Chapter 2
Baryon Asymmetry in the Early Universe

In this chapter the basic concepts and notation related to the physics of the early universe are introduced. To the best of our knowledge, the universe is evolving today from a very dense and hot phase. The Big-Bang cosmology and the thermal history of the universe are discussed in Sect. 2.1. The early universe sets the stage for many interesting phenomena, such as the dark matter production, the generation of the baryon asymmetry and the nucleosynthesis of light elements. In Sect. 2.2 we address in some detail the framework for a dynamical generation of the baryon asymmetry discussing the Sakharov conditions together with a toy model to show their implementation. Finally the baryon and lepton number violation within the SM is presented, which is induced by the sphaleron processes in the early universe. The discussion aim at showing why one has to invoke some new physics beyond the SM to quantitatively explain the observed baryon asymmetry in the universe.

2.1 Big-Bang Cosmology

At least on large scale our universe appears to us as isotropic and homogeneous, and this matter of fact is often attached to the so-called cosmological principle stating that the universe looks the same to all observers. The expansion of the universe is a natural consequence of any isotropic and homogeneous cosmological model based on General Relativity (GR). The very fact that the universe expands today implies that it was denser and warmer in the past. On the basis of GR and thermodynamics, we can extrapolate that matter had higher and higher temperature and density at earlier and earlier epochs, and that at most stages the entire system was in thermal equilibrium. The Big-Bang would then be the initial point in space-time from which we can start to study and address the early universe physics.

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S. Biondini, Effective Field Theories for Heavy Majorana Neutrinos in a Thermal Bath, Springer Theses, DOI 10.1007/978-3-319-63901-7_2
The formulation of the Big-bang model began in the 1940s with the idea that the abundances of light chemical elements had a cosmological origins. In their pioneering work [1, 2], George Gamow and his collaborators, Alpher and Herman, supposed that the universe was hot and dense enough to allow a nucleosynthetic processing of the hydrogen, and has expanded and cooled down to the present state. Later in 1948, Alpher and Herman predicted an important consequence of a hot universe [3, 4]: a transition from a plasma of baryons, electrons and photons to a gas of atoms and free electromagnetic radiation. At this stage the atomic gas gets transparent to photons, and a relic background radiation is expected to be associated with this transition. Indeed the Cosmic Microwave Background (CMB) was detected sixteen years after its prediction [5] and it has been the first experimental proof that our universe had a hot past.

2.1.1 Dynamics of an Expanding Universe

We address briefly the dynamics of an expanding universe by using GR. We aim at capturing the main features relevant to our discussion: in the past the universe was smaller, denser and hotter. We focus on the epoch in which the universe was filled with relativistic particles, namely with typical momenta much bigger than their mass. The present discussion follows standard text book derivations, such as [6].

Starting from the observation of an isotropic and homogeneous universe, its overall geometry can be described in terms of few independent parameters entering the Einstein equations of GR. In particular we start from the well known equation

$$ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, $$

(2.1)

that connects the space-time geometry with the energy content of the universe, where $R_{\mu\nu}$ is the Ricci tensor, $R$ is the Ricci scalar, $T_{\mu\nu}$ is the energy-momentum tensor and $G$ is the gravitational constant. Natural units $c = \hbar = 1$ are adopted throughout the thesis. One can find the explicit form of (2.1) for an isotropic and homogeneous metric, known as Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ ds^2 = dt^2 - a(t) \left[ \frac{dr^2}{1 - \kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], $$

(2.2)

which has a maximally symmetric 3-D subspace of a 4-D space-time. In Eq.(2.2) $t$ is the time variable, $(r, \theta, \varphi)$ are the polar coordinates, $\kappa$ is a constant related to the spacial curvature. Its possible values are $-1$, 0 and $+1$ accommodating a 3-hyperboloid, a 3-plane and a 3-sphere respectively and describing an open, flat or
close universe. The quantity $a(t)$ is called scale factor and it measures how rapidly the universe expands through the definition of the Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \quad (2.3)$$

where the dot stands for the time derivative. Assuming a FLRW geometry the left-hand side of Eq. (2.1) becomes (the 00 component)

$$R_{00} - \frac{1}{2} \dddot{a} = 3 \left( \frac{\ddot{a}^2}{a^2} + \frac{\kappa}{a^2} \right). \quad (2.4)$$

Let us now consider the energy momentum tensor on right-hand side in Eq. (2.1). We notice that, for cosmological epochs relevant to us, the content of the universe can be described as a homogeneous fluid with energy density $\varepsilon(t)$ and pressure $p(t)$. If we consider this fluid as a whole at rest with respect to a comoving reference frame, then the only non-zero component of the fluid velocity, $u_\mu$, is $u_0 = 1$. Hence, the 00 component of the energy momentum tensor gives

$$T_{00} = (\varepsilon + p)u_0u_0 - g_{00}p = \varepsilon. \quad (2.5)$$

Combining (2.4) and (2.5) we obtain the Friedmann equation:

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \varepsilon - \frac{\kappa}{a^2}, \quad (2.6)$$

that relates the rate of the cosmological expansion with the total energy density, $\varepsilon$, and space curvature, $\kappa$. The Friedmann equation has to be supplemented with an additional equation since two unknown functions of time appear: $a(t)$ and $\varepsilon(t)$. That equation can be obtain from the covariant conservation of the energy momentum tensor $T_{\mu\nu}$, that brings to

$$\dot{\varepsilon} + 3\dot{a}^2(\varepsilon + p) = 0. \quad (2.7)$$

Last but not the least, we add the equation of state of matter. This is necessary to close the system of equations that governs the universe expansion, and it can be written as follows

$$p = p(\varepsilon), \quad (2.8)$$

enforcing the pressure to be some function of the energy density. The equation of state (2.8) is not a consequence of GR.

Since we are going to deal with a heat bath of SM particles at high temperatures, it is instructive to inspect more closely the Friedmann equation in the case the universe consists, almost entirely, of relativistic degrees of freedom. Indeed we want to study the dynamics of very heavy particles inducing a baryon asymmetry in a background
of either massless particles or with a mass much smaller than the typical three-momentum scale, provided by the temperature of the plasma, $T$. This epoch in the early universe is often denoted as *radiation dominated era*. In the case of a plasma made almost entirely of relativistic particles, the equation of state in (2.8) reads:

$$ p = \frac{\varepsilon}{3}. \quad (2.9) $$

We further assume a flat geometry, $\kappa = 0$, which is indeed very close to the real universe, so that the Friedmann equation (2.6) becomes

$$ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \varepsilon. \quad (2.10) $$

Inserting the equation of state (2.9) into (2.7) we obtain for the energy density and Friedmann equation in (2.10) respectively

$$ \varepsilon = \frac{K}{a^4}, \quad (2.11) $$

$$ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} \frac{K}{G a^4}, \quad (2.12) $$

where $K$ is a constant that embeds the energy density and scale factor at some initial time $t_0$. One can easily find from (2.12) that $a(t) \propto \sqrt{t}$ and hence the Hubble rate is $H = 1/(2t)$. The energy density as a function of time can be obtained from the Friedmann equation (2.10), once the scale factor $a(t)$ has been eliminated in favour of $t$:

$$ \varepsilon = \frac{3}{8\pi G} H^2 = \frac{3}{32\pi G} \frac{1}{t^2}. \quad (2.13) $$

Already from this last simple relation we see that the smaller the age of the universe the bigger the energy density.

