Chapter 2
Definition of the Plasma State

"I can't tell you just now what the moral of that is, but I shall remember it in a bit."
"Perhaps it hasn't one", Alice ventured to remark.
"Tut, tut, child!" said the Duchess, "Everything's got a moral, if only you can find it."

Lewis Carroll, Alice in Wonderland

The plasma state is a gaseous mixture of positive ions and electrons. Plasmas can be fully ionized, as the plasma in the Sun, or partially ionized, as in fluorescent lamps, which contain a large number of neutral atoms. In this section we will discuss the defining qualities of the plasma state, which result from the fact that we have a huge number of charged particles that interact by electric forces. In particular we will see that the plasma state is able to react in a *collective* manner. Therefore, the plasma medium is more than the sum of its constituents.

2.1 States of Matter

Before going deeper into definitions of the plasma state, let us recall the characteristic properties of a neutral gas. A gas is characterized by the number of particles per unit volume, which we call the *number density* n. The unit of n is m^{-3}. The motion of the particles (in thermodynamic equilibrium) is determined by the *temperature* T of the gas. In an ideal gas, the product of number density and temperature gives the pressure, $p = nk_{\mathrm{B}}T$, in which k_{B} is Boltzmann's constant.

We will use the same terminology for plasmas, but in the plasma state we have a mixture of two different gases, light electrons and heavy ions. Therefore, we have

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to distinguish the electron and ion gas by individual densities, n_e and n_i. Moreover, plasmas are often in a non-equilibrium state with different temperatures, T_e and T_i of electrons and ions. Such two-temperature plasmas are typically found in gas discharges. The solar plasma (in the interior and photosphere), on the other hand, is a good example for an isothermal plasma with $T_e = T_i$.

Plasmas exist in an environment that provides for a large number of ionization processes of atoms. These can be photoionization by an intense source of ultraviolet radiation or collisional ionization by energetic electrons. Impact ionization is the dominant process in gas discharges because of the ample supply of energetic electrons. Photoionization is found in space plasmas where the electron and atom densities are low but a large number of ultraviolet (UV) photons may be present. These processes and their reciprocal processes can be written in terms of simple reaction equations, as summarized in Table 2.1.

Besides recombination by these volume processes, electrons and ions can effectively recombine at surfaces, which may be the walls of discharges or embedded microparticles. In thermodynamic equilibrium, each of these volume processes is balanced by the corresponding reciprocal process (i.e., photoionization and two-body recombination, or impact ionization and three-body recombination.) Because the ionization energy of neutral atoms lies between 3 and 25 eV, plasmas produced by impact ionization typically exist at high temperatures. Photoionized plasmas require short wavelength radiation, typically in the UV region. There are also situations, in hot and dilute plasmas, where collisional ionization is efficient but electrons are too few for three-body recombination. Then, a steady state can be reached, in which two-body recombination balances the impact ionization. The solar corona is an example for such a plasma that is in a non-thermodynamic equilibrium.

As a final remark, it is worth mentioning that some plasmas are not governed by local equilibria but by non-local processes. The properties of the solar wind at the Earth orbit, for example, are mostly determined by the emission process at the Sun's surface and by heating processes (e.g., shocks) during the propagation from Sun to Earth. We will see in Chap. 11 that a negative glow is also produced by electrons that have gained their energy at a different place.

Table 2.1 Ionization and recombination processes

$\mathrm{e} + \mathrm{A} \rightarrow \mathrm{A}^+ + 2\mathrm{e}$	Collisional ionization
$\mathrm{h}\nu + \mathrm{A} \rightarrow \mathrm{A}^+ + \mathrm{e}$	Photoionization
$\mathrm{A}^+ + 2\mathrm{e} \rightarrow \mathrm{A} + \mathrm{e}$	Three-body recombination
$\mathrm{A}^+ + \mathrm{e} \rightarrow \mathrm{A} + \mathrm{h}\nu$	Two-body recombination

2.1.1 The Boltzmann Distribution

Before we discuss the thermodynamic equilibrium of a plasma in more detail, it is meaningful to recall some elementary concepts of classical statistical mechanics. There, the relative population of different energy states is regulated by the Boltzmann factor. The relative population of the energy states W_i and W_k[1] is given by

$$\frac{n_i}{n_k} = \frac{g_i}{g_k} \exp\left(-\frac{W_i - W_k}{k_\mathrm{B} T}\right) . \tag{2.1}$$

g_i and g_k are the degeneracies of the states i and k, i.e., the number of substates with the same energy. The exponential of the form $\exp(-W/k_\mathrm{B}T)$ determines how many atoms have overcome the energy barrier W_i–W_k between the states i and k. Another example for a Boltzmann distribution is the Maxwell–Boltzmann velocity distribution of free particles

$$f_\mathrm{M}(v_x, v_y, v_z) = \frac{1}{Z} \exp\left(-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2\,k_\mathrm{B} T}\right) . \tag{2.2}$$

Z is a normalization factor. Here, the distribution of velocities is determined by the kinetic energy $W = m(v_x^2 + v_y^2 + v_z^2)/2$. The Maxwell distribution will be discussed in more detail in Sect. 4.1.

