Chapter 2
Standard Metals and the Fermi Liquid

One of the milestones and great results of the 20th century is *Landau's Fermi liquid theory*, which underlines our present understanding of the majority of the known states of matter, like normal metals, semi-conductors, superconductors and superfluids. To better understand the differences between the predictions of this great theory and the behaviour of the cuprate superconductors, strange states of matter discovered since the early 80s, it is necessary to recall its basic properties in this Introduction, referring the reader interested in the technical aspects to Appendix A or to standard condensed matter textbook (e.g. [1, 2]).

Let us start by recalling the basic properties of a system of free fermions in a box, where the Pauli exclusion principle controls everything. In the ground-state of this system all the single-particle states inside a sphere in momentum space with radius k_F[1] are filled, while the state outside the sphere are empty. The external surface of this sphere is called the *Fermi surface*. The system has two types of low energy excitations. One can in fact fill a state slightly outside the Fermi surface, creating a particle, or remove a fermion from a filled state slightly inside the Fermi surface, creating what is typically called a hole. These excitations are gapless by definition, and have linear dispersion (for $k - k_F \ll k_F$):

$$\varepsilon(k) = \frac{k^2}{2m} - \mu = \frac{k_F}{2m}(k - k_F) \equiv v_F(k - k_F) \ , \tag{2.1}$$

where m is the mass of the fermions, $\mu \equiv \frac{k_F^2}{2m}$ is the chemical potential, the quantity $v_F \equiv \frac{k_F}{m}$ is called the Fermi velocity and particles and holes are distinguished by the sign of $k - k_F$. Rephrasing the previous statements in a more formal language,

[1] k_F is fixed by the density of Fermions, as we will see below.

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the existence of these kind of excitations manifests itself as a pole in the complex frequency plane of the retarded Green's function $G_R(\omega, \vec{k})$ of the electron operator $\psi(\omega\vec{k})$:

$$G_R^0(\omega, \vec{k}) \equiv \psi(\omega, \vec{k})\psi(0,0) = \frac{1}{\omega - \varepsilon(k) + i0^+} . \tag{2.2}$$

Note that the retarded Green's function (2.2) describes the response of the system to the addition of a single electron. Fourier transforming the propagator (2.2) back in time, in fact, one can notice that the resulting expression is exactly the one for the usual Green's function for the propagation of a free particle of energy $\varepsilon(k)$ (2.1):

$$G_R^0(t, \vec{k}) = i\sqrt{2\pi}\theta(t)e^{-i\varepsilon(k)t} . \tag{2.3}$$

The previous picture might not be any more valid in the case in which interactions between electrons are turned on, since in this case the concept of particle is not well defined. Actually, one can naively expect that the qualitative picture of the non-interacting gas should remain valid if the interactions are weak, but there are no apparent reasons for this intuition to remain valid at strong coupling.

The basic assumption from which the phenomenological Landau theory is constructed, is that the qualitative picture for non-interacting Fermi gas remains actually valid for a generic interacting fermionic system, also in the case of strong interactions between the elementary constitutive fermions. Specifically, the Landau Fermi liquid theory has two fundamental starting assumptions:

- There exists a Fermi surface which characterizes the ground state of a generic interacting fermionic system. In momentum space, this surface lies at $\vec{k} = \vec{k}_F$ and is the locus at which the Green's function $G_R^0(\omega, \vec{k})$ has a simple pole.
- The low energy excitations around the Fermi surface are weakly interacting particles, called quasi-particles, despite the (possibly strong) interactions between the fundamental fermions. The quasi-particles are characterized by the same charge and statistics of the underlying fundamental fermions.

Given these basic assumptions, one has to verify that the theory is stable, namely that, when the interactions between quasi-particles are switched on, the quasi-particles life-time is long enough such that an approximate particle picture still applies. Eventually, it can be proven that, given a generic local interaction between quasi-particles, the decay rate of a quasi-particle obeys (see Appendix A for details)

$$\Gamma \sim \frac{\varepsilon^2}{\mu} \ll \varepsilon . \tag{2.4}$$

Thus, despite the potentially strong interactions, there is a region sufficiently near to the Fermi surface, where quasi-particles have long life-time and an approximate particle pictures still applies.

This implies that, near the Fermi surface, the retarded Green's function for quasi-particles acquires the following form:

$$G_R(\omega, \vec{k}) = \frac{Z}{\omega - v_F(k - k_F) + \Sigma(\omega, k)} , \tag{2.5}$$

where $Z < 1$ is the quasi-particle residue, which, as we will see later, measures the jump in the occupation number at the Fermi surface, and $\Sigma(\omega, k)$ is called the free energy. Finally, according to (2.1), the free energy Σ has the following low energy behavior:

$$\Im\Sigma(\omega, k) = \frac{i\Gamma}{2} \sim i\omega^2 . \tag{2.6}$$

Starting from these basic assumptions, it is possible to develop a general low energy theory, independently of the precise microscopic details of the system. Moreover, just introducing some phenomenological prescriptions, one can derive the behaviour of the thermodynamical quantities like the specific heat C_V and the chemical potential μ, the entropy s and the thermo-electric transport coefficients (see Appendix A for more details).