It is useful to relate the Hubble parameter with the temperature of the universe. This will help to clarify that earlier times correspond to higher temperatures. Considering a relativistic massless particle specie, labelled with the subscript $i$, as part of a heat bath in thermal equilibrium and neglecting chemical potentials, the corresponding energy density reads

$$ \varepsilon_i(T) = g_i \int \frac{d^3p}{(2\pi)^3} p f_i(p) = g_i \left\{ \begin{array}{ll}
\frac{7}{30} T^4, & \text{(boson)}, \\
\frac{7}{8} \frac{\pi^2}{30} T^4, & \text{(fermion)}. 
\end{array} \right. \quad (2.14) $$
In thermal equilibrium the distribution \( f_i(p) \) in (2.14) is either the Bose-Einstein or the Fermi-Dirac distribution, namely

\[
n_B = \frac{1}{e^{\beta E(p)} - 1}, \quad n_F = \frac{1}{e^{\beta E(p)} + 1},
\]

where \( E(p) \) is the energy of the particle, \( \beta = 1/T \) and written in a reference frame at rest with respect to the thermal bath. For highly relativistic particles the energy is \( E(p) = p \) and \( p \equiv |p| \) stands for the modulo of the three-momentum of the particle with internal degree of freedom \( g_i \) (for example spin polarizations). Hence for a thermal bath made of different relativistic particle species, the total energy density is

\[
\varepsilon = \left( \sum_i g_{b,i} + \frac{7}{8} \sum_i g_{f,i} \right) \frac{\pi^2}{30} T^4 = g_* \frac{\pi^2}{30} T^4,
\]

where we define the effective number of degrees of freedom, \( g_* \), as the sum over bosonic, \( g_{b,i} \), and fermionic, \( g_{f,i} \), degrees of freedom (the latter weighted for the statistical factor 7/8 coming from the integration of the Fermi-Dirac distribution). In general \( g_* \) is temperature dependent because the number of relativistic particle species may change during the universe evolution. Now we rewrite Eq. (2.13) substituting the expression for the energy density in (2.16) as follows

\[
H = \frac{T^2}{M_{Pl}^*},
\]

where we used \( G = M_{Pl}^{-2} \), where \( M_{Pl} \) is the Planck mass, and the definition of the effective Planck mass, which depends on the number of effective degrees of freedom:

\[
M_{Pl}^* = \sqrt{\frac{90}{8\pi^3 g_*}} M_{Pl} \approx \frac{1}{1.66\sqrt{g_*}} M_{Pl}.
\]

We notice that \( M_{Pl}^* \) is temperature dependent because it is a function of \( g_* \). This dependence is rather weak and it is a good approximation to take \( M_{Pl}^* \) as a constant discussing the early universe at some stage of its evolution. Finally by comparing Eq. (2.11) and (2.16) we obtain

\[
T(t) \propto \frac{1}{a(t)},
\]

where the relation holds exactly when the number of relativistic degrees of freedom does not change over the considered period of time. Due to the weak dependence on \( g_* \) with the temperature, the relation (2.19) provides an important observation: at a smaller scale factor corresponds a higher temperature. In summary we say that going back in time the universe was smaller, denser and warmer.
Let us conclude this section with a brief discussion about thermal equilibrium. We are going to consider processes that occur in an expanding universe filled with particles. The rates of interactions between these particles are often much higher than the expansion rate of the universe, so that the cosmic medium is in thermal equilibrium at any moment of time. However, we note that as a rule of thumb the most interesting periods in the cosmological evolution are those when one or another reaction goes out of equilibrium. In this case the abundance of some particle species freezes out and decouples from the heat bath. Nevertheless the laws of equilibrium thermodynamics are still useful since they enable us to estimate the time of departure from equilibrium and determine the direction of non-equilibrium processes. Moreover most of the constituents of the heat bath, understood as a background for a given process of interest deviating the equilibrium conditions, are in thermal equilibrium.

The thermodynamical description of a system with various particle species is usually made in terms of a chemical potential \( \mu \) for each type of particle. Given the reaction involving different particles labelled with \( A_i \) and \( B_j \) as follows

\[
A_1 + A_2 + \cdots + A_n = B_1 + B_2 + \cdots + B_m ,
\]

the corresponding chemical potentials in thermal equilibrium, or better in chemical equilibrium, obey to the following relation

\[
\mu_{A_1} + \mu_{A_2} + \cdots + \mu_{A_n} = \mu_{B_1} + \mu_{B_2} + \cdots + \mu_{B_m} .
\]

For example the chemical potential of the photon is zero and for a particle and its corresponding antiparticle the chemical potentials are the same but opposite in sign. Let us consider the process \( e^+ e^- \rightarrow 2\gamma \). We say that it is in equilibrium if it is equally likely as the back reaction \( 2\gamma \rightarrow e^+ e^- \).

Being the particle interactions in the thermal plasma fairly weak, we can take the equilibrium distributions to be the Bose–Einstein and Fermi–Dirac ones, as anticipated when writing (2.15). Upon integrating the distribution function over the three-momentum one obtains the corresponding number density of the particle species \( i \)

\[
n_i = g_i \int \frac{d^3 p}{(2\pi)^3} f_i(E(p)) ,
\]

where \( f_i \) can be either \( n_B \) or \( n_F \) in (2.15) and \( g_i \) are the internal degrees of freedom of the particle. For example for the photons one finds \( (m_\gamma = 0 \text{ and } \mu_\gamma = 0) \)

\[
n_\gamma = \frac{2T^3}{\pi^2} \zeta(3) ,
\]

where \( \zeta(3) = 1.202 \), being \( \zeta(x) \) the Riemann zeta function. More details on the thermodynamics of the early universe can be found e.g. in [6] or in the appendix of [7].
### 2.1.2 Brief Thermal History of the Universe

We discussed how the cosmological principle leads to an expanding universe with a hot past. Going back in time means looking at a smaller and smaller universe filled with particles at higher and higher temperatures. We can pinpoint some relevant periods in the universe evolution, shown in Fig. 2.1, and we aim at discussing them briefly in order to arrive at the topic of interest: the generation of the baryon asymmetry in the universe.

We start with the recombination period, also called photon decoupling or last scattering. The plasma of hadrons, mainly hydrogen, electrons and photons turns into a gas of atoms. Before recombination the temperature was too high to allow for bound states of nuclei and electrons, so that the photons were continuously scattered off the charged particles and trapped in the hot plasma. The transition temperature from the plasma to the gas of atoms can be naively estimated to be of order of $T \sim 10 \text{ eV}$, even though more accurate analysis give fraction of the eV scale, $T \sim 0.3 \text{ eV}$ [6]. From this moment onwards, the cross section with neutral atoms is so small that the average photon has not interacted with matter ever since: the medium became transparent to photons. The CMB carries information about this very moment, giving access to the universe when its temperature was about 3000 K ($T \sim 0.3 \text{ eV}$) and 370 000 years old. We have already mentioned that the high degree of CMB isotropy shows that the Universe was pretty much homogeneous at recombination: the density perturbations were comparable with temperature fluctuations and were roughly of order $\delta T / T \sim 10^{-5}$. Nevertheless, these perturbations have grown and have given rise to structures: first stars, then galaxies, then clusters of galaxies. The CMB provides the earliest direct probe of universe structure that we can study in great detail.