2.1.1.1 Derivation of the Boltzmann Distribution

The derivation of the Boltzmann distribution from statistical mechanics is given here for completeness. This paragraph may be skipped at first reading of this section.

We start with the concept of entropy, which attains a maximum value in a thermodynamic equilibrium. Already in 1866, the Austrian physicist Ludwig Boltzmann (1844–1906) introduced the logarithmic dependence of entropy on probability. The entropy of a classical system of N particles, having a total energy U, which can populate its different energy states W_i with N_i particles, is defined by

$$S = -k_\mathrm{B} \sum_i n_i \ln n_i . \tag{2.3}$$

Here, $n_i = N_i/N$ is the relative population of the energy state W_i. Letting S take a maximum value, we must take care of the constraining conditions

$$g(n_i) = \sum_i n_i W_i = U \quad \text{and} \quad h(n_i) = \sum_i n_i = 1 . \tag{2.4}$$

[1] To avoid confusion with the electric field E we denote energies by the symbol W.

A maximum with constraints is found by the method of Lagrange multipliers, which requires

$$\frac{\partial S}{\partial n_i} = \lambda \frac{\partial g}{\partial n_i} + \mu \frac{\partial h}{\partial n_i} \ . \tag{2.5}$$

Herefrom we immediately obtain

$$\begin{aligned} -\ln n_i - 1 &= \lambda W_i + \mu \\ n_i &= \exp\{-\mu - 1 - \lambda W_i\} \ . \end{aligned} \tag{2.6}$$

The two Lagrange-multipliers λ and μ are determined using the constraint $h = 1$

$$1 = \mathrm{e}^{-\mu-1} \sum_k \mathrm{e}^{-\lambda W_k} \ , \tag{2.7}$$

which gives

$$\mathrm{e}^{-\mu-1} = \left(\sum_k \mathrm{e}^{-\lambda W_k} \right)^{-1} \tag{2.8}$$

and finally

$$n_i = \frac{1}{Z} \mathrm{e}^{-\lambda W_i} \quad \text{and} \quad Z = \sum_k \mathrm{e}^{-\lambda W_k} \ . \tag{2.9}$$

The normalizing factor Z is called the partition function. The other Lagrange multiplier is found from the thermodynamic relationship $1/T = \partial S/\partial W$ and yields $\lambda = (k_\mathrm{B} T)^{-1}$ (cf. Problem 2.6). Then the relative population of the energy states is given by the Boltzmann distribution

$$n_i = \frac{1}{Z} \exp\left(-\frac{W_i}{k_\mathrm{B} T} \right) \ . \tag{2.10}$$

The exponential $\exp(-W_i/k_B T)$ is the *Boltzmann factor* and the Boltzmann distribution over energy states (2.1) follows immediately.

2.1.2 The Saha Equation

The Boltzmann factor (2.1) describes the distribution of the internal states of an atom or the free states of the Maxwell–Boltzmann gas. Now we seek for a thermodynamic description of the equilibrium between atoms and ions.

Thermal equilibrium conditions of a plasma are typically found in the interior of stars or in the electric arc discharges used for street and stadium illumination. The thermodynamic equilibrium state is characterized by the detailed balancing of each process with its reciprocal process. Here we consider the balance of electron impact ionization and three-body recombination

$$\mathrm{e} + \mathrm{A} \rightleftharpoons \mathrm{A}^{+} + 2\mathrm{e} \;, \tag{2.11}$$

which can be quantified by the balance of the reaction rates

$$n_{\mathrm{e}}\, n_{\mathrm{A}}\, S(T) = n_{\mathrm{e}}^{2}\, n_{\mathrm{A}^{+}}\, R(T) \;. \tag{2.12}$$

Here, n_{e} is the electron density, n_{A} the neutral atom density and $n_{\mathrm{A}^{+}}$ the ion density. The coefficients, $S(T)$ for ionization and $R(T)$ for three-body recombination are only dependent on temperature. (Rate coefficients will be discussed in more detail in Sect. 4.2.3.) Therefore, (2.12) can be rearranged into a *mass action law*

$$\begin{aligned} \frac{n_{\mathrm{e}}\, n_{\mathrm{A}^{+}}}{n_{\mathrm{A}}} &= \frac{S(T)}{R(T)} =: f_{\mathrm{Saha}}(T) \\ f_{\mathrm{Saha}}(T) &= \frac{2 Z_{\mathrm{A}^{+}}}{Z_{\mathrm{A}}} \exp\left(-\frac{W_{\mathrm{ion}}}{k_{\mathrm{B}} T}\right) \;. \end{aligned} \tag{2.13}$$

The function f_{Saha} was derived in 1920 by the Indian astrophysicist Megh Nad Saha (1893–1956) [57]. It contains the exponential $\exp(-W_{\mathrm{ion}}/k_{\mathrm{B}}T)$ that determines the probability to overcome the energy barrier W_{ion} and the *partition functions* $Z_{\mathrm{A}} = \sum_k g_k \exp(-W_k/k_{\mathrm{B}}T)$ of the atom [cf. (2.9)] and $Z_{\mathrm{A}^{+}}$ of the ion. The factor 2 is the degeneracy of the free electrons, which have two distinguishable spin states. The exponential in the Saha function has obvious similarities with the Boltzmann factor.

