Chapter 2
Introduction to the Sun

Abstract The Sun has a basic impact on human life; thus, during our existence man has viewed our star in many different ways. From the scientific point of view, the only one of concern here, the Sun is an analog of the Rosetta stone for stellar studies, thanks to its closeness to the Earth. This book is limited to the introduction of the structure and evolution of our star and neglects the solar magnetic field and related phenomena. As a consequence, we treat the Sun as a spherically symmetric star, with an atmosphere made only of the photosphere, which is constant for most of its lifetime. Even with this restriction we can address many fundamental questions which concern the global physical parameters of the Sun, the present model of its internal structure, its energy source and the way the energy flows outwards and, finally, the evolution of our star from its formation to its shutdown. This introductory chapter contains two sections, the first summarizes our approach and the second describes the basic solar data.

2.1 Our Approach

2.1.1 Motivation

No one can avoid forming a mental image of the Sun without it being strongly connected to our life. Naturally, this idea depends on a person's cultural level and power of imagination. For example, under the guise of irrationality and fear, the Sun may become a mysterious source of benign and malignant influences whose position in the sky affects people and their lives. Actually, this belief is called astrology and is still thriving today. Even in the past, the Sun was identified with a god who had to be propitiated with a suitable cult, as did primitive religions. From a scientific point of view, the Sun is interesting to many research fields in addition

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to solar physics. The Sun greatly affects the Earth and our lives here and in space. The influence of the Sun is first realized by its light radiation which it constantly emits and which supports life on the Earth, and secondly, to a lesser extent, by the intriguing phenomena of solar magnetic activity. The Sun represents an extraordinary laboratory of physics, where we can observe phenomena needing extreme conditions of temperature and density, and sizes that cannot be reached in terrestrial labs. Although there are billions of stars in the Universe that could, in principle, also serve as such laboratories, the Sun is the only star that is close enough for us to observe in detail. Hence, the Sun is a basic reference point for the study of the physical processes occurring in the other stars.

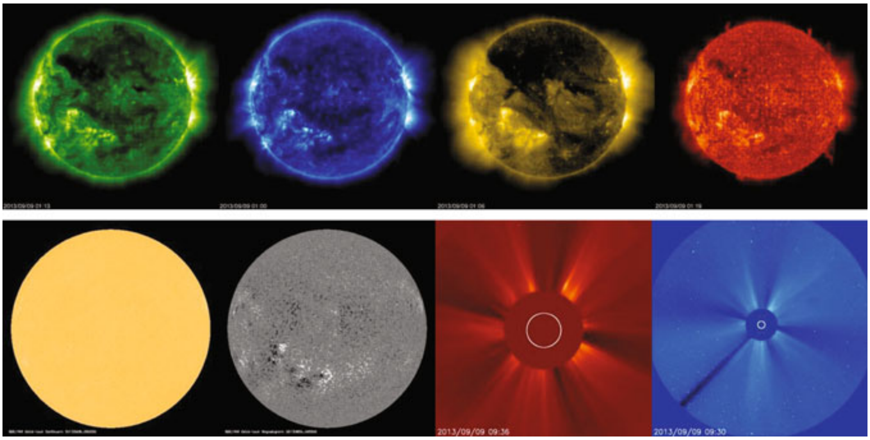

Fig. 2.1 The images of the Sun at the website of the SOHO space mission (http://sohowww.nascom.nasa.gov/data/realtime-images.html) for the day September 9th, 2013. From *top left to right*, the first four images represent the solar disk observed at different wavelengths in the extreme ultraviolet by the instrument EIT. Because the level of the solar atmosphere that we observe depends on the opacity of the solar gas and the opacity changes with wavelength, as we will see in detail in Chap. 3, these images correspond to four different levels in the solar transition region and corona, which are the outer parts of the solar atmosphere. Each atmospheric level has a different temperature, passing in the order from about 70 thousand K to 1, 1.5 and 2 million K degrees. The colors (*blue*, *green*, *yellow* and *red*), selected to represent the different temperatures, are arbitrary since they correspond physically to visible wavelengths and not to the ultraviolet wavelengths of observation. In the *last two images* of the *second row*, which have been obtained by the coronagraph in white light LASCO and are again represented with arbitrary colors, the corona is seen extending far beyond the solar disk. The visible solar disk is indicated by a white circumference in the images and was suitably occulted in the instrument to reach the same effect of a total eclipse. The *red image* extends outwards to 8.4 million km from the Sun's center, the *blue* to 45 million km. The photosphere is represented in the first image of the *second row*, which is the only one whose color has a real meaning. The magnetic field at the photospheric level is reported in the following image, where *black* and *white* denote opposite magnetic polarities

2.1.2 The Steady Sun

Nowadays the reader can easily access images of the solar atmosphere with high spatial resolution and at different wavelengths, which are provided by solar observations with ground based and space borne instruments. The images in Fig. 2.1 represent the solar atmosphere at different levels.

