Contents

Part I  Plenary Activities

Thirteenth International Congress on Mathematical Education:
An Introduction ............................................................... 3
Gabriele Kaiser

Uncovering the Special Mathematical Work of Teaching ............ 11
Deborah Loewenberg Ball

Mathematics, Education, and Culture: A Contemporary Moral
Imperative ................................................................. 35
Bill Barton

Mathematics Classroom Studies: Multiple Lenses and Perspectives .... 45
Berinderjeet Kaur

“What is Mathematics?” and why we should ask, where
one should experience and learn that, and how to teach it .......... 63
Günter M. Ziegler and Andreas Loos

International Comparative Studies in Mathematics: Lessons
and Future Directions for Improving Students’ Learning ............ 79
Jinfa Cai, Ida A.C. Mok, Vijay Reddy and Kaye Stacey

Transitions in Mathematics Education: The Panel Debate ......... 101
Ghislain Gueudet, Marianna Bosch, Andrea A. diSessa,
Oh Nam Kwon and Lieven Verschaffel

Part II  Awardees’ lectures

ICMI Awards Ceremony ................................................. 121
Carolyn Kieran and Jeremy Kilpatrick
Mathematics Discourse in Instruction (MDI): A Discursive Resource as Boundary Object Across Practices .......................... 125
Jill Adler

The Challenging Relationship Between Fundamental Research and Action in Mathematics Education ................................. 145
Michèle Artigue

Elementary Mathematicians from Advanced Standpoints—A Cultural Perspective on Mathematics Education ................... 165
Alan J. Bishop

Design and Development for Large-Scale Improvement .................... 177
Hugh Burkhardt and Malcolm Swan

Making Sense of Mathematics Achievement in East Asia: Does Culture Really Matter? ..................................... 201
Frederick K.S. Leung

Part III Reports of the Survey Teams

Digital Technology in Mathematics Education: Research over the Last Decade ......................................................... 221
Marcelo C. Borba, Petek Askar, Johann Engelbrecht, George Gadanidis, Salvador Llinares and Mario Sánchez Aguilar

Conceptualisation of the Role of Competencies, Knowing and Knowledge in Mathematics Education Research ........ 235
Mogens Niss, Regina Bruder, Núria Planas, Ross Turner and Jhony Alexander Villa-Ochoa

Assistance of Students with Mathematical Learning Difficulties—How Can Research Support Practice?—A Summary .................. 249
Petra Scherer, Kim Beswick, Lucie DeBlois, Lulu Healy and Elisabeth Moser Opitz

Mathematics Teachers Working and Learning Through Collaboration ................................................................. 261
Barbara Jaworski, Olive Chapman, Alison Clark-Wilson, Annalisa Cusi, Cristina Esteley, Merrilyn Goos, Masami Isoda, Marie Joubert and Ornella Robutti

Geometry Education, Including the Use of New Technologies: A Survey of Recent Research .......................................... 277
Nathalie Sinclair, Maria G. Bartolini Bussi, Michael de Villiers, Keith Jones, Ulrich Kortenkamp, Allen Leung and Kay Owens
Part IV  Reports from the Thematic Afternoon

European Didactic Traditions in Mathematics: Aspects and Examples from Four Selected Cases. .......................... 291
Werner Blum, Michèle Artigue, Maria Alessandra Mariotti, Rudolf Sträßer and Marja Van den Heuvel-Panhuizen

German-Speaking Traditions in Mathematics
Education Research .......................................... 305
Hans Niels Jahnke, Rolf Biehler, Angelika Bikner-Ahsbahs, Uwe Gellert, Gilbert Greefrath, Lisa Hefendehl-Hebeker, Götz Krummheuer, Timo Leuders, Marcus Nührenbörger, Andreas Obersteiner, Kristina Reiss, Bettina Rösken-Winter, Andreas Schulz, Andreas Vohns, Rudolf vom Hofe and Katrin Vorhölter