Proceeding back in time we find the Big-Bang Nucleosynthesis (BBN) [8–11]. The temperature is set by the nuclei binding energy, namely $T \sim 1 \text{ MeV}$. Accurate analysis provides somewhat smaller temperatures though, namely fractions of MeV. From an earlier phase where protons and neutrons were free in the hot plasma, as the temperature dropped during the universe expansion, neutron capture and thermonuclear reactions became possible. At this stage light elements were formed: mainly Deuterium, D, Helium isotopes, $^3\text{He}$ and $^4\text{He}$, and small amount of Lithium, $^7\text{Li}$. Quantitative calculations based on GR and kinetic equations provides the primordial abundances of the element species. These predictions depend on essentially a single parameter, called the baryon-to-photon ratio and defined as follows

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_{\gamma}}, \quad (2.24)$$

where $n_B$, $n_{\bar{B}}$ and $n_{\gamma}$ are the baryon, antibaryon and photon number densities. The final light-element abundances are highly sensitive to this parameter, which characterizes the baryon-photon plasma during the nucleosynthesis process. The population of D and $^3\text{He}$ depends on $\eta_B$, and the cross sections of the processes leading to the formation of the heavier elements, like the $^4\text{He}$, inherits the dependence on the
baryon-to-photon ratio. The larger $\eta_B$ the later the process generating the $^4$He will stop, and consequently the smaller the freeze-out abundances of the reacting elements D and $^3$He. Today the direct measurement of primordial abundances is pretty accurate, and this is a cornerstone of the early universe physics and the standard hot big bang cosmology. Indeed there is a range of $\eta_B$ which is consistent with all four abundances (D, $^3$He, $^4$He and $^7$Li), which at (95% CL) reads [9]

$$4.7 \times 10^{-10} \leq \eta_B \leq 6.5 \times 10^{-10}.$$  \hspace{1cm} (2.25)

From now on, going back in time requires educated extrapolations. We cannot infer solid statements on our universe when it was hotter than $T \sim \text{MeV}$. However, it is possible and desirable that higher temperatures occurred in our universe. From the theoretical point of view this offers a very interesting scenario to test the laws of particle physics to extreme conditions. As we shall see the explanation of a baryon asymmetry naturally asks for some higher temperature regimes. By assuming that temperatures of order of the GeV scale and higher were possible, we can list additional epoch comprising phase transitions. Briefly we can summarize them as follows

---

Fig. 2.1  Stages of the universe evolution from inflation (bottom) to the present era (top). Typical temperatures, on the left, and age of the universe, on the right, are shown. Figure from [6]
(1) A transition (better a crossover) from a hadron gas to a quark-gluon plasma where the chiral symmetry is possibly restored. The transition temperature can be estimated from the QCD non-perturbative scale, $\Lambda_{\text{QCD}} \sim 250 \text{ MeV}$, even though more accurate simulations from lattice QCD provide the crossover to occur at $T_c = 154 \pm 9 \text{ MeV}$ [12]. For $T \geq T_c$ quarks and gluons are not bounded any more in colourless hadrons, rather they interact as individual particles.

(2) Electroweak phase transition. Above the electroweak scale, $T_W \sim 100 \text{ GeV}$, the Higgs condensate is absent and the $W$'s and $Z$ boson are then massless. The gauge group would be an unbroken $\text{SU}(2)_L \times \text{U}(1)_Y$, and all the SM fermions are massless as well. Further elaborations on the subject will be provided in the next sections.

(3) A more speculative transition is that involving the grand unification scale. This is related to the hypothesis that at higher energies, $T_{\text{GUT}} \sim 10^{16}$, the fundamental strong, weak and electromagnetic forces are unified into a single force. The supersymmetric extension of the SM provides some motivation for such speculation.

The next cosmological period we can see in Fig. 2.1 is the reheating phase after inflation. Here two relevant processes might have occurred that represent a contemporary challenge in particle physics and cosmology: the generation of the baryon asymmetry in the universe and the production of dark matter. Since we are going to discuss the former in the upcoming section we spent some words here on the latter.

There are many experimental observations that suggest the presence of an additional component in the matter content of the universe. At galactic and sub-galactic scales, this evidence includes galactic rotation curves [13], the weak gravitational lensing of distant galaxies by foreground structure [14], and the weak modulation of strong lensing around individual massive elliptical galaxies [15]. Furthermore, velocity dispersions of stars in some dwarf galaxies imply that they contain as much as one thousand times more mass than can be assigned to their luminosity, and the same was observed quite some time ago at the scale of galaxy clusters in 1933 by Fritz Zwicky [16]. On cosmological scales, observations of the anisotropies in the cosmic microwave background have lead to a determination of the total matter density of $\Omega_{\text{mat}} h^2 = 0.1326 \pm 0.0063$ [17], where $h$ is the reduced Hubble constant. Moreover, this information combined with measurements of the light chemical element abundances leads to an accurate estimate of the baryonic density given by $\Omega_B h^2 = 0.02273 \pm 0.00062$ [17]. Taken together, these observations strongly suggest that more than 80% of the matter in the universe (by mass) consists of non-luminous and non-baryonic particles, called dark matter.

On the other hand, there is almost total lack of information on dark matter from the particle physics point of view leading to a difficult assessment of the production mechanism in the early universe. Besides the fact that dark matter does not interact with photons, our knowledge of its fundamental interaction is scarce. We demand dark matter to be generated in the early stages of the universe evolution because it is an essential ingredient for the clumping of matter in the primordial gravitational potential wells that eventually formed stars, galaxies and large scale structures.
The process of the formation of large scale structures through the gravitational clustering of collisionless dark matter particles can be studied using N-body simulations. When the observed structures in our universe are compared to the results of cold dark matter simulations good agreement has been found [18]. Here cold means the dark matter to be non-relativistic at time of structure formation. Many candidates has been put forward e.g. gravitinos and neutralinos from supersymmetry, axions and sterile neutrinos. We refer to [19–21] for extensive reviews on dark matter candidates, as well as for discussions on dark matter production mechanisms in the early universe.

Finally we comment on the epoch of reheating and how some of the issues related to the Big-Bang Cosmology are treated. This stage comes right after the inflationary stage. Many of the problems that affect the Big-Bang theory arise from the very special initial conditions one has to require. At a qualitatively level, the Big-Bang model cannot explain why our universe is so large, almost spatially flat, homogeneous and isotropic. Another issue refers to the primordial density perturbations detected in the CMB, which are the seeds for the generation of the matter structures we see today (stars, galaxies, clusters and so on and so forth). The hot Big-Bang theory does not contain a way to generate those perturbations and they have to be put “by hands”. The aforementioned problems find an elegant solution in the inflationary model, according to which the hot phase of the early universe was preceded by a phase of exponential expansion. An initially small region of typical length of the Planck scale, $l_{Pl} \sim 1/M_{Pl}$, was inflated to very large sizes even larger than those of the visible present universe horizon. This explains eventually the dilution of any initial anisotropy, the homogeneity and the flatness. Moreover the model introduces a new field, the inflaton, which drives the exponential expansion and after the inflation epoch ends, it transfers its energy into the ordinary matter that populate the early universe. This is usually called the reheating phase. The primordial matter and energy perturbations are understood as quantum fluctuations of the inflaton field. The basic ideas of inflation were originally proposed by Guth [22] and Sato [23] independently, which were reviewed and brought to the modern fashion by Linde [24], and Albrecht and Steinhardt [25]. The inflationary epoch plays an important role with respect to the baryon asymmetry in the universe, as we are going to discuss in the upcoming section, and more in general it provides a reasonable explanation for the existence of a thermal bath of particles in the very early stages of the universe evolution.