The photosphere is the base of the atmosphere and corresponds to the visible solar disk (first two images in the second row). The outer layers are the chromosphere, transition region and corona in order from inwards to outwards (first row and last two images in the second row). Apparently the photosphere is uniform and spherically symmetric, but it may locally present regions with a strong magnetic field, *sunspots*, where the field can reach values of the order of 3000 Gauss. The number and size of sunspots varies during a magnetic activity cycle, which is about 11 years long. The outer atmosphere reaches very high temperatures, extends outwards over the solar disk and is not at all spherically symmetric. The characteristics of the outer atmosphere are connected with the magnetic field, which, starting from its roots in the photosphere spreads outwards and plays a dominant role in determining the structure of these layers. Moreover, the Sun has a bipolar global magnetic field of few Gauss, similar to the Earth's mean magnetic field. We expect that the magnetic field cannot easily penetrate too deep inside the Sun, because the interior is highly ionized and electrically conductive, so that any change of the field induces a current that will oppose to it, according to the Lenz's law (Heinrich Lenz was a Russian physicist, 1804−1865).

The description and explanation of solar global structure and evolution, which is our goal, does not require us to go into the detail (even though fascinating), of the solar magnetic field and related phenomena. Therefore, we restrict ourselves to study the so-called *quiet*, or better, *steady* Sun. This is an ideal star having a perfect spherical symmetry, a visible surface, the photosphere, without spots, and is free from chromosphere, transition region and corona. The steady Sun is in uniform and stable equilibrium and constantly emits the same amount of energy per unit of time that is created inside its core by nuclear fusion reactions.

2.1.3 Methods

We can begin to understand how the steady Sun works once we start thinking about a number of basic questions. In a similar approach to how a child tries to understand a toy he or she is attracted to, by taking it apart and analyzing a piece at a time, we will use basic questions to guide us in dismantling the solar machine into suitable parts that we can more easily understand.

Wherever we live on the Earth, we enjoy solar light 6 months a year, and we all have a healthy natural instinct to not directly observe by eye the solar disk, which is so bright that it can seriously damage our retina. But, how much light does the Sun actually emit? And, more generally, what are the characteristic data for the Sun, like size, mass and temperature, and how are they derived? The answers to these questions are given later in this chapter. Details of the chemical elements forming the Sun are given in Chap. 3.

The Sun is continuously releasing energy into the region of space called the *Heliosphere*, and as such is the primary energy source for our Earth and the other bodies that are gravitationally linked to it. However, the primary use of solar energy is to keep the structure of the whole Sun in its present extraordinary equilibrium. So, naturally the second basic question refers to the global *structure* that characterizes the Sun at present. The answer to this question requires determining the ways in which energy is produced in the solar core and flows outwards. Energy generation and transfer in turn are directly related to the temperature and density stratification of the solar gas. Modeling the structure of the present Sun is the main goal of this book and makes up the next four chapters, each one devoted to a deeper layer, starting at the photosphere and going down to the convection zone, radiation zone and core. We will construct an analytical, layer by layer model of the Sun, which emphasizes the basic physical concepts and approximates the structure of the so called *solar standard model*, that is a current and accurate solar numerical model.

However large it may be, we expect that the source of solar energy will eventually become exhausted. Then, our third and last question, which is discussed in the seventh chapter, concerns the life of the Sun, or, as astronomers say, its *evolution*, starting from its formation, passing through the present stage, till the end of its activity.

Before scouting the structure and evolution of the Sun in the next chapters, in the following section we describe some basic solar properties.

2.2 The Characteristic Data

By characteristic data of the Sun we mean, first of all, its distance from the Earth, and then after this its global parameters like: radius, mass, luminosity and surface temperature. Table 2.1 summarizes the values of the solar data. Although the values are given to the numerical accuracy with which we know them at present, only the first few significant figures will often be sufficient for us.

Table 2.1 Characteristic data of the Sun

Sun–Earth distance	AU	$1.495978707 \times 10^{13}$ cm
Radius	R_S	6.960×10^{10} cm
Mass	M_S	1.9889×10^{33} g
Mean density	ρ_S	1.408 g cm^{-3}
Surface gravity	g_S	2.7398×10^{4} cm s^{-2}
Luminosity	L_S	3.844×10^{33} erg s^{-1}
Effective temperature	T_e	5778 K

For each quantity the name, symbol and value in CGS units are given

The Sun's distance from the Earth, radius and mass are determined by suitable measurements and using Kepler's laws that describe the motions of the planets around the Sun (Johannes Kepler was a German scientist, 1571, 1630).