The Saha-equilibria between different ionization stages of an atom can be described in a similar manner. An example for the resulting ionization states of a free-burning argon arc discharge is shown in Fig. 2.1. Because this electric arc is operated in pressure equilibrium with the ambient air, the calculation was performed at constant pressure rather than at constant atom number. Ionization reaches a few percent at $T > 10{,}000\,\mathrm{K}$ and full single ionization is established at $\approx 20{,}000\,\mathrm{K}$, where also the onset of double-ionization A^{++} is observed. Converting temperature to energy units $k_{\mathrm{B}}T$, the onset of ionization in argon occurs at about 1 eV (corresponding to 11,600 K, see Sect. 4.1.3).

The high operating temperature of an arc discharge lamp leads to a high energy efficiency, because the maximum of the Planck curve for black-body radiation, as given by Wien's displacement law

$$\lambda_{\max}(\mathrm{nm}) = \frac{2.898 \times 10^{6}}{T(K)} \;, \tag{2.14}$$

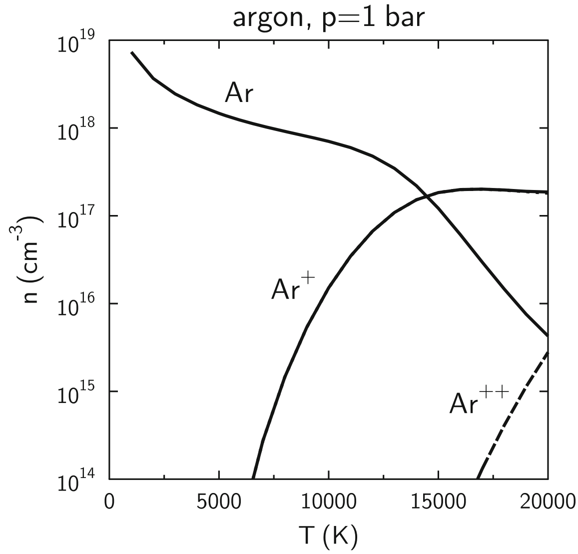

Fig. 2.1 Ionization states of an argon plasma in thermodynamic equilibrium at constant pressure calculated from Saha's equation

shifts to the blue end of the visible spectrum (414 nm at $T = 7000\,\mathrm{K}$). Arc discharge lamps used in data projectors have a sealed quartz discharge tube and develop operating pressures of (50–200) bar.

2.1.3 The Coupling Parameter

In the preceding paragraphs we have investigated how a rising temperature leads to population of excited atomic states, to ionization of atoms, and to equilibria with multiply ionized atoms. Now we focus our interest on the influence of particle density on the state of the plasma system. Then, the potential energy of the interacting particles becomes important.

The states of neutral matter, solid–liquid–gaseous, are determined by the degree of coupling between the atoms, which is described by the *coupling parameter* $\Gamma = W_{\mathrm{pot}}/k_{\mathrm{B}}T$, i.e., the ratio of the potential energy of nearest neighbors and the thermal energy. For the Coulomb interaction of singly charged ions, the coupling parameter becomes

$$\Gamma_{\mathrm{i}} = \frac{e^2}{4\pi\varepsilon_0 a_{\mathrm{WS}} k_{\mathrm{B}} T_{\mathrm{i}}} \,. \tag{2.15}$$

Here, a_{WS} is the Wigner–Seitz radius, a measure for the interparticle distance, defined by

$$n_\mathrm{i}\frac{4\pi}{3}\,a_\mathrm{WS}^3 = 1\ . \tag{2.16}$$

A similar coupling parameter can be defined for the interaction of the electrons or the interaction between electrons and ions. To give typical orders of magnitude for Γ, we can state that a gaseous state has $\Gamma < 1$ and is said to be weakly coupled. The liquid state is found between $1 < \Gamma < 180$. The solid phase exists at $\Gamma > 180$. Liquid and solid phase are called strongly coupled. The exact numbers depend on the system dimension and on the interaction with the electrons. Hence, a plasma is not necessarily in a gaseous state. Strongly coupled plasmas can behave like liquids or can even crystallize. Examples for the crystallization of a subsystem will be given in Chap. 10 on dusty plasmas.