What Is and What Might Be the Legacy of Felix Klein? ............. 321
Hans-Georg Weigand, William McCallum, Marta Menghini, Michael Neubrand, Gert Schubring and Renate Tobies

Part V  National Presentations

Argentinean National Presentation .............................. 337
Esther Galina and Mónica Villarreal

Teachers’ Professional Development and Mathematics
Education in Brazil ........................................... 345
Victor Giraldo

Mathematics Education in Ireland ............................... 347
Maurice OReilly, Thérèse Dooley, Elizabeth Oldham and Gerry Shiel

National Presentation of Japan .................................. 353
Toshiakira Fujii, Yoshinori Shimizu, Hanako Senuma and Toshikazu Ikeda

National Presentations of Lower Mekong Sub-region Countries .... 361
Fidel R. Nemenzo, Masami Isoda, Maitree Inprasitha, Sampan Thinwiangthong, Narumon Changsri, Nisakorn Boonsena, Chan Roth, Monkolsery Lin, Souksomphone Anothay, Phoutsakhone Channgakham, Nguyen Chi Thanh, Vũ Như Thu Hương and Phuong Thảo Nguyễn

Teaching and Learning Mathematics in Turkey .................... 367
Huriye Arikan
Part VI   Reports from the Topical Study Groups

Topic Study Group No. 1: Early Childhood Mathematics Education (Up to Age 7) ....................................... 375
Elia Iliada, Joanne Mulligan, Ann Anderson, Anna Baccaglini-Frank and Christiane Benz

Topic Study Group No. 2: Mathematics Education at Tertiary Level .................................................. 381
Victor Giraldo, Chris Rasmussen, Irene Biza, Azimehsadat Khakbaz and Reinhard Hochmuth

Topic Study Group No. 3: Mathematics Education in and for Work .................................................. 387
Geoff Wake, Diana Coben, Burkhard Alpers, Keith Weeks and Peter Frejd

Topic Study Group No. 4: Activities for, and Research on, Mathematically Gifted Students ..................... 391
Florence Mihaela Singer, Linda Jensen Sheffield, Matthias Brandl, Viktor Freiman and Kyoko Kakihana

Topic Study Group No. 5: Classroom Practice and Research for Students with Mathematical Learning Difficulties .................. 397
Lourdes Figueiras, Rose Griffiths, Karen Karp, Jens Holger Lorenz and Miriam Godoy Penteado

Topic Study Group No. 6: Adult Learning ........................ 401
Jürgen Maaß, Pradeep Kumar Misra, Terry Maguire, Katherine Safford-Ramus, Wolfgang Schlöglmann and Evelyn Süß-Stepancik

Topic Study Group No. 07: Popularization of Mathematics .......... 405
Christian Mercat, Patrick Vennebush, Chris Budd, Carlota Simões and Jens Struckmeier

Topic Study Group No. 8: Teaching and Learning of Arithmetic and Number Systems (Focus on Primary Education) ............. 413
Pi-Jen Lin, Terezinha Nunes, Shuhua An, Beatriz Vargas Dorneles and Elisabeth Rathgeb-Schnierer

Topic Study Group No. 9: Teaching and Learning of Measurement (Focus on Primary Education) ......................... 415
Christine Chambris, Barbara Dougherty, Kalyanasundaram (Ravi) Subramaniam, Silke Ruwisch and Insook Chung
Contents ix

Topic Study Group No. 10: Teaching and Learning of Early Algebra ............................................. 421
Carolyn Kieran, JeongSuk Pang, Swee Fong Ng, Deborah Schifter and Anna Susanne Steinweg

Topic Study Group No. 11: Teaching and Learning of Algebra .......... 425
Rakhi Banerjee, Amy Ellis, Astrid Fischer, Heidi Strømskag and Helen Chick

Topic Study Group No. 12: Teaching and Learning of Geometry (Primary Level) .............................................. 429
Sinan Olkun, Ewa Swoboda, Paola Vighi, Yuan Yuan and Bernd Wollring