2.2.1 The Distance from Earth

The Earth's orbit around the Sun is an ellipse with the Sun's position coinciding with one of the foci. Since the ellipse deviates only slightly from a circle (see Exercise 2.1), we are satisfied with the average distance of the Sun from the Earth. The *Sun–Earth mean distance* is about 150 million km. The astronomical unit, symbol AU, is a unit of length derived from this distance which is of convenient use in astronomy, in particular to express the distances in the solar system. Originally based on the measurement of the maximum and minimum distances from the Earth to the Sun, corresponding to the points in the orbit called aphelion and perihelion respectively, in 2012 the astronomical unit has been conventionally fixed to the exact value

$$1\,\text{AU} = 1.495978707 \times 10^{13}\,\text{cm} = 149.5978707\,\text{Gm}.$$

The light covers one AU in a time t such that

$$c = \text{AU}/t.$$

Then, since the light has a velocity of

$$c = 2.997950 \times 10^{10}\,\text{cm/s} = 0.2997950\,\text{Gm/ s};$$

we get

$$t = \text{AU}/c = 150\,\text{Gm}/0.300\,\text{Gm/s} = 500\,\text{s},$$

that is 8 min and 20 s.

Exercise 2.1 Denoting with a and p the distances that the Earth's center has from the Sun's center at the aphelion and perihelion, respectively and assuming that $(a + p)/2 = 1$ AU and $(a - p)/(a + p) = 0.016$, find the values of a and p in AU.

Worked Exercise 2.2 Measuring the Sun–Earth distance.

The method is a combination of an astronomical triangulation and a radar measurement of distance. The distance d from the planet Venus to the Earth can be determined from the time t_r that a radio wave impulse needs to reach Venus, be reflected by its surface and come back to Earth, as $d = ct_r$, where c is the light velocity. This implies that in the triangle EVS, whose vertices are the positions in the sky of the Earth (E), Venus (V) and Sun (S), the length of the side $EV = d$ is

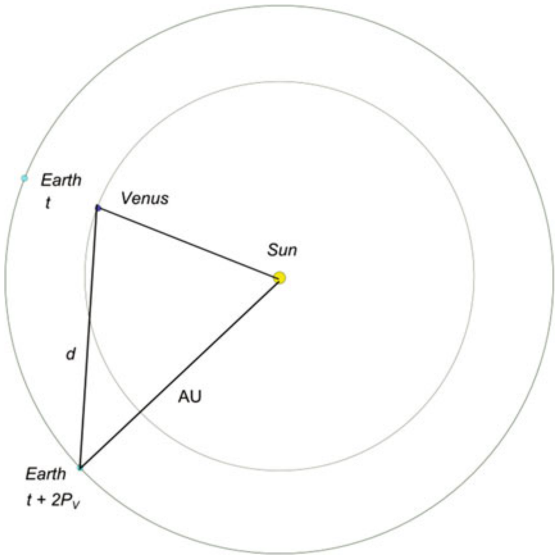

Fig. 2.2 The astronomical triangle used to determine the Sun–Earth distance. At time t, the Earth (E), Venus (V) and Sun (S) are aligned. Two revolution periods of Venus later, at time $t + 2P_V$, the Earth lies at the lower vertex of the triangle VSE. From observations and the radar measurement of the Venus–Earth distance (d), the astronomical unit (AU) can be determined

known (Fig. 2.2). The angle VES can be observed and the angle VSE is derived from the revolution period of Venus $P_V = 0.615$ years. In fact, the two positions of the Earth in the figure have been chosen to correspond respectively to the time t of a transit of Venus over the Sun, when the Earth, Venus and Sun are aligned, and the time $t + 2P_V$, two revolution periods later, when Venus is in the same position and the Earth is ahead on its orbit of the angle VSE. Because the time $2P_V = 2 \cdot 0.615 = 1.230$ y is elapsed, it is $VSE = 360° \cdot (1.230 - 1) = 82.8°$. According to the sine rule, $\text{AU}/\sin(EVS) = d/\sin(VSE)$ with $EVS = 180° - (VES + VSE)$ and $VES = 38.3°$ from observation. Thus, the astronomical unit is given by $\text{AU} = d \cdot \sin(EVS)/\sin(VSE)$, where $d = 174$ Gm from the radio measurement of distance.
Exercise end.

Exercise 2.3 Discuss the approximations which are implied in the solution of Exercise 2.2.

Exercise 2.4 Why the radar measurement of distance is not used directly with the Sun?

