Random Excitation Technique for Measurement of Acoustic Properties

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1 Introduction

The acoustic properties of a test specimen can be calculated using a randomly excited signal through a duct and measuring the properties of signal reflected from the test specimen. The various stages of the process include defining the objective, understanding relevant theory, identification of measurable parameters, formulation of theory and instrumentation for measurement, accuracy enhancement, controlling parameters and, most importantly, limitations of the used procedure. A random excitation technique for measurement of acoustic properties, with intermediate processes undergone, is presented in the following text.

To perform a study or analysis in a particular field, familiarity of the literature is required. For that, relevant literature review or Acoustic Wave Theory is presented for the reader which may be referred to, from time to time depending on the requirement in further sections. Also, the necessary mathematics is presented. The accuracy, cost, time, etc. of a particular analysis are highly influenced by the type of experimental set-up chosen. Various set-ups that can be used for the current analysis are described and compared, and a particular set-up is selected. The theory for the selected set-up is formulated according to the requirements.

Even after having an accurate set-up, proper selection of the measurement devices is required, or otherwise, the results obtained may be highly erroneous. So, various instrument parameters (specifications) are taken care of, while selecting the measurement devices. Some of them are discussed in detail. The signals can be pre-amplified to enhance signal characteristics. The signals acquired by most of the

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measurement devices are in the analog form. But, for analysis by computing devices, the signal needs to be converted into digital form. The wrong selection of A-to-D converters leads to introduction of error at this least expected stage. Depending on the type of data, the best suited A-to-D converter changes. The finally acquired data can be processed. But there are some common sources of error which need to be taken care of. Some basic precautions that can be helpful in eliminating these errors are mentioned.

2 Acoustic Waves

Acoustic waves or sound waves are longitudinal waves which travel by means of repetitive compression and expansion [1]. As the sound waves travel at sufficiently high speeds, the process is assumed to be adiabatic in nature. These waves exhibit phenomena like interference, diffraction and reflection which need to be quantified, so as to predict their behaviour on interaction with various materials and use them to modify the acoustic properties in various environments like reduction of reverberation time in large enclosures for audibility, design of audio instruments, muffler designs, sound proofing and many more applications.

A comparatively easier study of acoustic waves is possible, if wave front of the acoustic waves is planar. These types of wave are called plane waves.

2.1 Plane Waves

Plane waves may be referred to as waves with a planar wave front, i.e. the magnitude and phase of pressure perturbation (p) and particle velocity (u) are same on the plane perpendicular to the direction of wave propagation. This type of wave is generated in a tubular duct when the wave travels along its longitudinal axis. The basic equation (1D) for linearization can be achieved by taking elementary mass (Fig. 1), through which the acoustic wave is travelling,where

Elementary volume be dm
Ambient pressure p_0 and density ρ_0
Acoustic perturbations on pressure p and on density ρ
Control volume $\mathrm{d}V = \mathrm{d}x\mathrm{d}y\mathrm{d}z$
For acoustic waves $p/p_0 1, \rho/\rho_0 1$

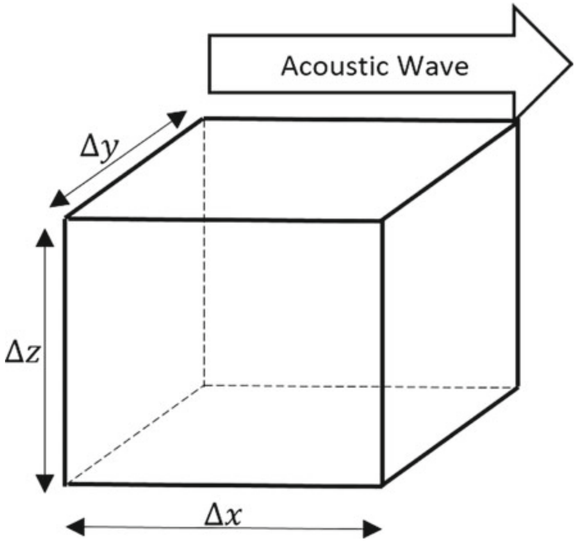

Fig. 1 Control volume showing acoustic wave flow in *x*-direction

- **Conservation of Mass**

Mass inflow

$$\mathrm{d}m_{in} = \rho_0 u \mathrm{d}y\mathrm{d}z$$

Mass outflow

$$\mathrm{d}m_{\mathrm{out}} = \left(\rho_0 u + \frac{\partial(\rho_0 u)}{\partial x}\mathrm{d}x\right)\mathrm{d}y\mathrm{d}z$$

Net mass increase in the control volume

$$\mathrm{d}m_{\mathrm{in}} - \mathrm{d}m_{\mathrm{out}} = \rho_0 u \mathrm{d}y\mathrm{d}z - \left(\rho_0 u + \frac{\partial(\rho_0 u)}{\partial x}\mathrm{d}x\right)\mathrm{d}y\mathrm{d}z$$

or

$$\mathrm{d}m_{\mathrm{in}} - \mathrm{d}m_{\mathrm{out}} = -\frac{\partial(\rho_0 u)}{\partial x}\mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{1}$$

Net change in density of control volume can be written as

$$\frac{\partial \rho}{\partial t}\mathrm{d}x\mathrm{d}y\mathrm{d}z = \mathrm{d}m_{\mathrm{in}} - \mathrm{d}m_{\mathrm{out}} \tag{2}$$

Equating Eqs. (1) and (2)

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = 0 \tag{3}$$

- **Conservation of Momentum**

Net change in momentum is equal to the net force acting on the control volume, i.e.

$$\frac{\partial(\rho_0 u \mathrm{d}x\mathrm{d}y\mathrm{d}z)}{\partial t} = p_0\mathrm{d}y\mathrm{d}z - (p_0 + \mathrm{d}p)\mathrm{d}y\mathrm{d}z$$

or

$$\rho_0 \mathrm{d}x\mathrm{d}y\mathrm{d}z \frac{\partial u}{\partial t} = -\mathrm{d}p\mathrm{d}y\mathrm{d}z$$

or

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\mathrm{d}p}{\mathrm{d}x}$$

or

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\mathrm{d}p}{\mathrm{d}x} = 0$$

As distance x and time t are mutually independent variables,

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \tag{4}$$

- **Conservation of Energy (Adiabatic Equation)**

For an adiabatic process,

$$\frac{p + p_0}{(\rho + \rho_0)^{\gamma}} = \text{constant (say } C)$$

Taking log both sides,

$$\log(p + p_0) - \gamma(\log(\rho + \rho_0)) = \log(C_1)$$

Differentiating with respect to ρ,

$$\frac{1}{p + p_0}\frac{\partial p}{\partial \rho} - \gamma\frac{1}{\rho + \rho_0} = 0$$

or

$$\frac{\partial p}{\partial \rho} = \gamma\frac{p + p_0}{\rho + \rho_0} \cong \gamma\frac{p_0}{\rho_0} = c_0^2\,(\text{say}) \tag{5}$$