Topic Study Group No. 13: Teaching and Learning of Geometry—Secondary Level ................................. 435
Ui Hock Cheah, Patricio G. Herbst, Matthias Ludwig, Philippe R. Richard and Sara Scaglia

Topic Study Group No. 14: Teaching Learning of Probability ........ 439
Carmen Batanero, Egan J. Chernoff, Joachim Engel, Hollylynne Stohl Lee and Ernesto Sánchez

Topic Study Group No. 15: Teaching and Learning of Statistics ...... 443
Dani Ben-Zvi, Gail Burrill, Dave Pratt, Lucia Zapata-Cardona and Andreas Eichler

Topic Study Group No. 16: Teaching and Learning of Calculus ...... 447
David Bressoud, Victor Martínez-Luaces, Imène Ghedamsi and Günter Törner

Topic Study Group No. 17: Teaching and Learning of Discrete Mathematics ................................................. 453
Eric W. Hart, James Sandefur, Cecile O. Buffet, Hans-Wolfgang Henn and Ahmed Semri

Topic Study Group No. 18: Reasoning and Proof in Mathematics Education ................................................... 459
Guershon Harel, Andreas J. Stylianides, Paolo Boero, Mikio Miyazaki and David Reid

Topic Study Group No. 19: Problem Solving in Mathematics Education ......................................................... 463
Peter Liljedahl, Manuel Santos-Trigo, Uldarico Malaspina, Guido Pinkernell and Laurent Vivier

Topic Study Group No. 20: Visualization in the Teaching and Learning of Mathematics .................................. 467
Michal Yerushalmy, Ferdinand Rivera, Boon Liang Chua, Isabel Vale and Elke Söbbeke
<table>
<thead>
<tr>
<th>Topic Study Group No.</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>Mathematical Applications and Modelling in the Teaching and Learning of Mathematics</td>
<td>471</td>
</tr>
<tr>
<td></td>
<td>Jussara Araújo, Gloria Ann Stillman, Morten Blomhøj, Toshikazu Ikeda and Dominik Leiss</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Interdisciplinary Mathematics Education</td>
<td>475</td>
</tr>
<tr>
<td></td>
<td>Susie Groves, Julian Williams, Brian Doig, Rita Borromeo Ferri and Nicholas Mousoulides</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Mathematical Literacy</td>
<td>481</td>
</tr>
<tr>
<td></td>
<td>Hamsa Venkat, Iddo Gal, Eva Jablonka, Vince Geiger and Markus Helmerich</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>History of the Teaching and Learning of Mathematics</td>
<td>487</td>
</tr>
<tr>
<td></td>
<td>Fulvia Furinghetti, Alexander Karp, Henrike Allmendinger, Johan Prytz and Harm Jan Smid</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>The Role of History of Mathematics in Mathematics Education</td>
<td>491</td>
</tr>
<tr>
<td></td>
<td>Constantinos Tzanakis, Xiaoqin Wang, Kathleen Clark, Tinne Hoff Kjeldsen and Sebastian Schorcht</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Research on Teaching and Classroom Practice</td>
<td>497</td>
</tr>
<tr>
<td></td>
<td>Yoshinori Shimizu, Mary Kay Stein, Birgit Brandt, Helia Oliveira and Lijun Ye</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Learning and Cognition in Mathematics</td>
<td>501</td>
</tr>
<tr>
<td></td>
<td>Gaye Williams, Wim Van Dooren, Pablo Dartnell, Anke Lindmeier and Jérôme Proulx</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Affect, Beliefs and Identity in Mathematics Education</td>
<td>507</td>
</tr>
<tr>
<td></td>
<td>Markku Hannula, Francesca Morselli, Emine Erktin, Maike Vollstedt and Qiao-Ping Zhang</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Mathematics and Creativity</td>
<td>511</td>
</tr>
<tr>
<td></td>
<td>Demetra Pitta-Pantazi, Dace Kūma, Alex Friedlander, Thorsten Fritzlar and Emiliya Velikova</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Mathematical Competitions</td>
<td>515</td>
</tr>
<tr>
<td></td>
<td>Maria Falk de Losada, Alexander Soifer, Jaroslav Svrcek and Peter Taylor</td>
<td></td>
</tr>
</tbody>
</table>
Contents