2.2.2 The Radius

The solar radius, R_S, is determined from the astronomical unit and the angular diameter α_S of the Sun. Because the Sun's size is small with respect to its distance from the Earth, the angle under which we see it, expressed in radians, is approximated by the ratio of the Sun's diameter to distance (Fig. 2.3). Then

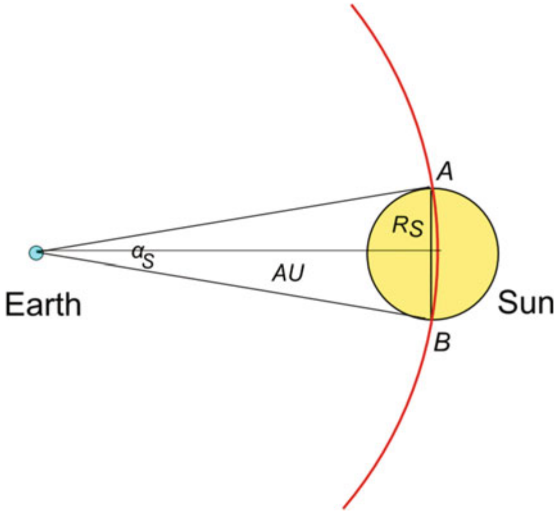

Fig. 2.3 The solar radius is more than 200 times smaller than the astronomical unit (in the figure it is only 1/6). This justifies that the solar diameter $2R_S$ is approximated with the arc *AB* (*red*) that the Sun occupies on a circle of radius *1*AU centered at the Earth. Therefore, the angular diameter is $\alpha_S = 2R_S/\text{AU}$

$$\alpha_S = 2R_S/\text{AU}$$

and

$$R_S = \text{AU} \cdot \alpha_S/2.$$

The Sun's disk in the sky as seen from the Earth has a diameter spanning in average an angle of a bit more than half degree, 0.53° or 1920″ in seconds of arc. The same angle is covered, for instance, by a football having a diameter of about 22 cm, placed at a distance of 23.78 m, or by a coin of 1 €, diameter 23 mm, at a distance of 2.486 m. To determine the solar radius, R_S, we have to transform the angular diameter of the Sun from degrees into radians. Since the degree to radian conversion's ratio is the same for all angles and a flat angle measures $\pi = 3.14159...$ in radians (π is a Greek letter to be read *pi*) and 180° in degrees, we can write

$$\alpha_{S(radian)} = \alpha_{S(degree)}\pi/180^\circ.$$

Therefore, the solar radius is

$$R_S = AU \cdot \alpha_S/2 = AU \cdot \alpha_{S(degree)}\pi/(180^\circ \cdot 2)$$
$$= 150\,\text{Gm} \cdot 0.53 \cdot 3.14/360^\circ = 0.696\,\text{Gm}$$

In particular, with this value of the solar radius, one second of arc on the Sun corresponds to

$$2R_S/1920 = 2 \cdot 0.696 \times 10^9\,\text{m}/1.92 \times 10^3 = 0.725 \times 10^6\,\text{m} = 725\,\text{km}.$$

As a matter of comparison, the mean radius of the Earth is 6371 km, about 110 times smaller than the solar radius, and one second of arc in km on the Sun corresponds to about the motorway distance between Naples and Venice, in Italy.

Worked Exercise 2.5 Measuring the solar angular diameter.

The solar disk has not a perfect sharp edge, like a solid sphere has. The apparent extension of the solar disk is related to the amount of emitted radiation and depends upon the radiation wavelength, as the images shown in Fig. 2.1 confirm. When light is restricted to the total emission in the visible spectrum, the edge is rather sharp because the emission at the solar limb decreases to about 15% in only 500 km. Therefore, the extremes of a solar diameter are defined as the points where the visible radiation is reduced to a factor $1/e$ of its value at the disk center. There are further issues concerning the way radiation is observed to vanish towards the limb. The instrumental apparatus and, in the case of ground based observations, the Earth's atmosphere, can degrade solar images. A significant part of these effects can be modeled by means of the so called *point spread function*, which accounts for the broadened and smoothed image of a single source point (Fig. 2.4).

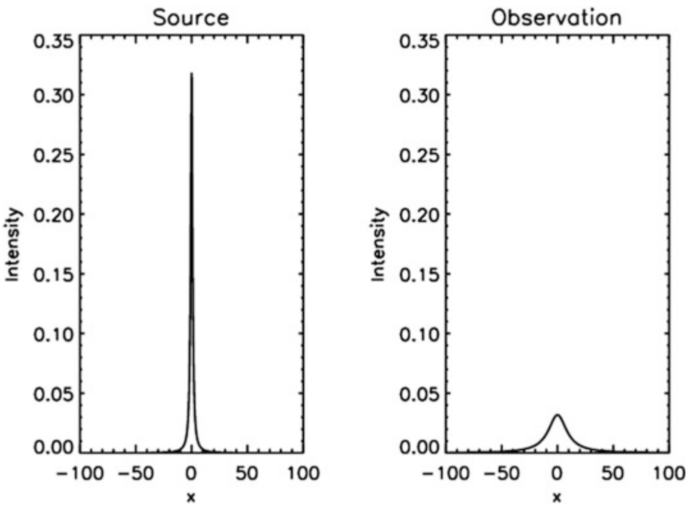

Fig. 2.4 The point spread function. An imaging system, which may include the Earth's atmosphere and telescope, degrades the observed image of a light source. The figure sketches the one dimensional case and the degradation effects are restricted to depression and broadening of the observed intensity. A single source point has an intensity profile (*left panel*) much thinner than the observed intensity profile (*right panel*), the ratio of their widths at half maximum being 1 to 10. Thus, the observed intensity profile is a direct approximation of the point spread function of the observing system