Equation (5) implies

$$\mathrm{d}\rho = \frac{\mathrm{d}p}{c_0^2}; \quad \frac{\partial \rho}{\partial t} = \frac{1}{c_0^2}\frac{\partial p}{\partial t}; \quad \frac{\partial \rho}{\partial x} = \frac{1}{c_0^2}\frac{\partial p}{\partial x} \tag{6}$$

Substituting Eq. (6) in (3),

$$\rho_0\frac{\partial u}{\partial x} + \frac{1}{c_0^2}\frac{\partial p}{\partial t} = 0$$

Differentiating with respect to t,

$$\rho_0\frac{\partial^2 u}{\partial x\partial t} + \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = 0 \tag{7}$$

Differentiating Eq. (4) with respect to x,

$$\rho_0\frac{\partial^2 u}{\partial x\partial t} + \frac{1}{c_0^2}\frac{\partial^2 p}{\partial x^2} = 0 \tag{8}$$

Subtracting Eq. (7) from (8),

$$\frac{\partial^2 p}{\partial t^2} - c_0^2\frac{\partial^2 p}{\partial x^2} \tag{9}$$

This type of equation with homogeneous form and constant coefficients (c_0) has a general form of solution

$$p(x,t) = C_1 f\left(t - \frac{x}{c_0}\right) + C_2 f\left(t + \frac{x}{c_0}\right) \tag{10}$$

For proceeding to the solution of the above differential equation, let us assume time dependence to be of the exponential form $e^{\imath\omega t}$. Then, the solution becomes

$$p(x,t) = C_1 e^{\imath\omega\left(t - \frac{x}{c_0}\right)} + C_2 e^{\imath\omega\left(t + \frac{x}{c_0}\right)} \tag{11}$$

The first part repeats itself at $x = c_0 t$, i.e. forward travelling wave with velocity c_0, and second part of the same as backward travelling wave with same velocity. Therefore, c_0 is the wave velocity and C_1 and C_2 are the amplitudes of the forward and backward travelling waves.

Equation (11) can be rearranged as

$$p(x,t) = [Ae^{-\imath kx} + Be^{\imath kx}]e^{\imath\omega t} \tag{12}$$

where $k = \omega/c_0$, A is the amplitude of forward travelling wave and B is the amplitude of backward travelling wave. From Eq. (4),

$$-\rho_0 \frac{\partial u}{\partial t} = \frac{\partial p}{\partial x} = -\rho \frac{\partial^2 \phi}{\partial x \partial t} \text{ (say)}$$

or

$$u = \frac{\partial \phi}{\partial x}; \quad p = -\rho_0 \frac{\partial \phi}{\partial t} \tag{13}$$

where ϕ is referred as velocity potential function. Therefore, the dependent variable in the wave equation may be ϕ or u instead of p. So that same solution can be generated for u, i.e.

$$u(x,t) = [C_3 e^{-\imath kx} + C_4 e^{\imath kx}]e^{\imath\omega t} \tag{14}$$

Substituting Eqs. (12) and (14) in (4),

$$C_3 = \frac{A}{\rho_0 c_0}, \quad C_4 = -\frac{B}{\rho_0 c_0}$$

And therefore,

$$u(x,t) = \frac{1}{Z_0}\left[Ae^{-\imath kx} - Be^{\imath kx}\right]e^{\imath\omega t} \tag{15}$$

where $Z_0 = \rho_0 c_0$, the characteristic impedance (later referred as Y) of the material and S is the cross section of tube.

$$Y = \begin{cases} \rho_0 c_0 & \text{for particle velocity } u, \\ \frac{(\rho_0 c_0)}{S} & \text{for volume velocity } \upsilon_\upsilon, \\ \frac{c_0}{S} & \text{for mass velocity } \upsilon. \end{cases} \tag{16}$$

For 3D waves in an inviscid medium,

$$\frac{\partial^2 p}{\partial t^2} - c_0^2 \nabla^2 p = 0 \tag{17}$$

where ∇^2 is the 3D Laplacian operator.

2.1.1 Waves in Absorptive Stationary Medium

Not going much further into detail, the particle motion with heat conduction from walls of circular tube induces radial motion of particles. Due to the presence of viscosity, the radial and axial motion of the particles can be assumed to be independent.

After a lot of mathematics [1], the final results for wave number for a progressive wave in a duct can be written as

$$k = k_0 + \alpha \tag{18}$$

where $k_0 = \omega / c_0$ is the wave number in free medium and α is the attenuation constant.

The standing wave solution for ducts can be shown as

$$p(x,t) = \left[A\mathrm{e}^{-\alpha x - \imath k x} + B\mathrm{e}^{\alpha x + \imath k x}\right]\mathrm{e}^{\imath \omega t} \tag{19}$$

and the acoustic mass velocity,

$$\upsilon(x,t) = \frac{1}{Y}\left[A\mathrm{e}^{-\alpha x - \imath k x} - B\mathrm{e}^{\alpha x + \imath k x}\right]\mathrm{e}^{\imath \omega t} \tag{20}$$

where Y is the characteristic impedance for the forward travelling wave, where Y_0 is the characteristic impedance for inviscid medium given in Eq. (16),

$$Y = Y_0\left(1 - \frac{\alpha}{k_0} + \imath \frac{\alpha}{k_0}\right) \tag{21}$$

3 Acoustic Waves

For the measurement of a property (acoustic impedance) in an experimental set-up, it is very essential to correlate it to the properties which can be measured easily, accurately and cost-effectively. In our case, the sound pressure level can be easily measured using a microphone and, when measured over a period of time, is sufficient to indicate the properties of an acoustic wave at a point. Various experimental set-ups exist to correlate the sound pressure level of a region to acoustic properties like impedance and reflection coefficient of the medium.

3.1 Set-up Selection

The experiment for the measurement of acoustic impedance of a sample surface or membrane can be done using various experimental set-ups, each having individual advantages and disadvantages.

3.1.1 Sliding Microphone Probe Experiment (SME)

A sketch of the B&K Standing Wave Apparatus is shown in Fig. 2 [2]. A loudspeaker is mounted on one end of the acoustic tube and is used to produce acoustic wave. This wave gets reflected from the surface of the test sample, mounted on the other end of the impedance tube. If rigid wall is present on the end surface of the sample, the magnitude of reflection coefficient is set to unity. The sliding microphone probe is moved along the impedance tube to measure minimum and maximum pressure corresponding to maxima and minima of the standing wave generated inside the tube due to incident and reflected wave components. The ratio of maximum to minimum pressure is known as SWR (standing wave ratio). The positions and magnitudes of these maxima and minima can be used to measure acoustic impedance and absorption coefficient of the test surface.

