 0.05)	218	219	220	221	222
Distr function	0.0040	0.0043	0.0047	0.0051	**0.0055**
NI (p = 0.05)	251	252	253	254	255
Distr function	0.0418	0.0444	0.0470	**0.0498**	**0.0527**
NI (p = 0.05)	337	338	339	340	341
Distr function	0.7287	0.7376	**0.7464**	0.7550	0.7634
NI (p = 0.05)	372	373	374	375	376
Distr function	0.9411	0.9443	**0.9473**	**0.9502**	0.9530

Source: own work

We wish to test hypotheses for years 2012 and 2013 firstly. We begin by identifying the null (*PI* means probability of inversion, *NI*- number of inversions) and alternative hypotheses.

$$H_0 : PI = 0.5$$

versus

$$H_1 : PI > 0.5$$

Our Test is Right-Tailed

Assuming significance level 0.05 and using tables of distribution for $PI = 0.5$ (under null hypothesis) we find the critical *NI* value of 374 from Table 5. $P(NI \geq 375) \leq 0.05$ but $P(NI \geq 374) \geq 0.05$. So we fail to reject H_0. Using p-value approach we obtain for $NI = 339$, p-value of 0.2536.

At the usual levels of significance hypothesis H_0 should not be rejected.

For years 2013 and 2014 we consider right tailed hypothesis too. Because 320 < 339 we fail to reject H_0. Therefore for these years the ranking by Sharpe measure seems to irrelevant.

To have significance of a ranking we expect $PI < 0.5$. In this case, the probability of inversion is lower than in a random situation. In addition, in such a situation, the ranking has a predictive value.

We will consider such case in following example related years 2014 and 2015 firstly. Let us consider the following hypotheses:

$$H_0 : PI = 0.5$$

versus

$$H_1 : PI < 0.5$$

Our Test is Left-Tailed

Let us assume the significance level as usual 0.05. In considered case $NI = 222$, (see Table 5) but the critical value (according Table 5) equals (left-tailed test) 254 because $P(NI \leq 255) \geq 0.05$ but $P(NI \leq 254) \leq 0.05$. Therefore H_0 should be rejected.

Using p-value approach we obtain for $NI = 222$, p-value of 0.005506. (see Table 5). At the usual levels of significance hypothesis H_0 should be rejected. It seems that in this case, the 2014 ranking is predictive for 2015.

The last case concerns the years 2015 and 2016. We will consider the following hypotheses:

$$H_0 : PI = 0.5$$

versus

$$H_1 : PI < 0.5$$

As before for significance level 0.05 the critical value is 254. The test statistic NI calculated from the sample is 213. Therefore H_0 should be rejected. Using p-value approach we obtain for $NI = 213$, p-value of 0.0026. At the usual levels of significance hypothesis H_0 should be rejected.

For the second time the classification based on the Sharpe measure seems to be non-random.

An analogous classification can be made using the Treynor measure, Table 6. In this case, the results of the sample suggest that the probability of inversion is less than 0.5 in the given years. Samples values range from about 0.29 to about 0.42. In all cases considered, both hypotheses will have the same form

$$H_0 : PI = 0.5$$

versus

$$H_1 : PI < 0.5$$

This time we will use the p-value approach firstly.

In the penultimate line of 6 table, the number of inversions was calculated, and in the last line the probability of inversion was estimated by frequency. The distribution of inversion is shown in Table 7.

The calculation results are in Table 8 for chosen number of inversions. The decisions are presented in Table 8 (assuming significance level 0.05).

None of the suggested methods did not bind into the power test problem. This will be the last part of the work.

Power of the Test

Using the cited theorem, the distribution of the number of inversions can be found. On Fig. 1. distribution functions for chosen values of p.

Table 6. Comparisons of ranking by Treynor's measure.