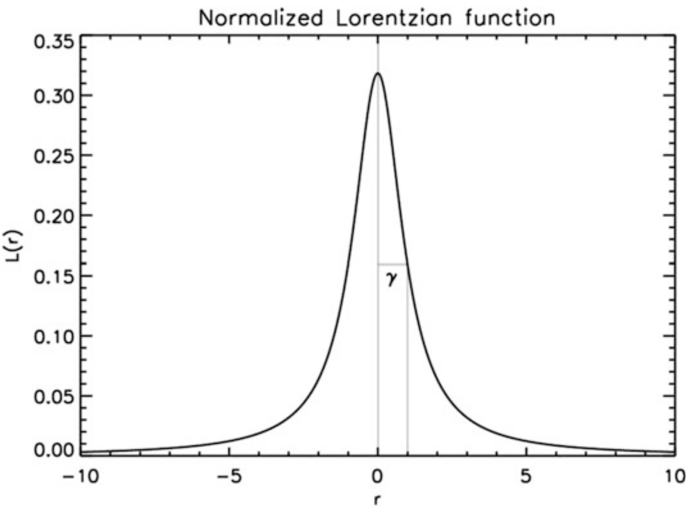

Fig. 2.5 The Lorentzian function $L(r) = \gamma/[\pi(r^2 + \gamma^2)]$ for $\gamma = 1$. This function is normalized, i.e. $\int_{-\infty}^{+\infty} L(r)dr = 1$. The horizontal segment labeled γ is the half width of the central peak at the level where the Lorentzian function is half of its maximum value. For $r \ll \gamma$ and $r \gg \gamma$ the Lorentzian function scales as $1/r^2$

If r is the coordinate along a solar diameter with $r = 0$ at disk center, $I(r)$ is the radiation intensity, and $P(r)$ the point spread function, the observed intensity $I_{obs}(r)$ is the integral

$$I_{obs}(r) = \int_{-\infty}^{+\infty} I(r')P(r' - r)dr'$$

The integral is the *convolution* of the intensity and point spread function. Because of this operation the observed intensity at the point r is the summation of the intensities from all the points r' in the neighborhood of r where the point spread function does not vanish. This implies that, even if the solar disk would have a step like edge, instrumental and possible atmospheric point spread functions make the observed intensity not vanish outside the limb (Exercise 2.7).
Exercise end.

Exercise 2.6 The Lorentzian function.
The normalized Lorentzian function is defined as

$$L(r) = \gamma/\left[\pi\left(r^2 + \gamma^2\right)\right]$$

where γ is a characteristic parameter (Fig. 2.5. Hendrik Lorentz was a Dutch physicist, 1893–1928). This function can be used as a point spread function for the *seeing*, which is the word to design the degradation that the Earth's atmosphere

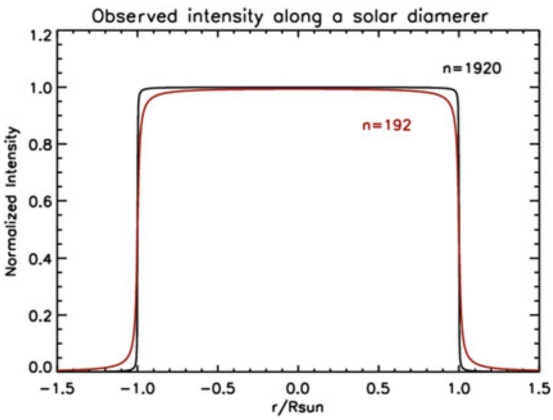

Fig. 2.6 The observed radiation intensity in the visible wavelength range along a solar diameter is modeled by the convolution of a rectangular intensity profile and a normalized Lorentzian point spread function. The result is plotted for two values of the Lorentzian half width γ corresponding to the values of $n = 1920$ (*black curve*) and $n = 192$ (*red curve*). Here $n = 2R_S/\gamma$, where R_S is the solar radius. Thus, $n = 1920$, that is the value of the solar angular diameter in arseconds, corresponds to $\gamma = 725$ km, or 1″, and $n = 192$ corresponds to $\gamma = 7250$ km, or 10″

induces on ground based observations. Find the maximum value of the Lorentzian function and show that γ is the *half width at half maximum*. Verify that this function is normalized, i.e. its integral between $r = -\infty$ and $r = +\infty$ is 1 [Hint Make use of d arctan $r/dr = 1/[(r^2 + 1)]$].

Exercise 2.7 Determine the intensity profile resulting by a convolution of a rectangular intensity profile (i.e. $I(r) = 0$ *for* $r < -R_S$ and $r > R_S$, and $I(r) = I_0$ *for* $-R_S < r < R_S$) and a Lorentzian point spread function with half width γ (Fig. 2.6).