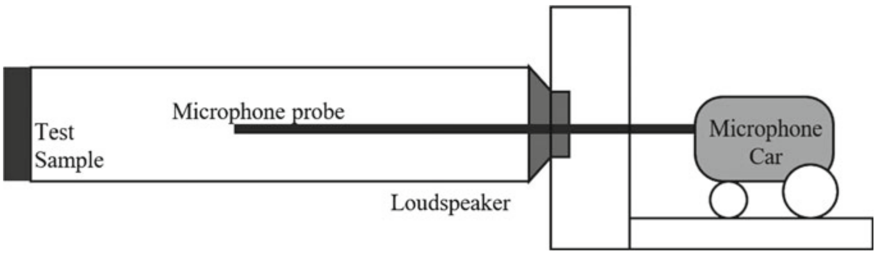

Fig. 2 Sliding microphone probe apparatus

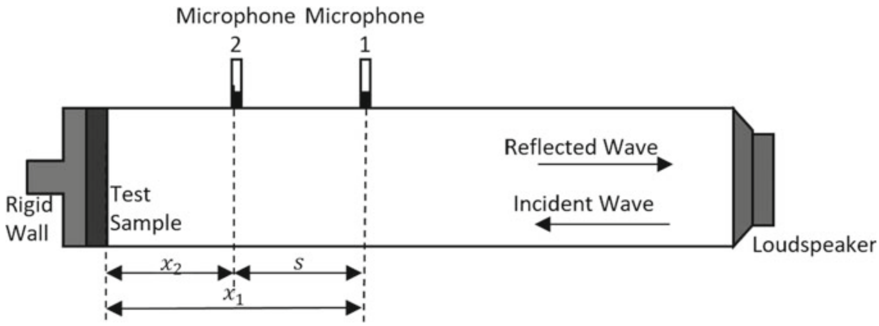

Fig. 3 Fixed microphone impedance tube apparatus

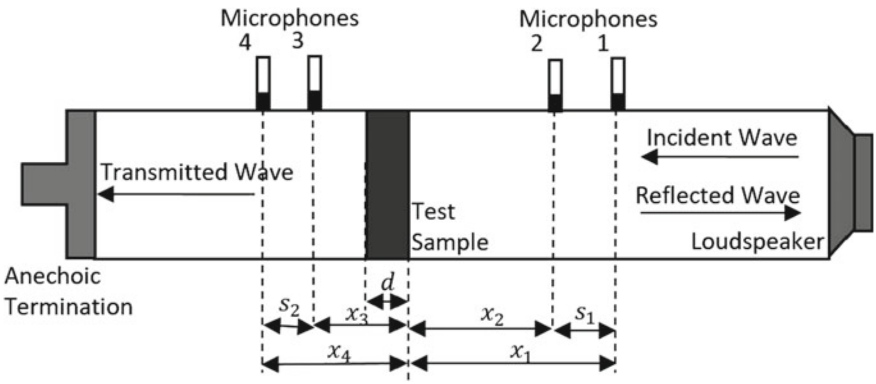

Fig. 4 Transfer matrix method apparatus

3.1.2 Fixed Two Microphone Impedance Tube Experiment (FME)

This method makes use of two microphones located at fixed positions [1]. The excitation can be random or discrete, depending on the requirement. A speaker is used to produce the acoustic wave inside the impedance tube. The signal picked by each microphone at the two fixed locations is used to calculate acoustic properties of the test sample. The auto- and cross-spectral densities of the individual signals at the two microphone locations are used to back-calculate the auto- and cross-spectral densities of the incident and reflected wave. These values are further processed to evaluate the required acoustic properties. A sample set-up diagram used for this experiment is shown in Fig. 3 [3].

3.1.3 Transfer Matrix Method (TMM)

This method is a slight modification of the fixed two microphone impedance tube experiment. An additional set of microphones is mounted on the other side of test sample [4]. The hard boundary or rigid wall is replaced with anechoic termination. A schematic is shown in Fig. 4. The readings from the four microphones are used to calculate the values of transfer matrix elements which can be further used to evaluate acoustic impedance and other properties of the test sample.

3.1.4 Comparison

The above methods have various advantages and disadvantages [1]:

- SME needs very simple instrumentation as only SPLs (sound pressure levels) need to be measured.
- SME is more accurate than other set-ups as the usual data acquisition and retrieval errors are absent.
- Any signal or combination of signals can be used in FME and TMM, but only sinusoidal signal is to be provided in SME with sufficient strength, to ensure SPL at pressure minima sufficiently higher than ambient noise.
- For different frequencies, in TMM and FME, the same set-up can be excited at different frequencies and data recorded can be used to measure impedance in a single experiment. But different set of readings are required for each frequency in case of SME.
- The acoustic field is likely to be disturbed by the moving probe tube in SME, while wall-mounted microphones and transient testing methods do not interfere with the acoustic field.
- In FME and TMM, tube length does not limit the measurement. But SME needs tube length at least one wavelength of the lowest frequency of interest.
- A more elaborate set-up is required for TMM than SME or FME, i.e. two extra microphones.
- The TMM approach is numerically more expensive than SME or FME.

For the current analysis, FME is used.

3.2 Theory

The sound waves are longitudinal waves which can be mathematically represented as [1]

$$p^{(r)} = p^{+} \mathrm{e}^{\imath\omega t}\left[\mathrm{e}^{\imath k x^{(r)}}\mathrm{e}^{\alpha x^{(r)}} + |R|\mathrm{e}^{-\imath\theta}\mathrm{e}^{-\imath k x^{(r)}}\mathrm{e}^{-\alpha x^{(r)}}\right] \tag{22}$$

and

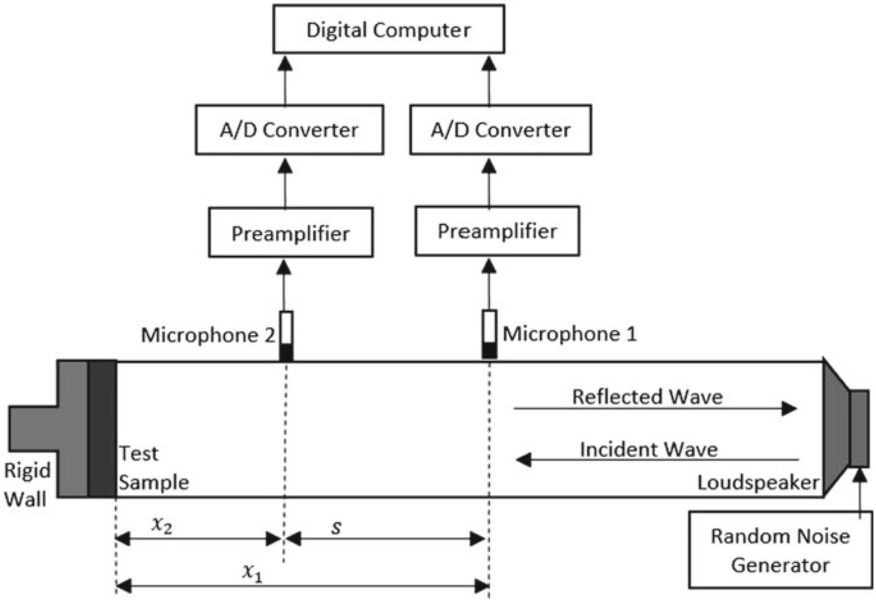

Fig. 5 Experimental set-up for fixed two microphone method

$$\left|p^{(r)}\right| = \left|p^{+}\right|\left[e^{2\alpha x^{(r)}} + |R|^{2}e^{-2\alpha x^{(r)}} + 2|R|\cos\left(\theta - 2kx^{(r)}\right)\right] \tag{23}$$

where

p^{+}	is the incident wave sound pressure level.
$x^{(r)}$	is the distance measured from reflective surface ($x = 0$) in the direction opposite to that of the incident wave at the rth position (Fig. 5).
$\|R\|$ and θ	are amplitude and phase of reflection coefficient of the test sample.
α	is the tube attenuation and k is the wave number for stationary medium in the tube.