### 2.2 Dynamical Generation of the Baryon Asymmetry

Observations suggest that the number of baryons in the universe is different from the number of antibaryons. The almost total absence of antimatter on Earth, in our solar system and in cosmic rays indicates that the universe is baryonically asymmetric. A more accurate reasoning could bring us to admit that matter and antimatter galaxies could coexist in clusters. However we would expect a detectable background of photon radiation coming from nucleus-antinucleus annihilation within the clusters [26]. This argument can be further generalized to large hypothetical domains of
matter an antimatter in the universe, but the missing observation of any induced
distortion on the CMB discards this possibility. As Cohen, de Rujula and Glashow
have compellingly argued, if there were to exist large amounts of antimatter in
the universe they could only be at a cosmological scale from us [27]. It therefore seems
that our universe is fundamentally matter-antimatter asymmetric.

There are observables to make this statement more quantitative. In particular we
refer to the baryon-to-photon ratio, already introduced in Sect. 2.1, and we recall it
here with the experimental value attached

\[ \eta_B = \frac{n_B - n_B}{n_\gamma} = (6.21 \pm 0.16) \times 10^{-10}. \] (2.26)

Such precise measurement comes from the study of the CMB anisotropies [28]. As
regards the CMB analysis, the parameter \( \eta_B \) plays a crucial role in determining the
relative amplitudes of even and odd peaks of the power spectrum of the microwave
background. This is in turn related to the acoustic oscillations of the baryon-photon
fluid at the time of recombination. It is astonishing the high level of agreement with
an independent prediction: the abundances of the light elements provided by BBN.
As discussed in the previous Sect. 2.1, the generation of elements like H, \(^3\)He, \(^4\)He
and \(^7\)Li occurred before the last scattering in a hot plasma. It is found that their
abundances can be obtained by an input of a single parameter, \( \eta_B \). The range for this
parameter, predicted by BBN and written in (2.25), agrees with the value extracted
from the CMB analysis in (2.26) establishing an extraordinary matching between
two independent measurements.

The challenge both from the cosmology and particle physics side is to explain
the observed value in (2.26). The standard cosmological model dramatically fails in
reproducing even only the order of magnitude of the baryon-to-photon ratio if we start
with a matter-antimatter symmetric phase at high temperatures. Let us consider the
reaction \( p + \bar{p} \leftrightarrow 2\gamma \), at temperature of the order of one GeV. Protons and neutrons
constitute the baryon content of the universe at this epoch. As the universe cools down
the process \( 2\gamma \rightarrow p + \bar{p} \) becomes ineffective due to Boltzmann suppression, and
therefore the annihilation process \( p + \bar{p} \rightarrow 2\gamma \) takes over. The same reactions stand
for neutrons and antineutrons. Eventually the number of baryons and antibaryons
is strongly reduced with respect to the photon number density, a straightforward
calculation provides [6, 29]

\[ \frac{n_B}{n_\gamma} \approx \frac{n_\bar{B}}{n_\gamma} \approx 10^{-18}, \] (2.27)

which is far too smaller than the value required for a successful nucleosynthesis, see
(2.25), and than the one in (2.26) from CMB analysis. It is hard to figure out processes
at temperatures below one GeV able to enhance the small ratio between baryon and
photon number densities induced by annihilations (an exception is provided by the
Affleck-Dine Baryogenesis [30]). Because of the strong disagreement between (2.26)
and (2.27), we come to the conclusion that a primordial matter-antimatter asymmetry had to exist already before BBN, and more specifically at temperatures of the GeV scale.

The observed baryon asymmetry could be set as an initial condition for the universe evolution. However, it would require a high fine tuning and the ad hoc baryon asymmetry would have not survived the huge dilution induced by the inflationary epoch. This is why the scenario of a dynamically generated baryon asymmetry is more appealing. The dynamical generation of a baryon asymmetry in the context of quantum field theory is called baryogenesis [31]. Indeed, quoting A. Riotto, “the guiding principle of modern cosmology aims at explaining the initial conditions required by standard cosmology on the basis of quantum field theories of elementary particle physics in a thermal bath” [26].

2.2.1 The Sakharov Conditions

Assuming a vanishing initial matter-antimatter asymmetry, it can be dynamically generated in an expanding universe if the particle interactions and the cosmological evolution satisfy the three Sakharov conditions [31]:

1. baryon number violation,
2. C and CP violation,
3. departure from thermal equilibrium.

In the following $B$ stands for the baryon number understood as the total baryonic charge of a given process. Any particle physics model that aims at generating an imbalance between matter and antimatter has to account for the aforementioned necessary conditions. Originally introduced in the framework of GUTs, we briefly discuss the Sakharov conditions also in relation with the SM of particle of physics in order to show that all the three requirements are fulfilled. However all attempts to reproduce quantitatively the observed baryon asymmetry have failed within the SM.

Since we start from a baryon symmetric universe, we need processes that violate the baryon (antibaryon) number to somehow evolve into a situation in which it holds $\eta_B \neq 0$. Processes are required that change the number density of baryons and antibaryons entering the definition of $\eta_B$. If C and CP are exact symmetries, then one can show that the rate for any process which produces a baryon excess is equal to the rate of the complementary process generating antibaryons. Hence no net imbalance can be produced. CP violation can be implemented in a model either introducing complex phases in the Lagrangian which cannot be reabsorbed by field redefinition (explicit breaking), or if some Higgs field generates a complex vacuum expectation value (spontaneous breaking).

Of the three Sakharov conditions, the first two can be investigated only after a particle physics model is specified. The third one, the departure from thermal equilibrium, can be discussed in a more general way. The baryon number $B$ is odd under C and CP discrete transformations. Using this property of $B$ together with
the requirement that the Hamiltonian of the system commutes with the combination CPT, where $T$ here is the time-reversal discrete symmetry, the thermal average of $B$ reads

$$
\langle B \rangle_T \equiv \frac{1}{Z} \text{tr} \left[ e^{-\frac{H}{T}} B \right] = \frac{1}{Z} \text{tr} \left[ (CPT)(CPT)^{-1} e^{-\frac{H}{T}} B \right] \\
= \frac{1}{Z} \text{tr} \left[ e^{-\frac{H}{T}} (CPT)^{-1} B (CPT) \right] = -\frac{1}{Z} \text{tr} \left[ e^{-\frac{H}{T}} B \right] \\
= -\langle B \rangle_T .
$$

(2.28)

In (2.28) $H$ and $Z$ are the Hamiltonian and the partition function of the system respectively (see also (5.2) in Chap. 5 for more details on the partition function). Therefore we see that in thermal equilibrium $\langle B \rangle_T = 0$, and the same stands for the antibaryon number. The outcome is the following: if we start with a baryonic symmetric phase, processes in thermal equilibrium cannot alter the initial value for the baryon and antibaryon number and hence $\eta_B$ remains zero. Put in other words, processes generating a net baryon number are equally likely as those destroying it.