Exercise 2.8 The normalized Gaussian function has the form $G(r) = [1/(\sigma\sqrt{\pi})]\mathrm{e}^{-r^2/\sigma^2}$. Show that a Lorentzian spread function with half width γ produces a larger solar angular diameter than a Gaussian point spread function with the parameter σ equal to the Lorentzian half width γ.

2.2.3 The Mass

The solar mass M_S can be derived from the Newton's law of gravitation. The gravity force provides the centripetal acceleration of the Earth orbiting around the Sun. Then, if m and v denote respectively the mass and speed of the Earth in a circular orbit of radius a, we can write

$$\frac{GmM_s}{a^2} = \frac{mv^2}{a}$$

and then

$$\frac{GM_s}{a} = v^2.$$

The orbital speed is the length of the circular orbit divided by the orbital period P

$$v = \frac{2\pi a}{P}.$$

Therefore

$$\frac{GM_S}{a} = v^2 = \left(\frac{2\pi a}{P}\right)^2$$

and

$$\frac{GM_S}{4\pi^2} = \frac{a^3}{P^2}.$$

This equation approximates the exact expression of Kepler's third law, where a is the semi-major axis of the elliptical orbit and the sum $M_S + m$ replaces M_S; this approximation is good enough since the Earth orbit is nearly circular and $M_S \gg m$ as we shall see soon. Thus, the solar mass is (with a in cm and P in seconds to be consistent with the units of G given in Table 1.5)

$$\begin{aligned} M_S &= \frac{4\pi^2 a^3}{GP^2} = \frac{4 \times 3.14^2 (1.5 \times 10^{13}\,\text{cm})^3}{6.67 \times 10^{-8}\,\text{dyn}\,\text{cm}^2\,\text{g}^{-2} \times (365 \times 24 \times 60 \times 60\,\text{s})^2} \\ &= 2.00 \times 10^{33}\ \text{g} \end{aligned}$$

An accurate value for the *mass* of the Sun is M_S = 1.9889 × 10^{33} g. The Sun has about 300 thousand times as much mass as the Earth whose mass is 5.976 × 10^{27} g.

Exercise 2.9 Show that the centripetal acceleration of a body moving along a circular orbit of radius a with constant speed v is given by $\frac{v^2}{a}$.

The radius and mass allow the evaluation of the *mean density,* ρ_S, and *surface gravity*, g_S. Since the Sun is a sphere of gas that is held together by the gravitational attraction of its component particles, its density is stratified and increases passing from the surface to progressively inner layers, as we will see in the next chapter. As a first step we can compute the mean density, which is the ratio of the solar mass to the volume, that is

$$\rho_S = M_S/\left(4\pi R_S^3/3\right) = 2.00 \times 10^{33} g\Big/\left[4 \cdot 3.14 \cdot \left(6.96 \times 10^{10}\text{cm}\right)^3/3\right]$$
$$= 1.41\,\text{g}\,\text{cm}^{-3}.$$

The solar mean density is nearly one and half the density of water and about a quarter of the mean density of the Earth.

The surface gravity is the acceleration impressed to a mass m which is on the Sun's surface, at 1 R_S from center. The surface gravity is inferred from the gravitational attraction that the Sun exercises on the mass m, which by virtue of the second law of dynamics is

$$F = g_S m = G M_S m/R_S^2,$$

where $G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2} is the gravitational constant.

Therefore, it results

$$g_S = G\,M_S/R_S^2$$
$$= 6.67 \times 10^{-8}\,\text{dyn}\,\text{cm}^2\,\text{g}^{-2} \cdot 2.00 \times 10^{33}\text{g}/\ (6.96 \times 10^{10}\,\text{cm})^2$$
$$= 2.74 \times 10^4\,\text{cm}\,\text{s}^{-2}.$$

The gravity at the surface of the Sun is more than 27 times the gravity at the surface of the Earth, where it reaches 9.78×10^2 cm s^{-2}.

Exercise 2.10 A satellite is in orbit around the Sun. Find the distance r at which the satellite would remain immobile relative to the solar surface, assuming that the Sun rotates around itself in 27 days. Notice that the rotation of the solar surface is actually differential, with the rotation period that is shorter at the equator (about 24 days) and longer at the poles (almost 38 days).

2.2.4 The Luminosity

The solar *luminosity*, which is the energy per second that the Sun emits into space as electromagnetic radiation, has the value

$$L_S = 3.844 \times 10^{33}\ \text{erg}\,\text{s}^{-1}.$$

To appreciate how large this number is, look at the Worked Exercise 2.11 which shows that the *solar constant*, C_S, that is the flux of solar energy at a distance of one astronomical unit, is given by

$$C_S = 1.4 \text{ kW m}^{-2}.$$

Worked Exercise 2.11 The solar constant.

