Preface

This is the second volume of a two-volume textbook on relation algebras. The first volume, *Introduction to Relation Algebras*, begins with the underlying motivation, going back to the calculus of relations of De Morgan, Peirce, and Schröder, and with the basic definitions, axioms, and examples of the subject. There follows a development of the arithmetic of relation algebras in which the most important laws are derived systematically from the axioms, with special emphasis on those laws that apply to, or even characterize, specific types of elements, such as as equivalence elements, functional elements, and ideal elements. The remainder of the first volume is devoted to an exposition of the algebraic side of the subject: subalgebras, homomorphisms, ideals, quotient algebras, simple algebras, direct products, and so on.

The purpose of this second volume is to make a systematic, cohesive, and detailed presentation of a selection of more advanced topics of the subject, topics that have been active areas of research over the last few decades, more accessible to readers, with the hope of bringing them to some of the frontiers of research on relation algebras and Boolean algebras with operators.

**Intended audience**

This volume is aimed at, but is not limited to, graduate students and professionals in a variety of mathematical disciplines, especially various branches of logic, universal algebra, and theoretical computer science. As regards the background needed to read this volume, it is helpful to have a general understanding of the basic notions and results of the theory of relation algebras, and some familiarity with the basic notions and results of universal algebra. The background provided in the
first volume, *Introduction to Relation Algebras*, is more than sufficient. Note that this second volume contains numerous, essential references to the first. The reader is strongly encouraged to secure at least electronic access to the first book in order to make use of the second. Any reference in this volume to material in Chapters 1–13 refers to the relevant result in the first volume.

Each chapter ends with a historical section and a substantial number of exercises. The exercises vary in difficulty from routine problems that help readers understand the basic definitions and theorems presented in the text, to intermediate problems that extend or enrich the material developed in the text, to difficult problems that often present important results not covered in the text. Hints and solutions to some of the exercises are available for download from the Springer book webpage. The main topics covered in this volume are canonical extensions, completions, representation theorems, varieties and universal classes, and atom structures.

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Loretta Bartolini an editor of the mathematical series *Graduate Texts in Mathematics*, *Undergraduate Texts in Mathematics*, and *Universitext* published by Springer, has served as the editor for these two volumes. She has given me a great deal of advice and guidance during the publication process, and I am very much indebted to her and her entire production team at Springer for pulling out all stops, and doing the best possible job in the fastest possible way, to produce these two companion volumes. Any errors or flaws that remain in the volumes are, of course, my own responsibility.

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