In order to illustrate the Sakharov conditions we choose a toy model, similar to the one in [32], that is inspired to GUTs. Baryon number violation occurs naturally in this class of models because quarks and leptons are embedded in the same irreducible representations. Then heavy gauge bosons and scalars are introduced that can mediate interactions leptons and quarks at the same vertex. The toy model consists of some exotic particles, the gauge bosons $\mathcal{X}$ and $\mathcal{Y}$, and four massless fermions, $f_1, \ldots, f_4$, each of the latter carrying a baryon number $B_1, \ldots, B_4$ respectively. The interaction Lagrangian of the toy model reads

$$
\mathcal{L}^{\text{toy int}} = g_1 \mathcal{X} \bar{f}_2 f_1 + g_2 \mathcal{X} \bar{f}_4 f_3 + g_3 \mathcal{Y} \bar{f}_1 f_3 + g_4 \mathcal{Y} \bar{f}_2 f_4 + h.c. ,
$$

(2.29)

where $g_1, \ldots, g_4$ are dimensionless complex coupling constants. The induced decay processes are

\[ \mathcal{X} \to \bar{f}_1 + f_2, \quad \mathcal{X} \to \bar{f}_3 + f_4, \]

(2.30)

\[ \mathcal{Y} \to \bar{f}_3 + f_1, \quad \mathcal{Y} \to \bar{f}_4 + f_2. \]

(2.31)

The tree level diagrams for the decay processes are shown in Fig. 2.2. Let us discuss the decay rates. At tree level we can parametrize the decay rate for the process $\mathcal{X} \to \bar{f}_1 + f_2$ as follows

$$
\Gamma^{(0)}(\mathcal{X} \to \bar{f}_1 + f_2) = |g_1|^2 A_{\mathcal{X}},
$$

(2.32)

where $A_{\mathcal{X}}$ contains the two-body phase space factor (the subscript stands for a decaying $\mathcal{X}$). For the charge conjugate process, that involves the particles $f_1$ and $\bar{f}_2$ in the final state, we have
Fig. 2.2 Tree level diagrams for the decay processes in (2.30) and (2.31). The heavy gauge bosons $X$ and $Y$ are the wiggled lines, solid lines stand for fermions. Similar diagrams for the charge conjugate processes are not shown.

$$\Gamma^{(0)}(\bar{X} \to f_1 + \bar{f}_2) = |g_1^*|^2 A_{\bar{X}} = |g_1|^2 A_X,$$

and we conclude that no asymmetry can be generated at tree level as the kinematic factors $A_X$ and $A_{\bar{X}}$ are equal. However the first Sakharov condition is already met: we start from a gauge boson with zero baryon number and we end up with a final state with a net baryon number $B_2 + \bar{B}_1 = B_2 - B_1$ for the first process in (2.30). Of course one has to require $B_1 \neq B_2$.

Clearly we have to go beyond tree level to obtain different rates for the decay of $X$. The one-loop diagrams describing the decay processes (2.30) are shown in Fig. 2.3, upper row. They are built by allowing for the exchange of a virtual heavy scalar $Y$. This time the decay width also comes from the interference between tree level and a one-loop amplitudes that give (we pick only the $O(g^4)$ terms)

$$\Gamma^{(1)}(X \to f_1 + f_2) = g_1 g_2^* g_3 g_4^* B_{X} + g_1^* g_2^* g_3^* g_4 B_X,$$

$$\Gamma^{(1)}(\bar{X} \to f_1 + \bar{f}_2) = g_1^* g_2 g_3^* g_4^* C_{\bar{X}} + g_1 g_2 g_3 g_4 C_X,$$

where $B_X$ and $C_{\bar{X}}$ comprise both the two-body phase space and the one-loop amplitude corresponding to the triangle topology in Fig. 2.3. In general the loop amplitude is a complex quantity, the imaginary part corresponding to the sum of the cuts that put
2.2 Dynamical Generation of the Baryon Asymmetry

\[ (a) \quad \chi' \rightarrow g_3^* f_1 \]

\[ (b) \quad \chi' \rightarrow g_3 f_3 \]

\[ (c) \quad \gamma \rightarrow g_4 f_1 \]

\[ (d) \quad \gamma' \rightarrow g_4^* f_4 \]

**Fig. 2.3** One-loop diagrams for the decay processes in (2.30) and (2.31). Similar diagrams for the charge conjugate processes can be drawn.

different particles simultaneously on shell. The explicit calculation gives \( B_{\chi'} = C_{\chi'} \).

We further elaborate the details of a very similar derivation in the case of leptogenesis in Chap. 7. Then we do find a non-vanishing difference in the decay rates

\[ \Gamma(\chi' \rightarrow f_1 + f_2) - \Gamma(\bar{\chi'} \rightarrow \bar{f}_1 + \bar{f}_2) = 4 \text{Im}(g_1 g_2^* g_3 g_4^*) \text{Im}(B_{\chi'}) , \]  

(2.36)

where the decay rates \( \Gamma(\chi' \rightarrow f_1 + f_2) \) and \( \Gamma(\bar{\chi'} \rightarrow \bar{f}_1 + \bar{f}_2) \) are understood as the sum of the tree-level and one-loop contributions as given in Eqs. (2.32) and (2.33), and in Eqs. (2.34) and (2.35) respectively. Similarly we have for the other decay mode the result

\[ \Gamma(\chi' \rightarrow f_3 + f_4) - \Gamma(\bar{\chi'} \rightarrow \bar{f}_3 + \bar{f}_4) = -4 \text{Im}(g_1 g_2^* g_3 g_4^*) \text{Im}(B_{\chi'}) , \]  

(2.37)

the derivation follows closely the one outlined. The loop amplitude in (2.37) is the same as in (2.36) because the very same particle content (the massless fermions and the intermediate gauge boson \( \gamma' \)) propagates in the triangle topologies \((a)\) and \((b)\) of Fig. 2.3. Besides the loop diagrams in the first raw in Fig. 2.3, one could also consider those with the \( \chi' \) as internal propagating gauge boson. However, this would lead to vanishing coupling combinations, such as \( \text{Im}(g_1 g_2^* g_3 g_4^*) = 0 \), and eventually provide a vanishing difference in (2.36) and (2.37). It is now clear how the second Sakharov condition enters: the decay rates for the process \( \chi' \rightarrow f_1 + f_2 \) and \( \bar{\chi'} \rightarrow \bar{f}_1 + \bar{f}_2 \) can be different due to the interference between tree-level and one-loop diagrams that involve C and CP violating processes. Moreover, there have to be two distinct heavy gauge bosons, coupling differently to the fermions and being heavier than the sum of the decaying products. The latter condition ensures the loop amplitude to have a non vanishing imaginary part, \( \text{Im}(B_{\chi'}) \).
The baryon asymmetry generated in the decays of the heavy gauge boson $\mathcal{X}$ can be expressed as follows

$$\epsilon_{\mathcal{X}} = \frac{(B_2 - B_1)\Delta \Gamma(\mathcal{X} \to \tilde{f}_1 + f_2) + (B_4 - B_3)\Gamma(\mathcal{X} \to \tilde{f}_3 + f_4)}{\Gamma_{\mathcal{X}}},$$