Let us start to transform the unit of measure of luminosity from erg/s into kW. Since

$$1 \text{ kW} = 10^3 \text{ W} = 10^3 \text{ J s}^{-1} = 10^3 \times 10^7 \text{ erg s}^{-1} = 10^{10} \text{ erg s}^{-1},$$

it is

$$1 \text{ erg s}^{-1} = 10^{-10} \text{ kW}$$

and

$$L_S = 3.844 \times 10^{33} \text{ erg s}^{-1} = 3.844 \times 10^{23} \text{ kW}.$$

The solar luminosity at the distance of 1 AU is distributed over the sphere centered in the Sun and with radius equal to 1 AU (Fig. 2.7). Then, the *solar constant*, C_S, is given by the luminosity divided by the surface of this sphere

$$C_S = L_S/\left(4\pi AU^2\right) = 3.84 \times 10^{23} \text{ kW}\Big/\left[4 \cdot 3.14 \cdot \left(1.50 \times 10^{11} \text{ m}\right)^2\right]$$
$$= 1.4 \text{ kW m}^{-2}.$$

Note that the solar radiation flux is $L_S/(4\pi R_S^2)$ at the solar surface, and is $L_S/(4\pi AU^2)$ at the radial distance of *1* AU, then it drops off with the square of the radial distance r from the Sun, as $1/r^2$. This scaling rule occurs when radiation undergoes neither absorption nor emission along its path.

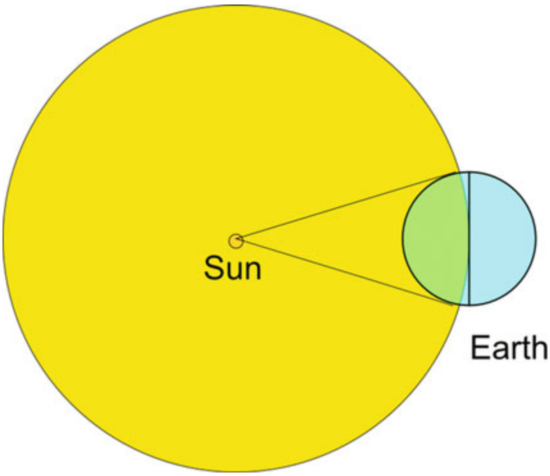

Fig. 2.7 The energy radiated by the Sun to the distance of one astronomical unit is distributed over the sphere centered in the Sun and with radius equal to 1 AU. The amount of solar energy that reaches the Earth is approximated by the radiation directed toward the Earth's section (represented by the vertical segment). Note that in reality the Earth's radius is much smaller than the astronomical unit and Sun's radius with respect to what the figure suggests

Finally, we consider the energy P that reaches the whole Earth in one second. Because solar rays hit a portion of the terrestrial surface, which is approximated by a section of the Earth, this power equals the solar constant multiplied by the area of the Earth's section S. The Earth's radius is $R_T = 6371$ km, hence

$$S = \pi R_T^2 = 3.14 \cdot (6371\,\mathrm{km})^2 = 1.3 \times 10^8\ \mathrm{km}^2.$$

Thus, we get for P

$$\begin{aligned} P = C_S S &= 1.4\ \mathrm{kW\,m^{-2}} \cdot 1.3 \times 10^8\ \mathrm{km}^2 \\ &= 1.4\ \mathrm{kW\,m^{-2}} \cdot 1.3 \times 10^{14} \mathrm{m}^2 = 1.7 \times 10^{14}\ \mathrm{kW}. \end{aligned}$$

The Sun in one hour delivers to the Earth 1.7×10^{14} kWh, that is about 10 million times as much energy as the first atomic bomb released in 1945.
Exercise end.

Exercise 2.12 Show that the flux that reaches the top of the Earth's atmosphere fluctuates around the value of the solar constant by about 6% during a year due to the Earth's varying distance from the Sun.

The solar constant includes all types of solar electromagnetic radiation, not just the visible light. It is measured at the top of the Earth's atmosphere and the measurement is adjusted to the distance of one AU scaling the flux by the inverse square of the distance (see Exercise 2.11).

The solar constant does not remain completely constant but typically varies by few parts per thousand or less because of solar magnetic activity. Fluctuations occur over a period of 11 years, due to the solar magnetic cycle, and over smaller timescales, from month to day, due to the rotation and evolution of solar magnetic regions.