Topic Study Group No. 31: Language and Communication in Mathematics Education .................................................. 521
Judit Moschkovich, David Wagner, Arindam Bose, Jackeline Rodrigues Mendes and Marcus Schütte

Topic Study Group No. 32: Mathematics Education in a Multilingual and Multicultural Environment ........................................ 525
Richard Barwell, Anjum Halai, Aldo Parra, Lena Wessel and Guida de Abreu

Topic Study Group No. 33: Equity in Mathematics Education (Including Gender) .................................................. 531
Bill Atweh, Joanne Rossi Becker, Barbro Grevholm, Gelsa Knijnik, Laura Martignon and Jayasree Subramanian

Topic Study Group No. 34: Social and Political Dimensions of Mathematics Education ........................................ 537
Murad Jurdak, Renuka Vithal, Peter Gates, Elizabeth de Freitas and David Kollosche

Topic Study Group No. 35: Role of Ethnomathematics in Mathematics Education .................................................. 543
Milton Rosa, Lawrence Shirley, Maria Elena Gavarrete and Wilfredo V. Alangui

Topic Study Group No. 36: Task Design, Analysis and Learning Environments Programme Summary ........................................ 549
Jere Confrey, Jiansheng Bao, Anne Watson, Jonei Barbosa and Helmut Linneweber-Lammerskitten

Topic Study Group No. 37: Mathematics Curriculum Development ................................................................. 555
Anita Rampal, Zalman Usiskin, Andreas Büchter, Jeremy Hodgen and Iman Osta

Topic Study Group No. 38: Research on Resources (Textbooks, Learning Materials etc.) ........................................ 561
Lianghuo Fan, Luc Trouche, Chunxia Qi, Sebastian Rezat and Jana Visnovska

Topic Study Group No. 39: Large Scale Assessment and Testing in Mathematics Education ........................................ 565
Rae Young Kim, Christine Suurtamm, Edward Silver, Stefan Ufer and Pauline Vos

Topic Study Group No. 40: Classroom Assessment for Mathematics Learning .................................................. 571
Denisse R. Thompson, Karin Brodie, Leonora Diaz Moreno, Nathalie Sayac and Stanislaw Schukajlow
Topic Study Group No. 41: Uses of Technology in Primary Mathematics Education (Up to Age 10) ........................... 575
Sophie Soury-Lavergne, Colleen Vale, Francesca Ferrara, Krongthong Khairiree and Silke Ladel

Topic Study Group No. 42: Uses of Technology in Lower Secondary Mathematics Education (Age 10–14) ............................. 577
Lynda Ball, Paul Drijvers, Bärbel Barzel, Yiming Cao and Michela Maschietto

Topic Study Group No. 43: Uses of Technology in Upper Secondary Education (Age 14–19) ........................................ 579
Stephen Hegedus, Colette Laborde, Luis Moreno Armella, Hans-Stefan Siller and Michal Tabach

Topic Study Group No. 44: Distance Learning, e-Learning, and Blended Learning ..................................................... 583
Rúbia Barcelos Amaral, Verónica Hoyos, Els de Geest, Jason Silverman and Rose Vogel

Topic Study Group No. 45: Knowledge in/for Teaching Mathematics at Primary Level ........................................... 585
Carolyn A. Maher, Peter Sullivan, Hedwig Gasteiger and Soo Jin Lee

Topic Study Group No. 46: Knowledge in/for Teaching Mathematics at the Secondary Level ....................................... 589
Ruhama Even, Xinrong Yang, Nils Buchholtz, Charalambos Charalambous and Tim Rowland

Topic Study Group No. 47: Pre-service Mathematics Education of Primary Teachers ............................................... 593
Keiko Hino, Gabriel J. Stylianides, Katja Eilerts, Caroline Lajoie and David Pugalee