(2.38)

where we define

$$\Delta \Gamma(\mathcal{X} \to \tilde{f}_1 + f_2) = \Gamma(\mathcal{X} \to \tilde{f}_1 + f_2) - \Gamma(\tilde{\mathcal{X}} \to f_1 + \tilde{f}_2),$$

(2.39)

and the total width reads

$$\Gamma_{\mathcal{X}} = \Gamma(\mathcal{X} \to \tilde{f}_1 + f_2) + \Gamma(\tilde{\mathcal{X}} \to f_1 + \tilde{f}_2) + \Gamma(\mathcal{X} \to \tilde{f}_3 + f_4) + \Gamma(\tilde{\mathcal{X}} \to f_3 + \tilde{f}_4).$$

(2.41)

Finally by using the results in (2.36) and (2.37) we obtain for the baryon asymmetry generated in the $\mathcal{X}$ decays

$$\epsilon_{\mathcal{X}} = \frac{4}{\Gamma_{\mathcal{X}}} \left[ (B_2 - B_1) - (B_4 - B_3) \right] \text{Im}(g_1 g_2^* g_3 g_4^*) \text{Im}(B_{\mathcal{X}}),$$

(2.42)

where we remind that $B_{\mathcal{X}}$ is not the baryon number of the heavy gauge boson, but the factor containing the loop amplitude. In order to have a non vanishing baryon asymmetry (2.42), both the couplings combination and the loop amplitude have to be complex quantities.

It is interesting to note that the two heavy gauge bosons cannot be degenerate in mass. Indeed the baryon asymmetry for the $\mathcal{Y}$ heavy gauge boson reads

$$\epsilon_{\mathcal{Y}} = \frac{4}{\Gamma_{\mathcal{Y}}} \left[ (B_1 - B_3) - (B_2 - B_4) \right] \text{Im}(g_1 g_2^* g_3 g_4^*) \text{Im}(B'_{\mathcal{Y}}),$$

(2.43)

where $B'_{\mathcal{Y}}$ is the loop amplitude for a decaying $\mathcal{Y}$ boson. If the gauge bosons are mass degenerate then the condition $B_{\mathcal{X}} = B'_{\mathcal{Y}}$ holds, and then $\epsilon_{\mathcal{X}} + \epsilon_{\mathcal{Y}} = 0$ holds as well.

This is because the two-particle phase space is the same in the decay processes for $\mathcal{X}$ and $\mathcal{Y}$, and the only difference in the corresponding loop amplitudes is the mass of the heavy intermediate boson (see Fig. 2.3), whereas all the fermions are massless.

Now we come to the third Sakharov condition: the departure from thermal equilibrium. In this toy model such condition is achieved as follows. Let us consider the heavy boson $\mathcal{X}$. If the decay rate $\Gamma_{\mathcal{X}}$ is smaller than the expansion rate of the universe, the particles $\mathcal{X}$ cannot decay on the time scale of the universe expansion. Then the interactions governing the $\mathcal{X}$ dynamics are so weak that they cannot catch up with the expanding system and the heavy gauge bosons $\mathcal{X}$ decouple from the thermal plasma. If the decoupling occurs when the particles are still relativistic, namely
\(M_X < T\), the heavy bosons remain as abundant as photons, \(n_X \approx n_{\bar{X}} \propto T^3\) (see Eq. (2.23)), also at later times. Therefore, at time such that \(M_X \simeq T\), they populate the universe with an abundance much larger than the equilibrium one:

\[
n_X \approx n_{\bar{X}} \approx (M_X T)^{3/2} e^{-M_X/T} \ll n_{\gamma}, \tag{2.44}
\]

which holds for \(T \leq M_X\) and it is Boltzmann suppressed when \(M_X < T\). The heavy particles are more abundant than their corresponding equilibrium population at temperature below \(M_X\): this is exactly what out-of-equilibrium dynamics means in this class of models. In other words, the heavy gauge bosons generate the baryon asymmetry through their CP violating decays and the back reactions, the inverse decays, are exponentially suppressed because the massless fermions populate the thermal plasma with mean energies much smaller than the heavy states mass, \(M_X\).

In general the out-of-equilibrium condition requires the typical interaction rate for the gauge boson \(X\) to be

\[
\Gamma_X < H|_{T=M_X}, \tag{2.45}
\]

where \(H\) stands for the Hubble rate as given in (2.17). Evaluating \(H\) at \(T = M_X\), one can obtain from (2.45) a condition on the model parameters. The decay rate goes like \(\Gamma_X \sim |g_i|^2 M_X\) and if the couplings are taken as spanning from \(10^{-2}\) to \(10^{-3}\), and \(g_*\) is taken at about \(10^2\), we obtain [26]

\[
M_X > \left[10^{-4}, 10^{-3}\right] M_{Pl} \approx \left[10^{15}, 10^{16}\right] \text{GeV}. \tag{2.46}
\]

Such energy scale window sets the typical mass of the heavy states in GUT models, within which the first convincing realization for baryogenesis has been proposed [31]. Quite recently it has been suggested that the reheating temperature after the inflation cannot be higher than \(10^{15}\) GeV as accounted for the CMB analysis [33]. The thermal production of these heavy particles predicted by GUT models seems then seriously affected, undermining the very basis of such scenario for a successful baryogenesis.

### 2.2.2 Baryogenesis: A Call for New Physics

Baryogenesis can already be implemented in SM framework, however, there are severe limitations in providing a quantitative solution for the baryon asymmetry generation. Indeed in order to reproduce the experimental value in (2.26) some new physics is needed together with an interesting and challenging overlap between cosmology and particle physics. As anticipated before, the SM contains all the ingredients required by the Sakharov conditions. The following discussion will also help to set some important and relevant aspects for the topic in the next chapter: baryogenesis via leptogenesis.
Let us start with the baryon number violation in the electroweak theory. In the SM the baryon and lepton number, $B$ and $L$, are called *accidental symmetries*. They are individually conserved at tree level but are violated at quantum level via Adler-Bell-Jackiw triangular anomalies [34, 35]. More specifically in 1976 t’Hooft realized that non-perturbative effects [36], called instantons, may induce processes which violate the combination $(B + L)$ but conserve $(B - L)$. The probability for these processes to occur today in our universe is pretty much low, being exponentially suppressed. However, in the early stages of the universe evolution, namely at much higher temperatures, baryon and lepton number violation processes could occur more likely enough to play a role in baryogenesis. Let us express the baryon and lepton numbers as follows

\[
B = \int d^3x \, J^B_0(x), \quad L = \int d^3x \, J^L_0(x),
\]

where the currents read

\[
J^B_\mu = \frac{1}{3} \sum_i \left( \bar{Q}_i \gamma_\mu P_L Q_i + \bar{U}_i \gamma_\mu P_R U_i + \bar{D}_i \gamma_\mu P_R D_i \right),
\]

\[
J^L_\mu = \sum_i \left( \bar{L}_i \gamma_\mu P_L L_i + \bar{E}_i \gamma_\mu P_R E_i \right).
\]

Table 2.1  SM fermions and their baryon and lepton numbers for the first generation. The particles are given as SU(2)$_L$ doublets and singlets

