Preface

The theory of relation algebras originated in the second half of the 1800s as a calculus of binary relations, in analogy with the calculus of classes (or unary relations) that was published around 1850 by George Boole in [15] and [16]. By 1900, it had developed into one of the three principal branches of mathematical logic. Yet, despite the intrinsic importance of the concepts occurring in this calculus, and their wide use and applicability throughout mathematics and science, as a mathematical discipline the subject fell into neglect after 1915.

It was revitalized and reformulated within an abstract axiomatic framework by Alfred Tarski in a seminal paper [104] from 1941, and since then the subject has grown substantially in importance and applicability. Numerous papers on relation algebras have been published since 1950, including papers in areas of computer science, and the subject has had a strong impact on such fields as universal algebra, algebraic logic, and modal logic. In particular, the study of relation algebras led directly to the development of a general theory of Boolean algebras with operators, an analogue for Boolean algebras of the well-known theory of groups with operators. This theory of Boolean algebras with operators appears to be especially well suited as an area for the application of mathematics to the theory and practice of computer science.

In my opinion, however, progress in the field and its application to other fields, and knowledge of the field among mathematicians, computer scientists, and philosophers, has been slowed substantially by the fact that, until recently, no systematic introductions to the subject existed. I believe that the appearance of such introductions will lead to a steady growth and influence of the theory and its applications, and to a much broader appreciation of the subject. It is for this reason that I have written these two volumes: to make the basic ideas and results of the subject (in this first volume), and some of the most important advanced areas of the theory (in the second volume, *Advanced Topics in Relation Algebras*), accessible to as broad an audience as possible.

Intended audience

This two-volume textbook is aimed at people interested in the contemporary axiomatic theory of binary relations. The intended audience includes, but is not limited to, graduate students and professionals in a variety of mathematical disciplines, logicians, computer scientists, and philosophers. It may well be that others in such diverse fields such as anthropology, sociology, and economics will also be interested in the subject. Kenneth Arrow, a Nobel Prize winning economist who in 1940 took a course on the calculus of relations with Tarski, said:

> It was a great course. . . . the language of relations was immediately applicable to economics. I could express my problems in those terms.

The necessary mathematical preparation for reading this work includes mathematical maturity, something like a standard undergraduate-level course in abstract algebra, an understanding of the basic laws of Boolean algebra, and some exposure to naive set theory. Modulo this background, the text is largely self-contained. The basic definitions are carefully given and the principal results are all proved in some detail.

Each chapter ends with a historical section and a substantial number of exercises. In all, there are over 900 exercises. They vary in difficulty from routine problems that help readers understand the basic definitions and theorems presented in the text, to intermediate problems that extend or enrich the material developed in the text, to difficult problems that often present important results not covered in the text. Hints and solutions to some of the exercises are available for download from the Springer book webpage.

Readers of the first volume who are mainly interested in studying various types of binary relations and the laws governing these relations might want to focus their attention on Chapters 4 and 5, which deal with the laws and special elements. Those who are more interested in the algebraic aspects of the subject might initially skip Chapters 4 and 5, referring back to them later as needed, and focus more on Chapters 1–3, which concern the fundamental notions and examples of relation algebras, and Chapters 6–13, which deal with subalgebras, homomorphisms, ideals and quotients, simple and integral relation algebras, relativizations, direct products, weak and subdirect products, and minimal relation algebras respectively.

The second volume—which consists of Chapters 14–19—deals with more advanced topics: canonical extensions, completions, representations, representation theorems, varieties and universal classes, and atom structures. Readers who are principally interested in these more advanced topics might want to skip over most of the material in Chapters 4–13, and proceed directly to the material in the second volume that is of interest to them.

Acknowledgements

I took a fascinating course from Alfred Tarski on the theory of relation algebras in 1970, and my notes for that course have served as a framework for part of the first volume. I was privileged to collaborate with him over a ten-year period, and during that period I learned a great deal more about relation algebras, about mathematics in general, and about the writing of mathematics. The monograph [113] is one of the fruits of our collaboration. Without Tarski's influence, the present two volumes would not exist.

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Savannah Smith read a draft of the first volume and called many typographic errors to my attention. Kexin Liu read the second draft of both volumes, caught numerous typographic errors, and made many

suggestions for stylistic improvements. Ian Hodkinson read through the final draft of the first volume, caught several typographic errors, and made a number of very perceptive and insightful recommendations. I am very grateful to all three of them.

Loretta Bartolini an editor of the mathematical series *Graduate Texts in Mathematics*, *Undergraduate Texts in Mathematics*, and *Universitext* published by Springer, has served as the editor for these two volumes. She has given me a great deal of advice and guidance during the publication process, and I am very much indebted to her and her entire production team at Springer for pulling out all stops, and doing the best possible job in the fastest possible way, to produce these two companion volumes. Any errors or flaws that remain in the volumes are, of course, my own responsibility.

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