2.2 Collective Behavior of a Plasma

What is the difference between a neutral gas and a plasma? In a neutral gas, particles interact only during a collision, i.e., when two gas atoms "feel" the short-range van der Waals force, which decays with the interparticle distance as r^{-6}. For most of the time, the gas atoms fly on a straight path independent of the other atoms. This is quite different in a plasma. The Coulomb force that describes the electrostatic interaction decays only slowly as r^{-2}, which makes it a long-range force. This means that each plasma particle interacts with a large number of other particles. Therefore, plasmas show a simultaneous response of many particles to an external stimulus. In this sense, plasmas show *collective behavior*, which means that the macroscopic result to an external stimulus is the cooperative response of many plasma particles. Mutual shielding of plasma particles or wave processes are examples of collective behavior.

2.2.1 Debye Shielding

The most important feature of a plasma is its ability to reduce electric fields very effectively. We can discuss this effect of *shielding* by placing a point-like extra charge $+Q$ into an infinitely large homogeneous plasma, which originally has equal densities of electrons and singly charged positive ions $n_{\mathrm{e}0} = n_{\mathrm{i}0}$. Let us assume that this extra charge $+Q$ is located at the origin of the coordinate system. We expect that electrons will be attracted and ions repelled by this extra charge as sketched in Fig. 2.2a. This gives rise to a net space charge in the vicinity of $+Q$, which tends to weaken the electric field generated by $+Q$.

The bending of the trajectory depends on the particle energy. The higher the energy of the electrons (ions) is, i.e., the higher the temperature of the electron (ion) gas is, the stiffer the trajectory becomes, as indicated in Fig. 2.2b. Therefore, a cold species of particles will be very effective in shielding the extra charge and we can conjecture

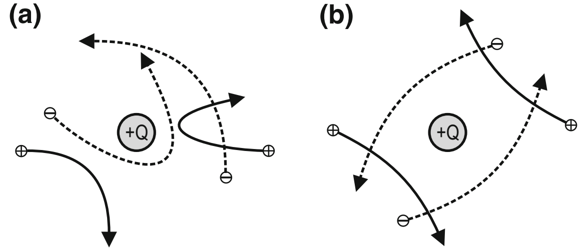

Fig. 2.2 **a** Shielding arises from a net attraction of electrons and repulsion of positive ions, leading to trajectory bending. **b** For higher energy the trajectories become stiffer and the shielding less efficient

that the size of the perturbed region is small, whereas the perturbation has a greater range for hotter electrons (ions).

Obviously, this shielding process is not static, but is governed by the thermal motion of the plasma electrons and ions. Therefore, we need a simple statistical description, for which we use the Boltzmann factor. For a quantitative description, we calculate the number of electrons and ions that are found at an enhanced electric potential in the vicinity of $+Q$. For a repulsive potential, the Boltzmann factor (2.1) gives the number of particles in a thermal distribution that have overcome a potential barrier Φ. For an attractive potential, the density can even become higher than the equilibrium value:

$$\begin{aligned} n_\mathrm{e}(\mathbf{r}) &= n_{e0} \exp\left(+\frac{e\Phi(\mathbf{r})}{k_\mathrm{B}T_\mathrm{e}}\right) \\ n_\mathrm{i}(\mathbf{r}) &= n_{i0} \exp\left(-\frac{e\Phi(\mathbf{r})}{k_\mathrm{B}T_\mathrm{i}}\right) \; . \end{aligned} \tag{2.17}$$

For simplicity, we assume that the perturbed potential Φ is small compared to the thermal energy, which allows us to expand the exponential and use only the first term in the Taylor-expansion

$$\begin{aligned} n_\mathrm{e}(\mathbf{r}) &\approx n_\mathrm{e0} \left(1 + \frac{e\Phi(\mathbf{r})}{k_\mathrm{B}T_\mathrm{e}}\right) \\ n_\mathrm{i}(\mathbf{r}) &\approx n_\mathrm{i0} \left(1 - \frac{e\Phi(\mathbf{r})}{k_\mathrm{B}T_\mathrm{i}}\right) \; . \end{aligned} \tag{2.18}$$

A self-consistent solution for the electric potential can then be obtained by using Poisson's equation

$$\Delta\Phi = -\frac{\rho}{\varepsilon_0} \; , \tag{2.19}$$

which links the electric potential Φ to the space charge distribution ρ. In the considered situation, the space charge consists of the additional point charge Q and the electron and ion contributions, yielding

$$\Delta\Phi = -\frac{\rho}{\varepsilon_0} = -\frac{1}{\varepsilon_0}\left[Q\delta(\mathbf{r}) - en_\mathrm{e}(\mathbf{r}) + en_\mathrm{i}(\mathbf{r})\right] \ . \tag{2.20}$$

Inserting the linearized densities from (2.18)

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{2}{r}\frac{\partial\Phi}{\partial r} = -\frac{1}{\epsilon_0}\left[Q\delta(r) - en_{\mathrm{e}0}\frac{e\Phi}{k_\mathrm{B}T_\mathrm{e}} - en_{\mathrm{i}0}\frac{e\Phi}{k_\mathrm{B}T_\mathrm{i}}\right] . \tag{2.21}$$