3.2.1 Spectral Density

The formulation of the experiment will later involve the calculation of spectral densities of the acquired signal. So, introductory text is provided for reference and better understanding of the current problem being analysed.

It is used to express the signal as distribution with the frequency variation [5]. The spectral density can be expressed as the distribution of energy or power of the signal with respect to frequency variation. The same can be classified on the basis of the quantity used for expressing distribution:

- **Energy Spectral Density** The total energy of the signal is decomposed into frequency components. The basic requirement is that the energy of the signal is finite, i.e. the signal is concentrated over a finite time interval.
- **Power Spectral Density** The energy of the signal per unit time interval is decomposed into frequency components. This is used for signals distributed over the whole time period of experimental measurement, i.e. if the signal is extended on the time axis, it is present for infinite time interval, and therefore, the total energy of the signal is infinite.

As the acoustic signal acquired through the microphone is continuous during the measured time interval, the power spectral density is used.

The power spectral density can be mathematically expressed as

Auto-Spectral Density

$$S_{AA}(f) = \frac{A(f)A^*(f)}{N^2} \tag{24}$$

Cross-Spectral Density

$$S_{AB}(f) = \frac{A(f)B^*(f)}{N^2} \tag{25}$$

where

N	is the number of points acquired in the time domain signal.
$A(f)$ and $B(f)$	are the fast Fourier transform (FFT) of time series signal $a(t)$ and $b(t)$, respectively, sampled for time period T. These are defined as

$$A(f) = \frac{1}{T}\int_0^T a(t)e^{\imath 2\pi ft} \tag{26}$$

and

$$B(f) = \frac{1}{T}\int_0^T b(t)e^{\imath 2\pi ft} \tag{27}$$

3.2.2 Evaluation of Attenuation Constant

For rigid case termination,

$$|R| = 1 \quad \text{and} \quad \theta = 0$$

The wave number can be calculated as

$$k = k_0 + \alpha = \frac{\omega}{c_0} + \alpha$$

where k_0 is the free medium wave number, $c_0 (= \sqrt{\gamma RT})$ is free medium wave velocity (velocity of sound) and ω is the dominant angular frequency of incident wave.

Substituting the above values in Eq. (23),

$$\begin{aligned} \left|p^{(r)}\right| = |p^+| \Big[& \mathrm{e}^{2\alpha x^{(r)}} + \mathrm{e}^{-2\alpha x^{(r)}} + 2\cos(2k_0 x^{(r)})\cos(2\alpha x^{(r)}) \\ & -2\sin(2k_0 x^{(r)})\sin(2\alpha x^{(r)}) \Big] \end{aligned} \tag{28}$$

where $r = 1, 2, 3\ldots$ As α is of the order 10^{-2} for a tube up to 1 m, one may simplify the above expression and retain only up to second-degree terms of $2\alpha x^{(r)}$, in the series expansion.

Thus,

$$\begin{aligned} \left|p^{(r)}\right|^2 = 2|p^+| \Big[& 1 + 2\alpha^2 (x^{(r)})^2 \Big\{1 - \cos(2k_0 x^{(r)})\Big\} + \cos(2k_0 x^{(r)}) \\ & - 2\alpha x^{(r)} \sin(2k_0 x^{(r)}) \Big] \end{aligned} \tag{29}$$

Defining

$$\delta_{i,j} = \mathrm{SPL}_i - \mathrm{SPL}_j = 10\log_{10}\left|\frac{p^{(i)}}{p^{(j)}}\right|^2 = 10\log_{10}\beta_{i,j} \tag{30}$$

where $i, j = 1, 2, 3\ldots$

Here, $\beta_{i,j}$ can be evaluated from actual experimental values of level difference $\delta_{i,j}$.

From Eq. (29) and (30),

$$\beta_{i,j} = \frac{2\alpha^2 (x^{(i)})^2 [1 - \cos(2k_0 x^{(i)})] - 2\alpha x^{(i)} \sin(2k_0 x^{(i)}) + [1 + \cos(2k_0 x^{(i)})]}{2\alpha^2 (x^{(j)})^2 [1 - \cos(2k_0 x^{(j)})] - 2\alpha x^{(j)} \sin(2k_0 x^{(j)}) + [1 + \cos(2k_0 x^{(j)})]} \tag{31}$$

This equation, on rearrangement, reduces to a quadratic equation as

$$\alpha^2 - b_{i,j}\alpha + \frac{1}{2}c_{i,j} = 0 \quad \text{for} \quad i, j = 1, 2, 3\ldots \tag{32}$$

where

$$b_{i,j} = \frac{\beta_{i,j}x^{(j)}\sin(2k_0x^{(j)}) - x^{(i)}\sin(2k_0x^{(i)})}{\beta_{i,j}(x^{(j)})^2[1 - \cos(2k_0x^{(j)})] - (x^{(i)})^2[1 - \cos(2k_0x^{(i)})]}$$

and

$$c_{i,j} = \frac{\beta_{i,j}[1 + \cos(2k_0x^{(j)})] - [1 + \cos(2k_0x^{(i)})]}{\beta_{i,j}(x^{(j)})^2[1 - \cos(2k_0x^{(j)})] - (x^{(i)})^2[1 - \cos(2k_0x^{(i)})]}$$

For the above set-up, $i = 1$ and $j = 2$.

For every pair of positions (i,j), this quadratic yields two possible roots for α. The positive root is the required one. If both the roots are positive, then solution of a number of such quadratic equations formed from SPL observations at a number of points arranged in pairs (i,j) would be necessary to pick out the correct value of α as the one that repeats itself consistently.

3.2.3 Normalized Impedance and Reflection Coefficient

After determination of α and k, the sound pressure can be denoted as

$$p(x,t) = A(f)\mathrm{e}^{\imath(\omega t - kx)} + B(f)\mathrm{e}^{\imath(\omega t + kx)} \tag{33}$$

and

$$\upsilon(x,t) = \frac{1}{Y}[A(f)\mathrm{e}^{\imath(\omega t - kx)} - B(f)\mathrm{e}^{\imath(\omega t + kx)}] \tag{34}$$

Therefore,

$$p(0,t) = \{A(f) + B(f)\}\mathrm{e}^{\imath\omega t} \tag{35}$$

and

$$\upsilon(0,t) = \frac{1}{Y}\{A(f) - B(f)\}\mathrm{e}^{\imath\omega t} \tag{36}$$

where A and B are functions of frequency (f) and attenuation is ignored.