Topic Study Group No. 48: Pre-service Mathematics Education of Secondary Teachers ........................................... 599
Marilyn Strutchens, Rongjin Huang, Leticia Losano, Despina Potari and Björn Schwarz

Topic Study Group No. 49: In-Service Education and Professional Development of Primary Mathematics Teachers .................... 605
Akihiko Takahashi, Leonor Varas, Toshiakira Fujii, Kim Ramatlapana and Christoph Selter

Topic Study Group No. 50: In-Service Education, and Professional Development of Secondary Mathematics Teachers ............... 609
Jill Adler, Yudong Yang, Hilda Borko, Konrad Krainer and Sitti Patahuddin
Contents

**Topic Study Group No. 51: Diversity of Theories in Mathematics Education** ................................. 613
Tommy Dreyfus, Anna Sierpinska, Stefan Halverscheid, Steve Lerman and Takeshi Miyakawa

**Topic Study Group 52: Empirical Methods and Methodologies** ........ 619
David Clarke, Alan Schoenfeld, Bagele Chilisa, Paul Cobb and Christine Knipping

**Topic Study Group No. 53: Philosophy of Mathematics Education** .... 623
Paul Ernest, Ladislav Kvasz, Maria Bicudo, Regina Möller and Ole Skovsmose

**Topic Study Group No. 54: Semiotics in Mathematics Education** .... 627
Norma Presmeg, Luis Radford, Gert Kadunz, Luis Puig and Wolff-Michael Roth

**Part VII  Reports from the Discussion Groups**

**Classroom Teaching Research for All Students** ......................... 635
Shuhua An, Steklács János and Zhonghe Wu

**Mathematical Discourse in Instruction in Large Classes** .............. 637
Mike Askew, Ravi K. Subramaniam, Anjum Halai, Erlina Ronda, Hamsa Venkat, Jill Adler and Steve Lerman

**Sharing Experiences About the Capacity and Network Projects**
**Initiated by ICMI** ......................................................... 639
Angelina Matinde Bijura, Alphonse Uworwabayeho, Veronica Sarungi, Peter Kajoro and Anjum Halai

**Mathematics Teacher Noticing: Expanding the Terrains**
**of This Hidden Skill of Teaching** ..................................... 641
Ban Heng Choy, Jaguthsing Dindyal, Mi Yeon Lee and Edna O. Schack

**Connections Between Valuing and Values: Exploring Experiences**
**and Rethinking Data Generating Methods** ............................ 643
Philip Clarkson, Annica Andersson, Alan Bishop, Penelope Kalogeropoulos and Wee Tiong Seah

**Developing New Teacher Learning in Schools**
**and the STEM Agenda** .................................................. 645
Pat Drake, Jeanne Carroll, Barbara Black, Lin Phillips and Celia Hoyles

**Videos in Teacher Professional Development** ........................... 647
Tanya Evans, Leong Yew Hoong and Ho Weng Kin
Mathematics for the 21st Century School: The Russian Experience and International Prospects ............................................. 675
Sergei A. Polikarpov and Alexei L. Semenov

Lesson/Learning Studies and Mathematics Education ................. 677
Marisa Quaresma and Carl Winsløw

Mathematics Houses and Their Impact on Mathematics Education .... 679
Ali Rejali, Peter Taylor, Yahya Tabesh, Jérôme Germoni and Abolfazl Rafiepour

An Act of Mathematisation: Familiarisation with Fractions .......... 681
Ernesto Rottoli, Sabrina Alessandro, Petronilla Bonissoni, Marina Cazzola, Paolo Longoni and Gianstefano Riva

The Role of Post-Conflict School Mathematics .......................... 683
Carlos Eduardo Leon Salinas and Jefer Camilo Sachica Castillo

Applying Contemporary Philosophy in Mathematics and Statistics Education: The Perspective of Inferentialism ............... 685
Maike Schindler, Kate Mackrell, Dave Pratt and Arthur Bakker

