

# Preface

Optimization is a very broad field of research with a wide spectrum of important applications. Until the 1950s, optimization was understood as a single-objective optimization, i.e., as the specification and computation of minimum/maximum of a function of interest taking into account some constraints for the solution. Such optimization problems were the focus of mathematicians from ancient times. The earliest methods of calculus were applied for the analysis and solution of optimization problems immediately following their development. Moreover, these applications gave way to important results in the basics of natural sciences. The importance of optimization for the understanding of nature is well formulated by Leonard Euler:

Since the fabric of the universe is most perfect and the work of a most wise Creator, nothing at all takes place in the universe in which some rule of maximum or minimum does not appear.<sup>1</sup>

Further developments of optimization theory and computational methods were successfully applied not only in natural sciences but also in planning and design. Let us mention that several Nobel Memorial Prizes in Economics were awarded for the application of optimization methods to the problems of economics. Nevertheless, in the middle of the last century, it was understood that the model of single-objective optimization is not universal. In many problems of planning and design, a decisionmaker aims to minimize/maximize not a single but several objective functions. As an example, in industry, when producing metal sheets, the objectives are to minimize energy consumption, maximize process speed, and maximize the strength of the product at the same time. The purpose of multi-objective

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<sup>1</sup>Cum enim Mundi universi fabrica sit perfectissima atque a Creatore sapientissimo absoluta, nihil omnino in mundo contingit, in quo non maximi minimive ratio quaequam eluceat; quomobrem dubium prorsus est nullum, quin omnes Mundi effectus ex causis finalibus ope Methodi maximorum et minimorum aequè feliciter determinari queant, atque ex ipsis causis efficientibus (L. Euler, Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes, sive solutio problematis isoperimetrici lattissimo sensu accepti, Lausanne and Geneva, 1744).

optimization is to give an understanding of how these objectives are conflicting and to provide the user the possibility to choose an appropriate trade-off between the objectives.

As another example, consider multi-objective path finding problems which have received a lot of attention in recent years. Routing problems are part of everyday activity. We move material through transportation networks, and we move huge amounts of data through telecommunication networks or the Internet. Many times, we are looking for the shortest path, but in real life, other objectives can be considered. Suppose we consider the problem to route hazardous materials in a transportation network. In addition to the minimum distance, we need to have objectives on minimizing environmental risks and risks to human populations.

When multiple objectives are present, the concept of an optimal solution as in the single-objective problems does not apply. Naturally, first of all, the well-known classical single-objective optimization methods were generalized for the multi-objective case, e.g., methods of multi-objective linear programming and of multi-objective convex optimization were developed. The single-objective non-convex optimization problems are known as the most difficult. The difficulties certainly increase in case of several objectives. The generalization of mathematical methods of single-objective global optimization to multi-objective case and the development of new methods present a real challenge for researchers. Heuristic methods are more prone to various modifications, and the generalization of heuristic methods of global optimization for the multi-objective case has been booming. Meanwhile, many multi-objective optimization problems of engineering can be solved by software implementing the heuristic methods. Nevertheless, the mathematical analysis of non-convex multi-objective optimization problems is urgent from the point of applications as well as of general global optimization theory. A subclass of those problems waiting for a more active attention of researchers is multi-objective optimization of non-convex expensive black box problems; this book is focused on the theoretically substantiated methods for problems of such a type.

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