On the l.h.s. of this equation we have used the spherical symmetry of the problem, which makes Φ independent of angular variables. On the r.h.s. we have used the assumed neutrality of the unperturbed system, $n_{\mathrm{e}0} = n_{\mathrm{i}0}$. Rearranging all contributions that contain Φ to the left side, we obtain a differential equation of the Helmholtz type

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{2}{r}\frac{\partial\Phi}{\partial r} - \frac{1}{\lambda_\mathrm{D}^2}\Phi = -\frac{Q}{\epsilon_0}\,\delta(r) \ . \tag{2.22}$$

The parameter λ_D has the dimension of a length and is defined by

$$\frac{1}{\lambda_\mathrm{D}^2} = \frac{e^2 n_{\mathrm{e}0}}{\varepsilon_0 k_\mathrm{B} T_\mathrm{e}} + \frac{e^2 n_{\mathrm{i}0}}{\varepsilon_0 k_\mathrm{B} T_\mathrm{i}} \ . \tag{2.23}$$

The differential equation (2.22) can be solved by assuming that the potential distribution is given by a modified Coulomb potential

$$\Phi(r) = \frac{a}{r}\,f(r) \ . \tag{2.24}$$

Then, for all $r > 0$, the function $f(r)$ is a solution of the differential equation

$$f'' - \lambda_\mathrm{D}^{-2} f = 0 \ , \tag{2.25}$$

which gives $f_1(r) = \exp(-r/\lambda_D)$. A second solution, $f_2(r) = \exp(+r/\lambda_\mathrm{D})$, is unphysical because the perturbed field would increase indefinitely with distance r. The normalization constant a is obtained by applying Gauss' theorem to a small sphere around the origin

$$\oint \mathbf{D}\cdot \mathrm{d}\mathbf{A} = 4\pi r^2 \varepsilon_0 E_r = Q \ . \tag{2.26}$$

Here we have assumed that for $r/\lambda_\mathrm{D} \to 0$ the sphere only contains the extra charge Q but no space charge from the perturbed distributions of electrons and ions. From (2.24) we obtain

$$E_r = \frac{a}{r^2}\left(1 + \frac{r}{\lambda_\mathrm{D}}\right)\mathrm{e}^{-r/\lambda_\mathrm{D}} \to \frac{a}{r^2} \ . \tag{2.27}$$

Hence, the normalization a is the same as for the Coulomb potential $a = Q\,(4\pi\varepsilon_0)^{-1}$. The complete solution

$$\Phi(r) = \frac{Q}{4\pi\varepsilon_0\, r}\, \mathrm{e}^{-r/\lambda_\mathrm{D}} \tag{2.28}$$

is called the *Debye–Hückel potential* after pioneering work of Pieter Debye (1884–1966) and Erich Hückel (1896–1980) on polarization effects in electrolytes [58]. A similar shielded Coulomb potential was later found in nuclear physics for interactions mediated by the exchange of a finite-mass particle like the pion by Nobel prize winner Hideki Yukawa (1907–1981). In classical weakly-coupled plasmas, the standard terminology is *Debye shielding* whereas the younger literature on strongly-coupled systems has a preference for *Yukawa-interaction*, a terminology also used in colloid science.

The parameter λ_D is the *Debye shielding length*, which describes the combined shielding action of electrons and ions. When we are interested in the individual contributions of electrons and ions we can define the *electron Debye length* λ_De and the *ion Debye length* λ_Di separately,

$$\lambda_\mathrm{De} = \left(\frac{\varepsilon_0 k_\mathrm{B} T_\mathrm{e}}{n_\mathrm{e0} e^2}\right)^{1/2} \qquad \lambda_\mathrm{Di} = \left(\frac{\varepsilon_0 k_\mathrm{B} T_\mathrm{i}}{n_\mathrm{i0} e^2}\right)^{1/2} . \tag{2.29}$$

Inspecting the dependence of the electron (ion) Debye length on temperature, we see that our initial conjecture is confirmed. The shielding length increases when the temperature rises, i.e., the size of the perturbed region becomes larger. The dependence on density $\propto n_\mathrm{e0}^{-1/2}$ $(n_\mathrm{i0}^{-1/2})$ means that an increasing number of shielding particles makes the shielding more efficient and diminishes the size of the perturbed volume.

Yet, why is the Debye length independent of the particle mass? We can gain insight into this property from a mathematically simpler situation, in which the electric field is homogeneous. Consider the cathode ray tube of a traditional oscilloscope. There, an electron of mass m, charge q and energy W enters the space between two deflection plates of length L (Fig. 2.3) and performs a free-fall motion in the electric field. Hence, the trajectory in the space between the deflection plates is a parabola.