If p and υ are random functions of time, then the required impedance $Z(\omega)$ or $Z(f)$ at $x = 0$, normalized with respect to the characteristic impedance of tube Y, is

$$\frac{Z(f)}{Y} = Z_n(f) = \frac{S_{p\upsilon}(f)}{YS_{\upsilon\upsilon}(f)} \tag{37}$$

where

$S_{pv}(f)$ cross-spectral density between p and υ at $x = 0$.
$S_{\upsilon\upsilon}(f)$ auto-spectral density of υ at $x = 0$.

And these, in turn, are related to FFT (fast Fourier transform) of p and υ at $x = 0$ as

$$S_{p\upsilon}(f) = \frac{1}{T} P_0(f,T) V_0^*(f,T), \tag{38}$$

$$S_{\upsilon\upsilon}(f) = \frac{1}{T} V_0(f,T) V_0^*(f,T), \tag{39}$$

$$S_{pp}(f) = \frac{1}{T} P_0(f,T) P_0^*(f,T) \tag{40}$$

where superscript * denotes complex conjugate and T is the finite time of the record used in Fourier Transform as

$$P_0(f,T) = \frac{1}{T} \int_0^T P(0,t) \mathrm{e}^{-\imath\omega t} \mathrm{d}t \tag{41}$$

Therefore, taking FFT of Eqs. (35) and (36),

$$P_0(f,T) = A(f,T) + B(f,T) \tag{42}$$

$$V_0(f,T) = \frac{1}{Y} (A(f,T) - B(f,T)) \tag{43}$$

Substituting Eqs. (42) and (43) in Eqs. (38), (39) and (40) yields

$$S_{p\upsilon}(f) = \frac{1}{Y} \{ S_{AA}(f) - S_{BB}(f) - 2\imath Q_{AB}(f) \} \tag{44}$$

$$S_{\upsilon\upsilon}(f) = \frac{1}{Y^2} \{ S_{AA}(f) - S_{BB}(f) - 2C_{AB}(f) \} \tag{45}$$

$$S_{pp}(f) = S_{AA}(f) - S_{BB}(f) - 2C_{AB}(f) \tag{46}$$

where

$$S_{AA}(f) = \frac{1}{T} A(f) A^*(f) \tag{47}$$

$$S_{BB}(f) = \frac{1}{T} B(f) B^*(f) \tag{48}$$

$$S_{AB}(f) = \frac{1}{T} A(f) B^*(f) = C_{AB}(f) + \imath Q_{AB}(f) \tag{49}$$

Again, doing FFT (fast Fourier transform) of Eqs. (33) and (34) at $x = 1$ and $x = 2$,

Pressures,

$$P_1(f, T) = A(f, T)\mathrm{e}^{-\imath kx_1} + B(f, T)\mathrm{e}^{\imath kx_1} \tag{50}$$

$$P_2(f, T) = A(f, T)\mathrm{e}^{-\imath kx_2} + B(f, T)\mathrm{e}^{\imath kx_2} \tag{51}$$

and velocities,

$$V_1(f, T) = \frac{1}{Y}\left[A(f, T)\mathrm{e}^{-\imath kx_1} - B(f, T)\mathrm{e}^{\imath kx_1}\right] \tag{52}$$

$$V_2(f, t) = \frac{1}{Y}\left[A(f, T)e^{-\imath kx_2} - B(f, T)e^{\imath kx_2}\right] \tag{53}$$

Now,

$$\begin{aligned} S_{11}(f) &= \frac{1}{T}P_1(f, T)P_1^*(f, T) \\ &= S_{AA}(f) + S_{BB}(f) + 2[C_{AB}\cos(2kx_1) + Q_{AB}\sin(2kx_1)] \end{aligned} \tag{54}$$

$$\begin{aligned} S_{22}(f) &= \frac{1}{T}P_2(f, T)P_2^*(f, T) \\ &= S_{AA}(f) + S_{BB}(f) + 2[C_{AB}\cos(2kx_2) + Q_{AB}\sin(2kx_2)] \end{aligned} \tag{55}$$

$$S_{12}(f) = \frac{1}{T}P_1(f, T)P_2^*(f, T) = C_{12}(f) + \imath Q_{12}(f) \tag{56}$$

where

$$\begin{aligned} C_{12} = {} & S_{AA}\cos(k(x_1 - x_2)) + S_{BB}\cos(k(x_1 - x_2)) \\ & + 2[C_{AB}\cos(k(x_1 + x_2)) + Q_{AB}\sin(k(x_1 + x_2))] \end{aligned} \tag{57}$$

$$Q_{12} = -S_{AA}\sin(k(x_1 - x_2)) + S_{BB}\sin(k(x_1 - x_2)) \tag{58}$$

If written in matrix form, where $s = x_1 - x_2$

$$\begin{bmatrix} S_{11} \\ S_{22} \\ C_{12} \\ Q_{12} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2\cos(2kx_1) & 2\sin(2kx_1) \\ 1 & 1 & 2\cos(2kx_2) & 2\sin(2kx_2) \\ \cos(ks) & \cos(ks) & 2\cos(k(x_1 + x_2)) & 2\sin(k(x_1 + x_2)) \\ -\sin(ks) & \sin(ks) & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{AA} \\ S_{BB} \\ C_{AB} \\ Q_{AB} \end{bmatrix} \tag{59}$$

i.e.

$$[P_{12}] = [T][P_{AB}]$$

If taken inverse of $[T]$, the values S_{11}, S_{22}, C_{12} and Q_{12} can be used to obtain the values of S_{AA}, S_{BB}, C_{AB} and Q_{AB} as follows:

$$[T]^{-1} = \frac{1}{6 + 2\cos(2k(x_1 + x_2))} \begin{bmatrix} 1 & 1 & -2\cos(k(x_1 - x_2)) & -\frac{3+\cos(2k(x_1+x_2))}{\sin(k(x_1-x_2))} \\ 1 & 1 & 2 - 2\cos(k(x_1 - x_2)) & -\frac{3+\cos(2k(x_1+x_2))}{\sin(k(x_1-x_2))} \\ -\cos(2kx_2) & -\cos(2kx_1) & 2\cos(k(x_1 + x_2)) & 0 \\ -\sin(2kx_2) & -\sin(2kx_1) & 2\sin(k(x_1 + x_2)) & 0 \end{bmatrix} \tag{60}$$

$$[P_{AB}] = [T]^{-1}[P_{12}]$$

The above acquired matrix $[P_{AB}]$ can be used to calculate normalized impedance

$$\frac{Z(f)}{Y} = Z_n(f) = \frac{S_{AA}(f) - S_{BB}(f) - 2\imath Q_{AB}(f)}{S_{AA}(f) + S_{BB}(f) - 2C_{AB}(f)}$$

And the reflection coefficient

$$R(f) = |R|e^{\imath\theta} = \frac{Z_n(f) - 1}{Z_n(f) + 1} = \frac{-S_{BB}(f) + C_{AB}(f) - \imath Q_{AB}(f)}{S_{AA}(f) + C_{AB}(f) - \imath Q_{AB}(f)}$$

4 Measurement (Microphones and Transducers)

A microphone acts as a converter of acoustic property at the point of measurement to a more scalable quantity for computation, like voltage signal. The obtained voltage signal can be further used for analysis and computation. While selecting the type of microphone, one should pay special attention to the various specifications of the microphone and the experimental requirements like sensitivity, sampling rate, dynamic range (signal min-max), operation requirements (operating voltage and frequency), precision, accuracy and other parameters. The mismatch in any of these quantities may lead to inaccurate results; e.g. sampling rate should be at least twice the maximum measured frequency [6]; piezoelectric transducers may be used for robust applications; a high-voltage input may be required for high sensitivity measurements.