Teaching Linear Algebra .................................................. 687
Sepideh Stewart, Avi Berman, Christine Andrews-Larson and Michelle Zandieh

Creativity, Aha!Moments and Teaching-Research ...................... 689
Hannes Stoppel and Bronislaw Czarnocha

White Supremacy, Anti-Black Racism, and Mathematics Education: Local and Global Perspectives ............................... 691
Luz Valoyes-Chávez, Danny B. Martin, Joi Spencer and Paola Valero

Research on Non-university Tertiary Mathematics ...................... 693
Claire Wladis, John Smith and Irene Duranczyk

Part VIII Reports from the Workshops

Flipped Teaching Approach in College Algebra: Cognitive and Non-cognitive Gains ............................................. 697
Maxima J. Acelajado

A Knowledge Discovery Platform for Spatial Education: Applications to Spatial Decomposition and Packing ..................... 699
Sorin Alexe, Cristian Voica and Consuela Voica
Designing Mathematics Tasks for the Professional Development of Teachers Who Teach Mathematics Students Aged 11–16 Years .............................. 701
Debbie Barker and Craig Pournara

Contributing to the Development of Grand Challenges in Maths Education ................................................................. 703
David Barnes, Trena Wilkerson and Michelle Stephan

The Role of the Facilitator in Using Video for the Professional Learning of Teachers of Mathematics ............................ 705
Alf Coles, Aurelie Chesnais and Julie Horoks

Making Middle School Maths Real, Relevant and Fun ......................... 707
Kerry Cue

“Oldies but Goodies”: Providing Background to ICMI Mission and Activities from an Archival Perspective .......................... 709
Guillermo P. Curbera, Bernard R. Hodgson and Birgit Seeliger

Using Braids to Introduce Groups: From an Informal to a Formal Approach ................................................................. 711
Ester Dalvit

Curious Minds; Serious Play ............................................. 713
Jan de Lange

International Similarities and Differences in the Experiences and Preparation of Post-Graduate Mathematics Students as Tertiary Instructors .......................................................... 717
Jessica Deshler and Jessica Ellis

Using LISP as a Tool for Mathematical Experimentation .......................... 719
Hugo Alex Diniz

Mathematics Teachers’ Circles as Professional Development Models Connecting Teachers and Academics ......................... 721
Nathan Borchelt and Axelle Faughn

Exploring and Making Online Creative Digital Math Books for Creative Mathematical Thinking ............................................. 723
Pedro Lealdino Filho, Christian Bokhove, Jean-Francois Nicaud, Ulrich Kortenkamp, Mohamed El-Demerdash, Manolis Mavrikis and Eirini Geraniou

The Shift of Contents in Prototypical Tasks Used in Education Reforms and Their Influence on Teacher Training Programs .... 725
Karl Fuchs, Christian Kraler and Simon Plangg
Workshop on Framing Non-routine Problems in Mathematics for Gifted Children of Age Group 11–15 ......................... 751
Sundaram R. Santhanam

Enacted Multiple Representations of Calculus Concepts, Student Understanding and Gender ............................. 753
Ileana Vasu

Using Inquiry to Teach Mathematics in Secondary and Post-secondary Education .......................... 755
Volker Ecke and Christine von Renesse

Making of Cards as Teaching Material for Spatial Figures. ............ 757
Kazumi Yamada and Takaaki Kihara

Creative Mathematics Hands-on Activities in the Classroom. ......... 759
Janchai Yingprayoon

Part IX  Additional Activities

Teachers Activities at ICME-13 .................................................. 763
Nils Buchholtz, Marianne Nolte and Gabriele Kaiser

Early Career Researcher Day at ICME-13 ................................. 765
Gabriele Kaiser, Thorsten Scheiner and Armin Jentsch
Proceedings of the 13th International Congress on Mathematical Education
ICME-13
Kaiser, G. (Ed.)
2017, XVIII, 766 p. 55 illus., Hardcover
ISBN: 978-3-319-62596-6