Rank 2012	2013	Rank 2013	2014	Rank 2014	2015	Rank 2015	2016
1	8	1	5	1	2	1	2
2	4	2	4	2	8	2	29
3	3	3	3	3	5	3	34
4	1	4	2	4	6	4	4
5	2	5	33	5	4	5	10
6	18	6	35	6	7	6	1
7	31	7	25	7	1	7	5
8	33	8	1	8	3	8	9
9	9	9	6	9	16	9	15
10	21	10	26	10	9	10	33
11	19	11	27	11	22	11	12
12	14	12	36	12	25	12	19
13	24	13	21	13	26	13	7
14	25	14	9	14	15	14	6
15	17	15	14	15	28	15	8
16	20	16	17	16	23	16	18
17	15	17	32	17	14	17	14
18	36	18	10	18	34	18	16
19	10	19	12	19	35	19	11
20	29	20	13	20	27	20	17
21	30	21	11	21	20	21	13
22	5	22	30	22	29	22	22
23	22	23	22	23	33	23	27
24	12	24	16	24	30	24	3
25	16	25	34	25	17	25	23
26	28	26	31	26	21	26	26
27	7	27	24	27	18	27	21
28	6	28	20	28	31	28	25
29	23	29	15	29	36	29	28
30	32	30	18	30	24	30	32
31	26	31	7	31	32	31	24
32	27	32	23	32	11	32	30
33	11	33	8	33	10	33	20
34	13	34	19	34	13	34	31
35	34	35	28	35	12	35	35
36	35	36	29	36	19	36	19
NI	234		266		211		183
p	0.371		0.422		0.334		0.290
	0.429		0.222		0.921		0.476

Source: own work

Table 7. Distribution of inversions. Selected values.

NI (p = 0.05)	181	182	**183**	184	185
Distr function	0.00009	0.00010	**0.00012**	0.00013	0.00015
NI (p = 0.05)	209	210	**211**	212	213
Distr function	0.0018	0.0020	**0.0021**	0.0023	0.0026
NI (p = 0.05)	232	233	**234**	235	236
Distr function	0.0119	0.0128	**0.0138**	0.0148	0.0159
NI (p = 0.05)	264	265	**266**	267	268
Distr function	0.0768	0.0809	**0.0850**	0.0894	0.0939

Table 8. Results for chosen number of inversions.

Years	p-value	Decision
2012/13	0.01378	REJECT
2013/14	0.08505	FAIL TO REJECT
2014/15	0.00214	REJECT
2015/16	0.00012	REJECT

Source: own work

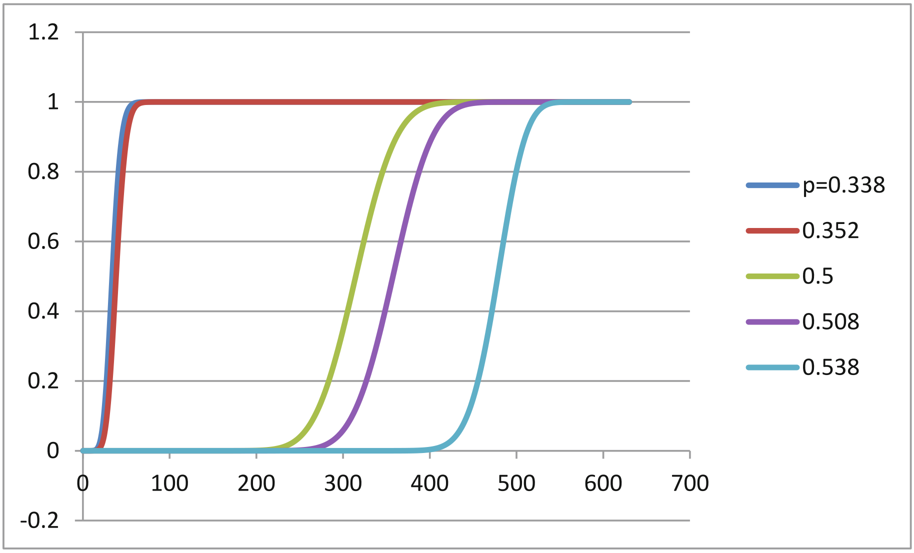

Fig. 1. Distribution function of the number of inversions (chosen values of p) Source: own work