The initial velocity is $v = (2W/m)^{1/2}$ and the electron needs a transit time $\tau = L/v$ to traverse the deflection plates. In this time, it has fallen a distance

$$s = \frac{qE}{2m}\tau^2 = \frac{qEL^2}{4W} , \tag{2.30}$$

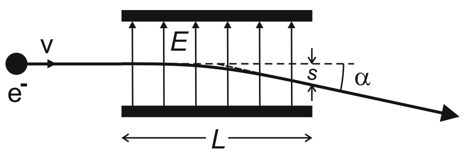

Fig. 2.3 Deflection of an electron beam by a transverse electric field E in a traditional cathode ray oscilloscope tube

which is independent of the mass m but depends on the energy W. Noting that the tangent to the parabola at the exit point intersects the undeflected orbit at $x = L/2$, the deflection angle becomes $\alpha = \arctan(2s/L)$. This independence of mass is the reason why a transverse electric field can be used as an *energy filter* to sort out particles of same energy independent of their mass.

The shielding length λ_D is often called the *linearized Debye length*. The reader may also recognize an analogy between the structure of (2.23) and the parallel circuit of two resistors in electricity:

$$\frac{1}{\lambda_D^2} = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2} \quad \leftrightarrow \quad \frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} . \tag{2.31}$$

In the shielding process, electrons and ions work in parallel. Attracting electrons and repelling ions both results in a net negative charge in the vicinity of the extra charge. Similar to the total resistance R_{total} of the parallel circuit, which is smaller than any of the two resistors R_1 and R_2, the linearized Debye length is smaller than λ_{De} and λ_{Di}. A comparison between a Coulomb potential and a shielded potential is shown in Fig. 2.4. For $r > \lambda_D$, the potential decays much faster than a Coulomb potential.

Summarizing, the perturbed electric potential around an extra charge Q decays exponentially for $r > \lambda_D$. This observation has two consequences. When we require that a cloud of electrons and ions behaves as a plasma, the cloud must have a size of several Debye lengths. Moreover, any deviation from equal densities of electrons and ions tends to be smoothed by Debye shielding. Therefore, a plasma has the natural tendency to become quasineutral.

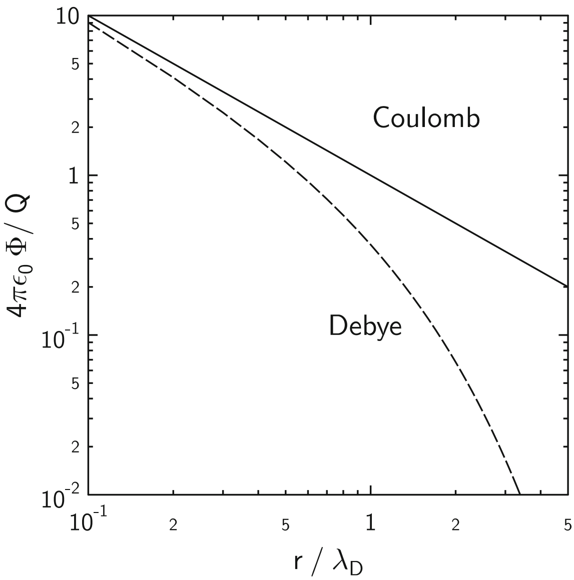

Fig. 2.4 Comparison of a Coulomb and shielded (Debye–Hückel or Yukawa) potential. Note the stronger decay for $r/\lambda_D > 1$

2.2.2 Quasineutrality

As we have learned in the preceding paragraph, a plasma is not a strictly neutral mixture of electrons and positive ions but deviations from neutrality can develop on short scales, the Debye length. Therefore, we define that on length scales larger than the Debye length the plasma must be quasineutral

$$\left| \sum_j Z_j \, e \, n_{i0,j} - n_{e0} \, e \right| \ll n_{e0} \, e \; . \tag{2.32}$$

Here, the sum is extended over all (positive) ion species j of charge number Z_j. The charge number is a positive quantity, hence negative ions will have a minus-sign before the charge number. For a single ion species of charge $q = +e$ we often use the short-hand notation of the quasineutrality condition $n_{i0} = n_{e0}$. In this way, quasineutrality can be used as a defining quality of a *classical* plasma, in Langmuir's parlance, to distinguish the plasma region from space-charge regions.

There are, however, systems consisting of one polarity of charges only, like electrons or ions in potential traps, which show similar collective behavior as plasmas. In these *nonneutral* plasmas (cf. Sect. 3.1.6), as they are called, the potential trap takes the role of the neutralizing species of the other polarity.

2.2.3 Response Time and Plasma Frequency

The response to an external electric perturbation was established by the combined action of many particles. Therefore, Debye shielding is one example for collective behavior of a plasma. The second aspect of collective behavior is the time scale, after which the electrons establish a shielded equilibrium. The heavier ions will take a much longer time to reach their equilibrium positions.