4.1 Sensitivity

Microphone sensitivity (S) is typically measured with a 1 kHz sine wave at a 94 dB sound pressure level (SPL), or 1 Pascal (Pa) pressure. The magnitude of the analog or digital output signal from the microphone with that input stimulus is a measure of its sensitivity. It is usually expressed as decibels (*dB*) with respect to a reference sensitivity of 1 V/Pa. Numerically [7],

$$S = 20 \times \log_{10} \frac{V \times p_0}{V_0 \times p} \text{dB} \quad \text{re} \quad \frac{1\,\text{V}}{\mu\text{Pa}}$$

Here, $p_0 = 1\,\mu\text{Pa}$ and $V_0 = 1\,\text{V}$.

4.2 Frequency Response

Frequency response is the magnitude and phase of the measured quantity as a function of frequency, with respect to input. For an acoustic system, the signal recorded should be undistorted, i.e. the frequency response line should be flat. But in practical case scenario, it is not so. Therefore, it needs to be studied for the region of concern and a particular microphone is selected accordingly.

Sample data for a typical microphone is provided in Fig. 6 [8].

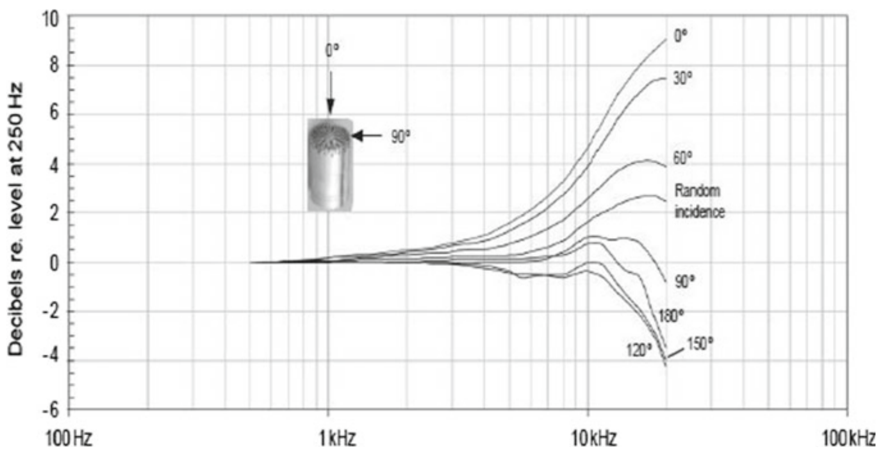

Fig. 6 Typical data for a 1/2″ microphone

4.3 Dynamic Range

It is the ratio of the largest to smallest value of the quantity being measured on decibel scale. A typical transducer has its own operating noise which distorts the very low values of the signal deciding the lower cut-off and very high values leading to saturation of the signal at peaks, thus deciding the higher cut-off.

Therefore, highly sensitive microphones are to be used for measuring very low signal values and low sensitivity microphones for high-level signals like impact noise.

5 Pre-amplification

The signal strength of microphone output is very small and if it is used to perform further processing, the SNR (signal-to-noise ratio) of that signal may get distorted. SNR is defined as the ratio of the strength of electrical signal output from microphone to that of unwanted interference. Mathematically,

$$\mathrm{SNR} = 10 \times \log_{10} \frac{P_{\mathrm{S}}}{P_{\mathrm{N}}}$$

where

P_{S} is the signal strength and
P_{N} is the noise strength.

As the noise is introduced mostly by the environmental disturbances and generally remains constant for a particular environment, P_{N} remains constant. The SNR can be improved simply by increasing P_{S}. So, pre-amplification is done to improve the SNR and thus accuracy of the measured signal. Due to this reason, the pre-amplifier is installed near the microphone, so as to minimize the distortion during transmission.

Most of the measurement devices come with inbuilt set-up for pre-amplification, and there is no need to add additional pre-amplifiers in the set-up.

6 Analog to Digital Conversion

As most of our processing and simulation is done through computers, the analog signal recorded by microphones is converted to digital signal via analog to digital converters. This signal is fed to the computer for further calculations and generation of the results.

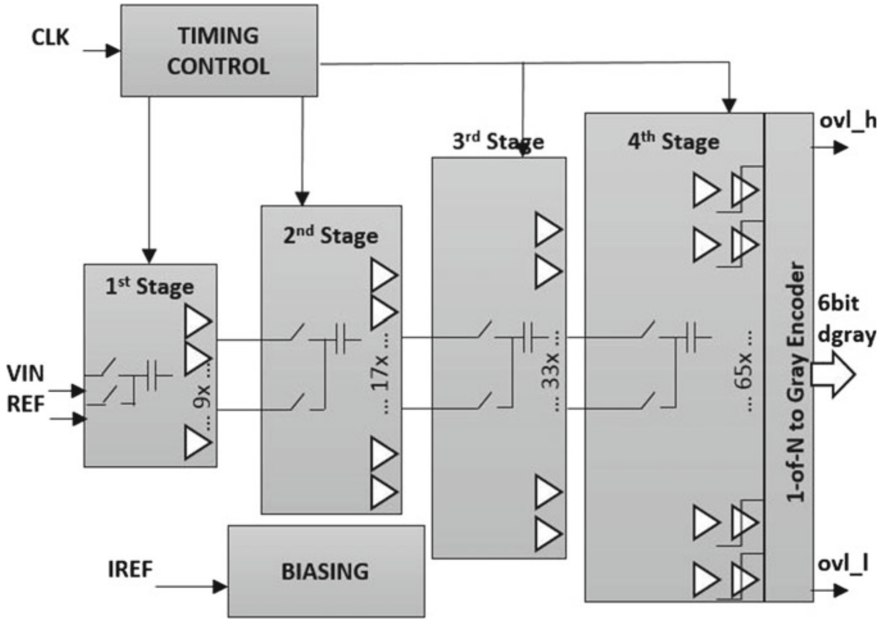

Fig. 7 Flash ADC

There are many methods for conversion of analog signal to digital signal given below.

6.1 Flash ADC

It is mostly used, where high sampling rate with low to moderate resolution is required. But high power consumption, size and cost are associated for higher precisions. A typical ADC is shown in Fig. 7 [9].

6.2 Sigma-Delta ADC

It encodes the analog signal to digital with at higher frequency, then applies digital filter to form high-resolution low sampling frequency digital output as shown in Fig. 8 [10]. It provides high resolution without the need of external precision equipment, but is slower due to oversampling.