2.2.5 The Temperature

In order to determine a *temperature* we approximate the Sun with a black body. The assumption of thermodynamic equilibrium is justified because the solar luminosity is a negligible amount of energy lost with respect to the total available energy and, moreover, the Sun is the seat of a large number of light emission and absorption processes. In this way the emission spectrum is close to the Planck function and we can infer a temperature from the Stefan–Boltzmann law. The outward radiation flux F from a unit surface element of a black body at temperature T is

$$F = \sigma T^4,$$

where $\sigma = 5.66956 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1} is the Stefan–Boltzmann constant. In the case of the Sun, the emitted flux is given by the ratio of the luminosity to the area of the solar surface and, hence, is computed in terms of the luminosity and radius as

$$F_S = L_S / \left(4\pi R_S^2\right).$$

Therefore, the Stefan–Boltzmann law allows us to define a particular temperature of the Sun, which is called the *effective temperature*, T_e, and is given by

$$F_S = \sigma T_e^4 = L_S / \left(4\pi R_S^2\right).$$

Thus

$$T_e^4 = F_S/\sigma = L_S / \left(4\pi R_S^2 \sigma\right)$$

and

$$\begin{aligned} T_e &= \left[L_S / \left(4\pi R_S^2 \sigma\right)\right]^{-4} \\ &= \left\{3.844 \times 10^{33}\,\mathrm{erg\,s^{-1}} \middle/ \left[4 \cdot 3.1415 \cdot \left(6.96 \times 10^{10}\mathrm{cm}\right)^2 \cdot 5.66956 \times 10^{-5}\mathrm{erg\,cm^{-2}K^{-4}s^{-1}}\right]\right\}^{-4} \\ &= 5778\,\mathrm{K}. \end{aligned}$$

The effective temperature is the surface temperature of the Sun since it is a characteristic of the visible solar disk, or *photosphere*, while, as we will see later, the internal layers of the Sun have temperatures much higher than the value of the effective temperature. The structure of the photosphere is discussed in the next chapter.

A characteristic, which is related to the surface temperature of a star, is its *color*. We know that the color of an incandescent solid body changes with the temperature to which it is heated, passing, with increasing temperature, from red to orange, yellow, white and, finally, blue. The Sun is a star of yellow color. Note that the

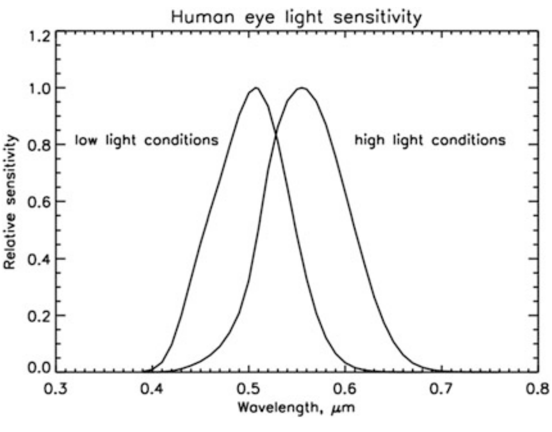

Fig. 2.8 The sensitivity of the human eye to light. The response to color depends on the level of the external light. The two curves give the fraction of light which is perceived at each wavelength in low (*left curve*) and high (*right curve*) light conditions. The maximum sensitivity is set to unity

body's color that we perceive depends not only on the amount of its emission at different wavelengths, but also on the sensitivity of the human eye, which acts as a light filter. The curves representing our standard sensitivity to light of different wavelengths have the shapes of bells covering the visible spectrum with their maxima in the yellow green wavelength range (Fig. 2.8).

2.2.6 Further Exercises

Exercise 2.13 Find the distance from the Earth at which a satellite in orbit is immobile relative to the ground.

Exercise 2.14 Evaluate the gravitational potential energy of the Earth in orbit around the Sun.

Exercise 2.15 Find the orbital velocity of the Earth around the Sun.

Exercise 2.16 Determine the escape velocity from the solar system at the position of the Earth. This escape velocity is the minimum velocity necessary to overcome the solar attraction and leave the solar system.

Exercise 2.17 Show that the escape velocity from the Sun is a factor 3.76 greater than the escape velocity from the Earth.

Exercise 2.18 The semi-major axes of the orbits for Venus, Mars, Jupiter and Saturn are 0.723, 1.524, 5.204, and 9.580 AU, respectively. Find the corresponding orbital periods from the Kepler's third law.

Exercise 2.19 Show that at a height h, which is small relative to the Earth's radius, the gravitational potential energy of a body with mass m can be written $E = mg_E h$, where g_E is the Earth surface gravity acceleration and the Earth's surface is taken as the zero energy level.

Exercise 2.20 A meteorite with mass $m = 1$ kg hits the Earth's atmosphere with a velocity of 40 km/s at a height of 10 km and starts to burn. If all the energy of the meteorite is transformed into red light at a wavelength $\lambda = 0.65$ μm, how much energy and how many photons are produced?

Exercise 2.21 An incandescent solid body, which is approximated with a black body, emits most of his light in the red at $\lambda = 0.65$ μm. What is its temperature? How much energy is required to shift the body's emission peak to the yellow at $\lambda = 0.57$ μm, if the mass is $m = 10$ kg and the specific heat is $C_s = 4 \times 10^6$ erg $g^{-1}K^{-1}$?

Exercise 2.22 Knowing that the Moon's rotation period around the Earth is 27.3 days, estimate the Moon-Earth distance.

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