<table>
<thead>
<tr>
<th></th>
<th>$Q_1 = \begin{pmatrix} u \ d \end{pmatrix}_L$</th>
<th>$u_R$</th>
<th>$d_R$</th>
<th>$L_1 = \begin{pmatrix} \nu_e \ e \end{pmatrix}_L$</th>
<th>$e_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$1/3$</td>
<td>$1/3$</td>
<td>$1/3$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$L$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

The fields $Q_i$ stand for the SU(2)$_L$ doublet quarks, $U_i$ and $D_i$ for the SU(2)$_L$ singlet quarks, then $L_i$ refers to the SU(2)$_L$ lepton doublets and $E_i$ for the SU(2)$_L$ lepton singlets. The left- and right-handed chiral projectors are $P_L = (1 - \gamma^5)/2$ and $P_R = (1 + \gamma^5)/2$, the index $i$ refers to the fermion generation. For example we have $E_1 = e$, $E_2 = \mu$ and $L_3 = (\nu_\tau, \tau)^T$ for leptons, $U_1$, $U_2$ and $U_3$ are the SU(2)$_L$ singlet up, charm and top quarks respectively. We summarize the $B$ and $L$ numbers in Table 2.1 for the first generation (they read the same for the second and third generation). The baryon and lepton number are classically conserved but the divergences of the currents in Eqs. (2.48) and (2.49) do not vanish at quantum level

\[
\partial^\mu J^B_\mu = \partial^\mu J^L_\mu = \frac{N_f}{32\pi^2} \left( g^2 W^a_{\mu\nu} \tilde{W}^{a,\mu\nu} - g^2 F_{\mu\nu} \tilde{F}^{\mu\nu} \right),
\]
where $W^a_{\mu\nu}$ and $F_{\mu\nu}$ are the SU(2)$_L$ and U(1)$_Y$ field strength tensors respectively, with corresponding gauge couplings $g$ and $g'$, and $N_f$ is the number of the fermion generations, $\tilde{W}^a_{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ the dual field strength tensors. From (2.50) it is clear that
\[ \partial\mu(J_B^\mu - J_L^\mu) = 0 , \] (2.51)
so that $(B - L)$ is conserved. On the other hand, the combination $(B + L)$ is violated and we have
\[ \partial\mu(J_B^\mu + J_L^\mu) = 2N_f \partial\mu K^\mu , \] (2.52)
with
\[
K^\mu = -\frac{g^2}{32\pi^2} 2\epsilon^{\mu\nu\rho\sigma} W^a_\nu (\partial_\rho W^a_\sigma + \frac{g}{3} \epsilon^{abc} W^b_\rho W^c_\sigma) \\
+ \frac{g'^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} B_\nu B_\rho B_\sigma .
\] (2.53)

It is important to notice that the violation of the current combination (2.52) is related to the vacuum structure of the electroweak theory. There are infinite degenerate ground states separated by a potential barrier as shown in Fig. 2.4, and a topological charge called Chern-Simons number, $N_{cs}$, is attached to each of the vacua. The change of the baryon (lepton) number with time can be then associated with the change in the Chern-Simons number, that is in turn due to a change from a vacuum state to another:
\[ \Delta B = B(t_f) - B(t_i) = N_f \left[ N_{cs}(t_f) - N_{cs}(t_i) \right] = N_f \Delta N_{cs} , \] (2.54)
where $t_i$ and $t_f$ are the initial and final time respectively and $N_f$ the number of fermion generations. Going from one ground state to another implies having $\Delta N_{cs} = \pm 1, \pm 2, ...$ as also shown in Fig. 2.4. In the SM there are three fermion generations,
so that $\Delta B = \Delta L = N_f \Delta N_{CS} = \pm 3n$, with $n$ a positive integer. That is to say that a vacuum to vacuum transition changes $\Delta B$ and $\Delta L$ by multiples of three units, and each transition generates 9 left-handed quarks (3 colors for each generation) and 3 left-handed leptons (one per generation).

In a semi-classical view, the probability of going to one vacuum state to another is determined by an *instanton* configuration. The transition rate has a very different form whether it is calculated at zero temperature or at finite temperature. In the former case, the probability of baryon and lepton non-conserving processes has been computed by t’Hooft and it is highly suppressed by a factor $e^{-4\pi/\alpha_W} \approx O(10^{-165})$ [36], where $\alpha_W = g^2 / (4\pi)$. The instantons do not threaten the stability of the proton [36]. In a thermal bath the situation may be quite different. It was suggested by Kuzmin, Rubakov and Shaposhnikov that transitions between vacua can be induced by thermal fluctuations of the electroweak field configurations [38]. So instead of tunnelling from one vacuum to another we may have a transition induced by thermal fluctuations over the barrier (see Fig. 2.4). In the case temperatures are larger than the typical barrier height the exponential $T = 0$ suppression is weakened and the $(B+L)$ violating processes may profuse and be in equilibrium in the expanding universe.

Finite temperature transitions among different ground states of the electroweak theory are governed by the sphaleron configurations which are static configurations corresponding to unstable solutions for the equation of motion of the theory [39]. The transition rate is quite different according to the corresponding temperature to be higher or lower than $T_W$, the temperature of the electroweak phase transition. In particular for $T < T_{EW}$ one finds the transition rate per unit volume [40]

$$\Gamma_{B+L} = \mu M_W^4 \left( \frac{M_W}{\alpha_W T} \right)^3 e^{-E_{sph} / (\alpha_W T)}, \quad (2.55)$$

where $M_W$ is the W boson mass, $\mu$ a constant of order one and $E_{sph} \equiv M_W(T) / \alpha_W$ is the sphaleron energy. The latter is temperature dependent through the finite temperature expectation value of the Higgs boson. The rate is still pretty much suppressed at temperatures below the electroweak scale. However the exponential suppression is expected to vanish when the electroweak symmetry is restored. In the symmetric phase, $T > T_{EW}$ the same rate has been found to be [41]

$$\frac{\Gamma_{B+L}}{V} \sim \alpha_5^5 T^4 \ln \frac{1}{\alpha_W} \quad . \quad (2.56)$$

Hence at temperature of order $T \sim 10^2$ GeV, the baryon number violating processes are not suppressed and are in equilibrium up to temperature of order $O(10^{12})$ GeV [42]. The first Sakharov condition is satisfied in the early universe already within the SM.
Let us come to the C and CP violation in the SM. It is known that C is maximally violated since only left-handed fermions couple to the SU(2) gauge fields. The CP violation was observed in the quark sector, more specifically in strange and beauty mesons decays [43–45]. Then the second Sakharov condition is also fulfilled. However the CP phases provided within the quark sector are far too small to account for $\eta_B \sim \mathcal{O}(10^{-10})$. In short, the only CP phase in the SM originates in the CKM matrix, connecting the mass and interaction (electroweak) eigenstates of the left-handed quarks [46]. There is a more quantitative way to express the amount of CP violation by means of the Jarlskog invariant that comes out to be $J \sim \mathcal{O}(10^{-20})$ [47]. Being not present any significant enhancement of the baryon asymmetry due to processes within the SM in the early universe [48, 49], it seems impossible to fill the many orders of magnitude gap to reproduce the baryon-to-photon ratio in Eq. (2.26).