Regarding the thermodynamical quantities, considering only the quasi-particle contribution, we obtain:

$$C_V = T\left(\frac{\partial s}{\partial T}\right)_V = s = \frac{\pi^2}{3} N(0) k_B^2 T , \tag{2.7}$$

$$\mu(n, T) = \mu(n, 0) - \frac{\pi^2}{4} k_B \left(\frac{1}{3} + \frac{n}{m^*}\frac{\partial m^*}{\partial n}\right)\frac{T^2}{T_F} . \tag{2.8}$$

where k_B is the Boltzmann constant, $N(0)$ is the density of carriers at the Fermi surface, n is the total density of quasi-particles and $T_F = k_F^2/(2m^* k_B)$ is the Fermi temperature.

The results previously described are extremely powerful and do not depend on the microscopic details of the system at hand. However, when we have to compare the theoretical predictions with experiments, we have to keep in mind that the experimental results provide us not only with the electronic contribution of the thermodynamical quantities (2.7), but with the total contribution, which takes into account also the effect of lattice vibrations, i.e. *phonons*, and defects. Since, as we will see in the next Section, it is not always an easy task to extrapolate the electronic contribution from the experimental data, in what follows we will try, if possible, to specify how the external degrees of freedom like lattice vibrations and impurities modifies the properties of the electronic plasma. As an example, regarding the specific heat C_V, it is known [2] that the phonons contribution to this quantity can be expanded in series of odd powers of the temperature T, namely:

$$C_{ph} = B_{ph} T^3 + E_{ph} T^5 + \dots . \tag{2.9}$$

Then, the total specific heat in a normal metal should scale as:

$$C_V = \gamma T + C_{ph} . \tag{2.10}$$

Table 2.1 Transport coefficients temperature dependence predicted by the Fermi liquid theory

	Low T, ($T \ll T_D$)	High T, ($T \gg T_D$)
ρ	$A_{imp} + B_{e,e}T^2 + C_{e,ph}T^{d+2}$	$\mathscr{A}_{e,ph}T$
S	$DT + E_{phd}T^d$	$\mathscr{D}_{e,ph}T + \mathscr{F}_{phd}\frac{1}{T}$
κ	$H_{ee}\frac{1}{T} + L_{imp}T + G_{e,ph}T^{d-1}$	$\mathscr{H}_{e,ph}$

As regards the thermo-electric transport coefficients (see Chap. 1 for their definition), the relevant quantities in experiments are the electric resistivity ρ, the thermal conductivity κ and the Seebeck coefficient S, which measure the voltage generated due to the presence of an applied external thermal gradient. Keeping into account the presence of phonons and defects, the Fermi liquid theory predicts the following behaviours for these quantities:

where d is the number of spatial dimensions in the system. In deriving the previous temperature scalings we have taken into account four different kind of scattering processes: the electron-impurity scattering (subscript imp), the electron-electron scattering (subscript ee), the electron-phonon scattering (subscript e-ph) and the phonon-drag mechanism (subscript phd), namely the process according to which the heat transfer in the metal causes a flux of phonons which carries the electron with it. This mechanism is relevant only if one considers the Seebeck effect S and can be neglected in the analysis of the electric and thermal transport. In normal metals, the phonon-phonon scattering process is sub-dominant with respect to the previously outlined scattering mechanism and can be neglected in the analysis of the thermoelectric transport coefficients (see Appendix A).

We have divided the whole temperature range into two intervals separated by the Debye temperature T_D. The coefficients which multiply the powers of the temperature T in Table 2.1, are constant which depends on the specific parameters of the metal under consideration. In the region $T \gg T_D$ the electron-phonon scattering mechanism largely dominates the transport. In the opposite regime, at very low T, the scattering mechanisms are dominated by the effect of impurities. However, in the transition region between $T \sim 0$ and $T \sim T_D$, since the scattering rate of the electron-phonon processes decreases faster than that of electron-electron processes as the temperature is decreased, there may be a region in which the transport properties are dominated by the electron-electron interactions. In this region the resistivity of the Fermi liquid scales as T^2. The T^2 scaling of the resistivity is considered a standard evidence of the presence of the Fermi liquid in the experimental measurements (see Chap. 3).

Finally, let us make some comments about the celebrated Wiedemann-Franz law. This law states that in a Fermi liquid in the presence of elastic scattering processes, the ratio $\kappa/(\sigma T)$ is constant in temperature and assumes the value $L_0 = \frac{\pi^2}{3e^2}$. In the analysis of Table 2.1, the scattering processes considered are all elastic with the exception of the electron-phonon interaction, under which a fraction of the energy of the quasi-particles is transferred to the lattice. This is definitely true in the low-T

region where, as one can see from the table, the Wiedemann-Franz law is not satisfied. However, in the high-T regime the fraction of energy transferred to the lattice is very small and also the electron-phonon scattering can be considered as elastic. In this region the Wiedemann-Franz law holds exactly.

References

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