When the potential perturbation is small, $|e\Phi| \ll k_B T$, the electron energy is not much changed from its thermal value. Hence, the typical electron velocity remains close to a thermal velocity $v_e \approx (k_B T_e/m_e)^{1/2}$ (see Sect. 4.1 for a more thorough definition). For the establishment of the new equilibrium, the electron must be able to reach its new position at a typical distance λ_{De}. This time can be estimated as $\tau \approx \lambda_{De}/v_e$. The reciprocal of this response time is called the *electron plasma frequency*

$$\omega_{pe} = \frac{v_e}{\lambda_{De}} = \left(\frac{n_{e0} e^2}{\varepsilon_0 m_e} \right)^{1/2} \; . \tag{2.33}$$

We will see in Sect. 8.1 that a deviation from thermodynamic equilibrium may excite oscillations of the plasma close to the electron plasma frequency. Such oscillations or waves are the natural collective modes of the electron gas.

In summary, we can state that a plasma of size L must be sufficiently large, i.e., $L \gg \lambda_D$, to behave in a collective manner. This would disqualify a typical candle flame as a plasma, because it is too small though it may have some ionization. Although not so obvious, a plasma must also exist for a period of time larger than the response time, $T \gg \omega_{pe}^{-1}$, to behave in a collective manner.

2.3 Existence Regimes

Plasmas are found in a huge parameter space, which covers seven orders of magnitude in temperature and twenty-five orders of magnitude in electron density (see Fig. 2.5). The temperature scale refers to the electron temperature. Typical examples are marked for astrophysical situations, some technical plasmas and the regime of controlled nuclear fusion.

2.3.1 *Strong-Coupling Limit*

The usual definition of a weakly coupled or *ideal* plasma is the requirement that there are many electrons inside the electron Debye sphere. This ensures that Debye shielding is a collective process and that the statistical derivation of a Debye length was correct. For this purpose we define the number of electrons inside the electron Debye sphere as the *plasma parameter* N_{De}

$$N_{De} = \frac{4\pi}{3}\lambda_{De}^3 n_e \ . \tag{2.34}$$

Because of the different temperatures, we can have different coupling states of electrons and ions $N_{De} \neq N_{Di}$. We will see in Chap. 10 that the dust system can be strongly coupled because of the high charge number on a dust grain whereas the electron and ion gas remain weakly coupled.

The border line between weakly and strongly coupled plasmas is defined by $N_{De} = 1$ (see dotted line in Fig. 2.5), which gives the equation of the border line

$$n_e = \left(\frac{4\pi\varepsilon_0}{3}\right)^2 \left(\frac{\varepsilon_0 k_B T_e}{e^2}\right)^3 \ . \tag{2.35}$$

This line has the slope 3 in the $\log n$–$\log T$ representation. From (2.15) and (2.16) we obtain a relation between the plasma parameter N_{Di} and the coupling parameter Γ_i (see Problem 2.5)

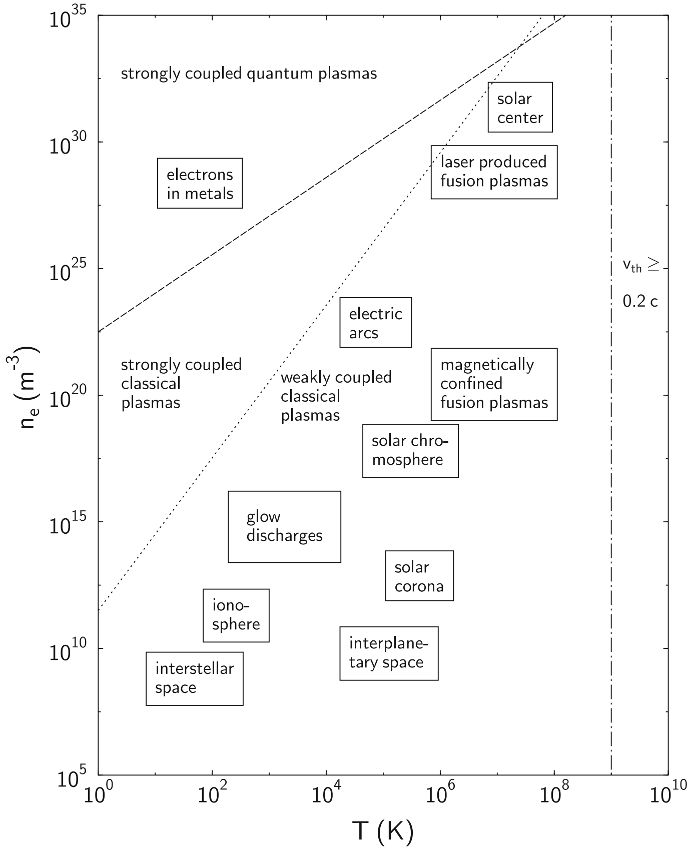

Fig. 2.5 Existence diagram of various plasmas. The *dotted line* marks the border for strong coupling, the *dashed line* the onset of quantum effects. Relativistic effects play a role for $T > 10^9$ K

$$\Gamma_{\mathrm{i}} = \frac{1}{3} N_{\mathrm{Di}}^{-2/3} \ . \tag{2.36}$$

Hence, a larger number of particles in the Debye sphere ensures that the coupling strength is small. In other words, the electric field is the average field of many particles, whereas in a strongly coupled system the field of the nearest neighbor dominates. Weakly coupled plasmas are found at high temperature and low electron density. On the border line, $N_{\mathrm{Di}} = 1$, we have $\Gamma = 1/3$.