Let us come to the third Sakharov condition. The departure from thermal equilibrium in the SM is provided by the electroweak phase transition. This mechanism gives the name to a class of models, which the SM belongs to, that provides the generation of the baryon asymmetry: *electroweak baryogenesis*. However in order to provide a sufficient deviation from equilibrium, the electroweak transition is required to be strongly first order and this sets a severe bound on the Higgs mass, $m_\phi \leq 72$ GeV [50]. Thus, viable models of electroweak baryogenesis need a modification of the scalar potential such that the nature of the electroweak phase transition is modified, together with new sources of CP violation (for example see [51, 52]).

In summary, despite the Sakharov conditions are comprised in the SM, we cannot achieve a successful baryogenesis. Additional sources of CP violation are invoked, together with some alternative mechanism for a strong enough departure from thermal equilibrium: the generation of the observed baryon asymmetry requires some new physics. Besides GUT baryogenesis, briefly discussed in the toy model in Sect. 2.2, alternatives comprise Affleck–Dine mechanism [30] and spontaneous baryogenesis [53]. Another interesting and appealing framework is baryogenesis via leptogenesis [54] (see Chap. 3). In this class of models an asymmetry is generated in the leptonic sector. Then due to the connection between baryon and lepton number provided by the sphaleron transitions, the lepton asymmetry is partially reprocessed into a baryon one. We already set the basis for leptogenesis discussing the toy model for GUT baryogenesis. Indeed new heavy states are added to the SM particle content: in its original formulation, heavy neutrinos with a large Majorana mass. In the following we discuss how baryon and lepton asymmetries can be related to each other.

### 2.2.3 Relating Baryon and Lepton Asymmetries

In this section we deal with the relation between baryon and lepton number at high temperatures. Beside being an interesting application of sphaleron transitions and equilibrium dynamics, such discussion introduces a fundamental ingredient for
leptogenesis. Our aim is to show that a matter-antimatter imbalance stored in the baryon sector implies a lepton asymmetry and vice versa. In the present discussion we stick to the SM particle content and the derivation follows the one given in [37, 55].

Let us consider a weakly coupled plasma at temperature $T$. We can assign a chemical potential $\mu_i$ to each of the quarks, leptons and Higgs field in the heat bath. Since there are left-handed lepton and quark SU(2) doublets, right-handed quarks and lepton SU(2) singlets (see Table 2.1) and one Higgs doublet, we can assign $5N_f + 1$ chemical potentials, where $N_f$ stands for the number of fermion generations. If we consider the degrees of freedom in the thermal bath as massless, the asymmetries in the number densities of particle and antiparticles read

$$n_i - \bar{n}_i = \frac{g_i T^3}{6} \begin{cases} \beta \mu_i + \mathcal{O}((\beta \mu_i)^3), \\ 2\beta \mu_i + \mathcal{O}((\beta \mu_i)^3), \end{cases} \quad (2.57)$$

where the first line holds for fermions, whereas the latter for bosons and $g_i$ stands for the internal degrees of freedom of the particle (antiparticle). The key observation is that one can deduce the particle-antiparticle asymmetries from the chemical potentials. We can find some relations among the chemical potentials of the different particles participating the interactions in the early universe [56]. Quarks, leptons and Higgs bosons interact via Yukawa and gauge couplings and, in addition, via the non-perturbative sphaleron processes. In thermal equilibrium all these processes yield constraints between the various chemical potentials. The effective 12-fermion interactions induced by the sphalerons lead to

$$\sum_i (3\mu_{Q_i} + \mu_{L_i}) = 0. \quad (2.58)$$

where the sum runs over the quark and lepton generations (the meaning of the index is the same as given in Sect. 2.2.2). The SU(3) QCD instanton processes [57], which generate an effective interaction between left- and right-handed quarks, provide the following relation

$$\sum_i (2\mu_{Q_i} - \mu_{U_i} - \mu_{D_i}) = 0. \quad (2.59)$$

A third condition, valid at all temperatures, is obtained by requiring that the total hypercharge of the plasma vanishes. From Eq. (2.57) and the known hypercharges one derives

$$\sum_i \left( \mu_{Q_i} + 2\mu_{U_i} - \mu_{D_i} - \mu_{L_i} - \mu_{E_i} + \frac{2}{N_f} \mu_{\phi} \right) = 0, \quad (2.60)$$
where \( \mu_\phi \) is the chemical potential of the Higgs doublet (all the components have the same chemical potential). The Yukawa interactions yield relations between the chemical potentials of left-handed and right-handed fermions (with different flavours)

\[
\mu_{Q_i} - \mu_{D_j} - \mu_\phi = 0, \quad \mu_{Q_i} - \mu_{U_j} + \mu_\phi = 0, \quad \mu_{L_i} - \mu_{E_j} - \mu_\phi = 0.
\]

The relations (2.58)–(2.61) hold if the corresponding interactions are in thermal equilibrium. In the temperature range \( 10^2 \text{ GeV} < T < 10^{12} \text{ GeV} \), gauge interactions are in equilibrium. On the other hand, Yukawa interactions are in equilibrium in a more restricted temperature range that depends on the strength of the Yukawa couplings [56]. We ignore this slight complication in the present discussion.

We define the baryon- and lepton-asymmetries number density as follows according to (2.57)

\[
n_{\Delta B} = \frac{g_B}{6} \Delta B T^2, \quad n_{\Delta L} = \frac{g_L}{6} \Delta L T^2,
\]

with

\[
\Delta B = \sum_i (2\mu_{Q_i} + \mu_{U_i} + \mu_{D_i}),
\]

\[
\Delta L = \sum_i (2\mu_{L_i} + \mu_{E_i}),
\]

and we assume that the asymmetry in each generation is the same, e.g. \( \mu_{L_e} = \mu_{L_\mu} = \mu_{L_\tau} \equiv \mu_L \). Then \( g_B \) and \( g_L \) are the degrees of freedom of the baryons and leptons. The relations (2.58)–(2.61) can be solved them in terms of a single chemical potential. If one takes \( \mu_L \) the baryon and lepton asymmetries are found to be [55]

\[
\Delta B = -\frac{4}{3} N_f \mu_L,
\]

\[
\Delta L = \frac{14N_f^2 + 9N_f}{6N_f + 3} \mu_L.
\]

This implies the important connection between the \( \Delta B, \Delta(B-L) \) and \( \Delta L \) asymmetries, that reads [58]

\[
\Delta B = c_s \Delta(B-L),
\]

\[
\Delta L = (c_s - 1) \Delta(B-L),
\]

with

\[
c_s = \frac{8N_f + 4}{22N_f + 13}.
\]

Looking at (2.67) one finds that, in order to have a baryon asymmetry, \( B - L \) violating interactions have to occur in the early universe. Moreover, since the \( B - L \)
combination is conserved by sphaleron interactions, the baryon asymmetry today is the same as the one present at the freeze-out of the sphaleron processes. There is another way to look at the relations (2.67) and (2.68). An asymmetry generated in the lepton sector induces automatically a baryon asymmetry when sphalerons are in equilibrium:

$$\Delta B = \frac{c_s}{c_s - 1} \Delta L.$$  (2.70)

A baryon asymmetry can be achieved also in those models where only lepton number is violated. This welcome the possibility to explain the generation of a matter-antimatter imbalance via lepton violating processes, namely baryogenesis via leptogenesis.

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Effective Field Theories for Heavy Majorana Neutrinos in a Thermal Bath
Biondini, S.
2017, XIII, 215 p. 86 illus., 65 illus. in color., Hardcover
ISBN: 978-3-319-63900-0