In the case of Sharpe measure, simple hypotheses were considered:

$$H_0 : PI = 0.5$$

(a) $H_1 : PI = 0.508$
(b) $H_1 : PI = 0.538$

(c) $H_1 : PI = 0.352$
(d) $H_1 : PI = 0.338$

Four pairs of simple hypotheses will be considered, $\alpha = 0.05$. Let β be type II error.

$$\beta = P(acceptH_0|p = 0.508) = P(PI \leq 374/p = 0.508) = 0.677$$
$$\beta = P(acceptH_0|p = 0.538) = P(PI \leq 374/p = 0.538) = 0.0002$$
$$\beta = P(acceptH_0|p = 0.352) = P(PI \geq 255/p = 0.352) \approx 0$$
$$\beta = P(acceptH_0|p = 0.338) = P(PI \geq 255/p = 0.338) \approx 0$$

Probability of type II error is very small except the case of p = 0.508. This result is not surprising since the value of 0.508 is very close to the value 0.5. In other cases, the β value is close to zero which indicates a high power of the test.

In the case of Treynor's measure, simple hypotheses were considered.

In all the cases we have left-tailed tests.

For $\alpha = 0.05$ critical value is 255.

$$\beta = P(PI \geq 255|p = 0.29) \approx 0$$
$$\beta = P(PI \geq 255|p = 0.33) \approx 0$$
$$\beta = P(PI \geq 255|p = 0.37) \approx 0$$
$$\beta = P(PI \geq 255|p = 0.42) \approx 0$$

4 Conclusions

The Treynor measure seems to be more useful. In the examples presented above, it was more often distinguishable from randomness, although the studies concerned the same sample using different indicators. In the case of the Sharpe measure, it even occurred that the reverse predicted ranking seemed more likely (estimate probability of inversion greater than 0.5). The problem requires further investigation, but the analysis attempted to favor the measure of Treynor. In both cases the test showed great power. For the inversion probability values analyzed, the test showed practically zero probability of type II error.

References

1. Czekała, M., Bukietyńska, A.: Distribution of inversions and the power of the τ- Kendall's test. In: Information Systems Architecture and Technology: Proceedings of 37th International Conference on Information Systems Architecture and Technology – ISAT 2016 – Part III, Springer (2017)
2. David, F.N., Kendall, M.G., Barton, D.E.: Symmetric Function and Allied Tables, Cambridge, p. 241 (1966)

3. Feller, W.: An Introduction to Probability Theory and its Application. Wiley, New York (1961)
4. Ferguson, S., Genest, Ch., Hallin, M.: Kendall's tau for autocorrelation. Can. J. Stat. **28**, 587–604 (2000)
5. Ferguson, S., Genest, Ch., Hallin, M.: Kendall's tau for autocorrelation. Department of Statistics Papers, UCLA (2011)
6. Hallin, M., Metard, G.: Rank-based test for randomness against first order dependence. J. Am. Stat. Assoc. **83**, 1117–1128 (1988)
7. Janjic, M.: A generating function for numbers of insets. J. Integer Seq. **17**, #14.9.7 (2014)
8. Kendall, M.G., Buckland, W.R.: A Dictionary of Statistical Terms. Oliver and Boyd, Edinburgh (1960)
9. Netto, E.: Lehrbuch der Combinatorik, 2nd edn. Teubner, Leipzig (1927). p. 96
10. Reilly, F.K., Brown, K.C.: Analiza inwestycji i zarządzanie portfelem. PWE, Warszawa (2001)
11. Wilimowska, Z., Kwias, M., Wilimowski, M.: Efficiency of investment funds investing in the polish market. In: International Conference on Business Risk in Changing Dynamics of Global Village, 26–27 April 2017, Nysa (2017)

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