2.3.2 Quantum Effects

Quantum effects come into play when the interparticle distance of the electrons becomes comparable with their thermal de Broglie wavelength

$$\lambda_B = \frac{h}{m_e v_{Te}} \, . \tag{2.37}$$

Here, $v_{Te} = (2k_B T_e/m_e)^{1/2}$ is the most probable speed of a Maxwell distribution (see Sect. 4.1). In this limiting case, the Pauli exclusion principle becomes important and we must use Fermi-Dirac statistics. Such a plasma is called degenerate and the conditions of a cold dense plasma are typically found in dead stars, like White Dwarfs. It is worth mentioning that the exclusion principle prevents the final collapse of such a burnt-out star.

The second border line for degeneracy of the electron gas, $\lambda_B = n_e^{-1/3}$ is also shown as dashed line in Fig. 2.5. Here, the slope is 3/2 in the $\log n$–$\log T$ diagram. Note that the electrons in a metal form a strongly coupled degenerate system. Relativistic effects for the electrons become important for $T > 10^9$ K as marked by the dot-dashed line in Fig. 2.5. The marked regions of typical plasmas can all be treated by non-relativistic models. This simplifies the plasma models in the subsequent chapters.

The Basics in a Nutshell

- Plasmas are quasineutral: $n_e = \sum_k Z_k n_k$.
- Quasineutrality can be violated within a Debye length λ_D

$$\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{(\lambda_{De}^2 + \lambda_{Di}^2)^{1/2}} \quad , \quad \lambda_{De,Di} = \left(\frac{\varepsilon_0 k_B T_{e,i}}{n_{e,i}\, e^2}\right)^{1/2} \, .$$

- Quasineutrality can be established by the electrons within $\tau = \omega_{pe}^{-1}$, with the plasma frequency

$$\omega_{pe} = \left(\frac{n_e e^2}{\varepsilon_0 m_e}\right)^{1/2} \, .$$

- The coupling parameter Γ determines the state of each plasma component (electrons, ions, dust)

$$\Gamma = \frac{q^2}{4\pi\varepsilon_0 a_{WS}\, k_B T} \, .$$

Γ may be different for the components, depending on the individual temperatures and densities. A gaseous phase is found for $\Gamma \ll 1$, the liquid state for $1 < \Gamma < 180$ and the solid phase for $\Gamma > 180$.

Problems

2.1 Prove that the electron Debye length can be written as

$$\lambda_{\mathrm{De}} = 69\,\mathrm{m}\left[\frac{T(\mathrm{K})}{n_{\mathrm{e}}(\mathrm{m}^{-3})}\right]^{1/2}$$

2.2 Calculate the electron and ion Debye length
(a) for the ionospheric plasma ($T_{\mathrm{e}} = T_{\mathrm{i}} = 3000\,\mathrm{K}$, $n = 10^{12}\,\mathrm{m}^{-3}$).
(b) for a neon gas discharge ($T_{\mathrm{e}} = 3\,\mathrm{eV}$, $T_{\mathrm{i}} = 300\,\mathrm{K}$, $n = 10^{16}\,\mathrm{m}^{-3}$).

2.3 Consider an infinitely large homogeneous plasma with $n_{\mathrm{e}} = n_{\mathrm{i}} = 10^{16}\,\mathrm{m}^{-3}$. From this plasma, all electrons are removed from a slab of thickness $d = 0.01\,\mathrm{m}$ extending from $x = -d$ to $x = 0$ and redeposited in the neighboring slab from $x = 0$ to $x = d$. (a) Calculate the electric potential in this double slab using Poisson's equation. What are the boundary conditions at $x = \pm d$? (b) Draw a sketch of space charge, electric field and potential for this situation. What is the potential difference between $x = -d$ and $x = d$?

2.4 Show that the equation for the shielding contribution (2.25) results from (2.22) and (2.24).

2.5 Derive the relationship between the coupling parameter for ion-ion interaction Γ (2.15) and N_{D} (2.34) under the assumption that $T_{\mathrm{e}} = T_{\mathrm{i}}$.

2.6 Show that the second Lagrange multiplier in (2.6) is $\lambda = (k_{\mathrm{B}}T)^{-1}$.
Hint: Start from

$$\frac{1}{T} = \frac{\partial S}{\partial \lambda}\frac{\partial \lambda}{\partial U}$$

and use $\sum n_i = 1$.

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