

# Contents

<b>Preface</b> .....	vii
<b>Introduction</b> .....	xi
<b>1 The calculus of relations</b> .....	1
1.1 Binary relations .....	1
1.2 Operations on relations .....	5
1.3 Relational laws .....	10
1.4 Relational properties .....	12
1.5 Boolean matrices .....	16
1.6 Relations and Boolean matrices .....	22
1.7 Historical remarks .....	27
Exercises .....	28
<b>2 Relation algebras</b> .....	35
2.1 Fundamental notions and axioms .....	36
2.2 Boolean algebras with operators .....	41
2.3 Verifying axioms .....	51
2.4 Language of relation algebras .....	57
2.5 Language of relations .....	64
2.6 Historical remarks .....	65
Exercises .....	67
<b>3 Examples of relation algebras</b> .....	71
3.1 Set relation algebras .....	71
3.2 Matrix algebras .....	74
3.3 Formula relation algebras .....	75
3.4 Boolean relation algebras .....	77

3.5	Group complex algebras .....	79
3.6	Geometric complex algebras .....	81
3.7	Lattice complex algebras .....	88
3.8	Small relation algebras .....	94
3.9	Independence of the axioms .....	100
3.10	Historical remarks .....	103
	Exercises .....	105
<b>4</b>	<b>Arithmetic</b> .....	<b>113</b>
4.1	Basic laws and duality principles .....	114
4.2	De Morgan-Tarski laws .....	119
4.3	Definability of converse .....	123
4.4	Consequences of the De Morgan-Tarski laws .....	124
4.5	Historical remarks .....	133
	Exercises .....	136
<b>5</b>	<b>Special elements</b> .....	<b>141</b>
5.1	Symmetric elements .....	141
5.2	Transitive elements .....	144
5.3	Equivalence elements .....	149
5.4	Right- and left-ideal elements .....	164
5.5	Ideal elements .....	177
5.6	Domains and ranges .....	185
5.7	Rectangles .....	190
5.8	Functions .....	198
5.9	Historical remarks .....	217
	Exercises .....	222
<b>6</b>	<b>Subalgebras</b> .....	<b>229</b>
6.1	Subuniverses and subalgebras .....	229
6.2	Properties preserved under subalgebras .....	230
6.3	Generators of subalgebras .....	233
6.4	Lattice of subalgebras .....	236
6.5	Regular and complete subalgebras .....	240
6.6	Atomic subalgebras .....	245
6.7	Minimal subalgebras .....	249
6.8	Elementary subalgebras .....	254
6.9	Historical remarks .....	262
	Exercises .....	264

<b>7</b>	<b>Homomorphisms</b> .....	271
7.1	Homomorphisms .....	271
7.2	Properties preserved under homomorphisms .....	273
7.3	A class of examples .....	281
7.4	Complete homomorphisms .....	282
7.5	Isomorphisms .....	285
7.6	Atomic monomorphisms .....	288
7.7	Exchange Principle .....	297
7.8	Automorphisms .....	299
7.9	Elementary embeddings .....	301
7.10	Historical remarks .....	302
	Exercises .....	304
<b>8</b>	<b>Ideals and quotients</b> .....	313
8.1	Congruences .....	313
8.2	Properties preserved by congruences .....	314
8.3	Generators of congruences .....	315
8.4	Lattice of congruences .....	316
8.5	Quotients .....	319
8.6	Ideals .....	324
8.7	Generators of ideals .....	332
8.8	Lattice of ideals .....	337
8.9	Ideal elements and ideals .....	344
8.10	Maximal ideals .....	347
8.11	Homomorphism and isomorphism theorems .....	353
8.12	Filters .....	366
8.13	Historical remarks .....	371
	Exercises .....	372
<b>9</b>	<b>Simple algebras</b> .....	385
9.1	Simple relation algebras .....	386
9.2	Integral relation algebras .....	391
9.3	Directly and subdirectly indecomposable algebras ....	396
9.4	Historical remarks .....	398
	Exercises .....	399
<b>10</b>	<b>Relativizations</b> .....	403
10.1	Relativizations to equivalence elements .....	403
10.2	Relativizations to ideal elements .....	406
10.3	Properties preserved under relativizations .....	408

10.4 Relativizations and simple algebras ..... 412

10.5 Generalized relativizations ..... 414

10.6 Historical remarks ..... 416

Exercises ..... 417

**11 Direct products** ..... 419

11.1 Binary external products ..... 419

11.2 Properties preserved under binary products ..... 420

11.3 Binary external decompositions ..... 429

11.4 Binary internal products ..... 433

11.5 Binary internal decompositions ..... 439

11.6 General external products ..... 444

11.7 Properties preserved under general products ..... 445

11.8 General external decompositions ..... 450

11.9 Internal products of systems ..... 459

11.10 General internal decompositions ..... 464

11.11 Total decompositions ..... 470

11.12 Atomless Boolean algebras of ideal elements ..... 476

11.13 Decompositions of homomorphisms ..... 480

11.14 Historical remarks ..... 486

Exercises ..... 489

**12 Subdirect products** ..... 499

12.1 Weak direct products ..... 500

12.2 Ample internal products ..... 507

12.3 Subdirect products ..... 509

12.4 Historical remarks ..... 516

Exercises ..... 517

**13 Minimal relation algebras** ..... 519

13.1 Types ..... 519

13.2 Type decomposition ..... 525

13.3 Classification of minimal relation algebras ..... 527

13.4 Classification of algebras of types one and two ..... 529

13.5 Historical remarks ..... 536

Exercises ..... 536

**References** ..... 541

**Index** ..... 551



<http://www.springer.com/978-3-319-65234-4>

Introduction to Relation Algebras

Relation Algebras, Volume 1

Givant, S.

2017, XXXII, 572 p. 25 illus., Hardcover

ISBN: 978-3-319-65234-4