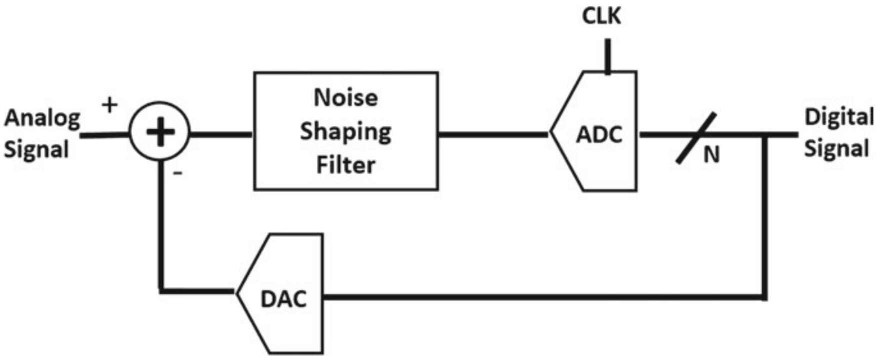

Fig. 8 Sigma-delta ADC

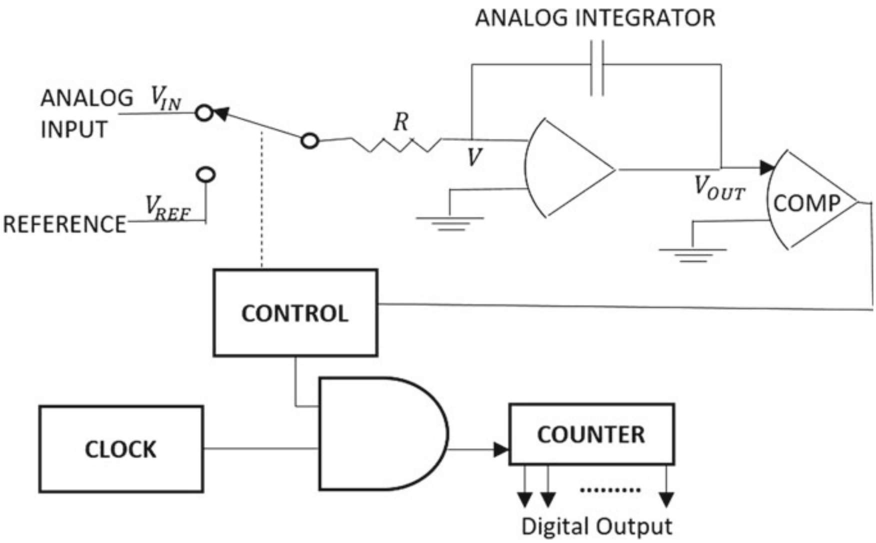

Fig. 9 Dual slope ADC

6.3 Dual Slope ADC

The sampled signal charges the capacitor for a fixed amount of time [11]. By integrating over time, noise integrates out of the conversion. Then, the ADC discharges the capacitor for a fixed rate, while a counter counts the ADCs output bits (Fig. 9). A longer discharge time results in a higher count. The signal is more immune to noise and has high accuracy, but it is slow and requires high precision external components to achieve accuracy.

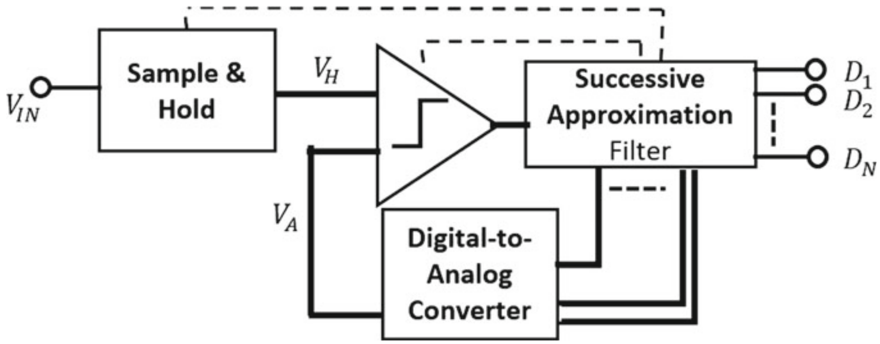

Fig. 10 Successive approximation converter

6.4 Successive Approximation Converter

It sets MSB (most significant bit) and converts it to analog form using DAC [12]. Then, it compares guess to input, sets next MSB and tests again. It is capable of high speed and lower cost, but has medium accuracy. Also, the speed is limited to 5 Msps and high resolution leads to slower speed. A sample SAC is shown in Fig. 10.

7 Analysis

The obtained signal in digital form is fed to the computer, and the formulation is done according to the above theory. The calculation and result generation can be done using various methods like

- Direct feeding to the custom designed circuits which use various arithmetic and logic gate circuits, and measuring the output signal for obtaining the required results. A circuit can be generated using operational amplifiers.
- Coding on software like MATLAB so that on feeding the digital signal in matrix form, it generates result.
- Making a block diagram using signal processing approach for the required framework calculations on software like LABVIEW.

8 Precautions

While performing an analysis, it is important to know what is to be done in the experiment, but it is equally and sometimes more important to know the things which are not to be done or taken care of, during the analysis. A little negligence in the procedure may lead to drastically erroneous results, which may become useless even after such an elaborate set-up and hard work. Some of them include the following:

8.1 Deviation from Assumptions

In the current analysis, there is an assumption that the travelling wave is planar in nature. But the acoustic waves remain planar in a duct, only up to a specific upper bound frequency. Above this frequency, the wave becomes non-planar and the results tend to include much higher noise content.

The planar wave frequency [2] for a cylindrical duct waveguide of diameter d in metre is

$$f_{\text{planewave}} \leq \frac{202}{d}$$

Taking two different frequencies for a given diameter pipe, one in the planar wave region and the other in the non-planar wave region, the following output has very high noise content for deviation from the assumption.

As seen in the graphs in the end (Figs. 11 and 12), as the frequency is increased above the given limit, the non-planar wave is generated. As the theory above is formulated with respect to planar waves, the non-planar part is treated as noise. The resulting analysis is full of noise. As seen in the graphs, the planar wave gives much smoother curve than the non-planar wave, when plotted for wave number for the same tube.

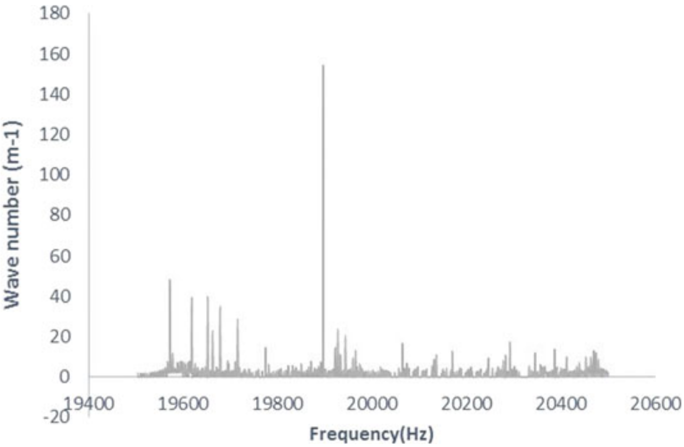

Fig. 11 Wave number for frequency corresponding to non-planar waveform

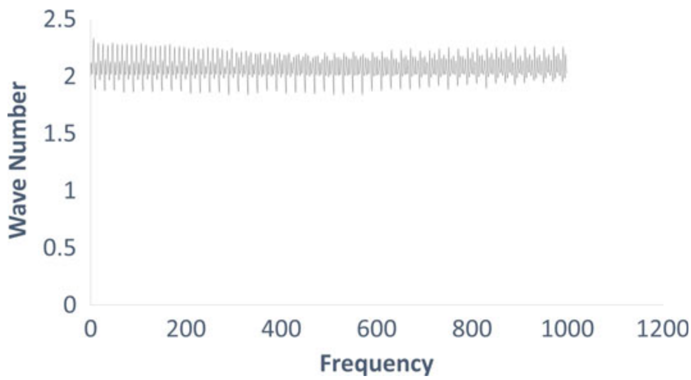

Fig. 12 Wave number for frequency corresponding to planar waveform

8.2 Environmental Effects

Always keep track of all the variables involved in the analysis, which tend to modify with the variation of environmental factors. One may calculate a specific result in a particular scenario or use it from some other source, but environmental conditions involved in the analysis of the source are to be taken care of. In our case, the speed of sound c_0 depends on surrounding temperature and altitude [13].

$$c_0 = \sqrt{\frac{\gamma R_0 T}{m_0 \left(1 + \frac{\gamma m_0 g^2 z^2}{C_p R_0 T}\right)}}$$

Some sample data values are provided in Table 1 to show the variation of the acoustic wave velocity with respect to the environmental factors.

8.3 Special Anomalies

Sometimes, the property under test involves some special or exceptional configurations which may produce some strange results, even when the experimental conditions are well taken care of. A similar special case scenario is experienced when the distance between the two microphones reaches a particular value. The reflection coefficient becomes indeterminate at that particular frequency [14]. Mathematically,

$$s \leq \frac{c_0}{2 f_{\text{mean}}}$$

It can be seen in the graphs plotted (Figs. 13 and 14) that there is a huge discontinuity in the curve at about 2800 Hz. This is obtained due to microphone

Table 1 Acoustic wave velocity

Atmospheric temperature T(K)	Altitude z(m)	Acoustic wave velocity c_0(m/s)
288.15	0	340.3
287.82	50	340.11
287.5	100	339.91
286.85	200	339.5
286.2	300	339.06
285.55	400	338.61
284.9	500	338.13
284.25	600	337.64
283.6	700	337.12
282.95	800	336.59
282.3	900	336.03
281.65	1000	335.45
275.15	2000	328.47
268.65	3000	319.34
262.15	4000	308.12
255.65	5000	295.04
249.15	6000	280.42
242.65	7000	264.66
236.15	8000	248.18
229.65	9000	231.38
223.15	10,000	214.65
216.65	11,000	198.26

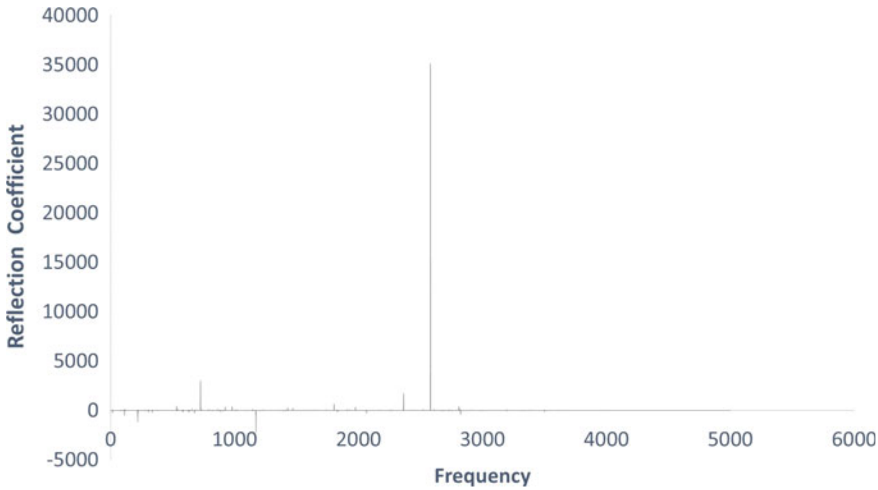

Fig. 13 Reflection coefficient when microphone spacing is integral multiple of half wavelength of test signal

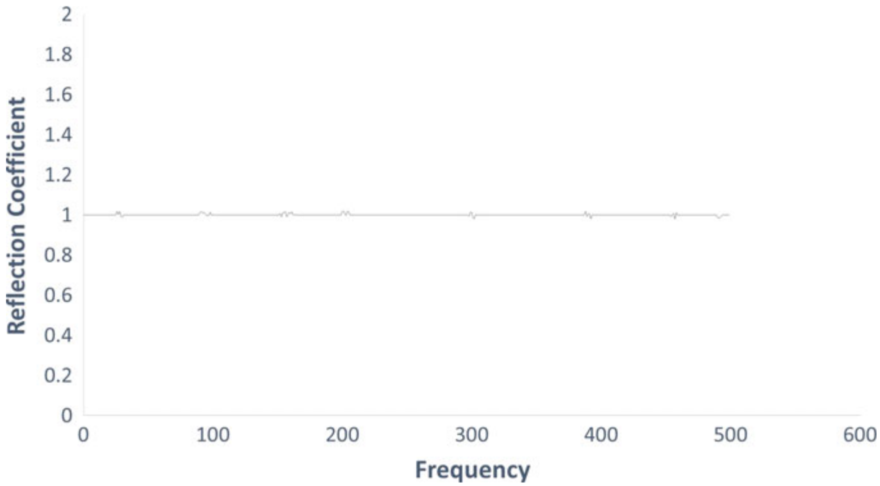

Fig. 14 Reflection coefficient when microphone spacing is less than half wavelength of test signal

spacing of 6 cm. If the lower frequencies are used or the microphone spacing is reduced, the problem can be completely eliminated.

8.4 Methods to Enhance Accuracy

Most of the assumptions are made on the basis of approximation and at the cost of accuracy. These approximations are made to reduce the computational complexity and sometimes additional set-up requirements for the required analysis. In our case, as a transient signal is being recorded, there is an instrument phase mismatch [15] introduced while acquiring the data from two different sources. There is no additional calculations done while formulation, to eliminate this error. But this error can be eliminated by introducing a little modification in the transfer function between the two microphones. This transfer function is proportional to cross-spectral density. So, the method suggests to use the original and switched microphone configurations to generate the two different transfer functions. The geometric mean of these transfer functions is the original transfer function required with instrument phase mismatch eliminated. Mathematically,

$$H_{12} = \sqrt{H_{12}^{\text{original}} H_{12}^{\text{switched}}}$$

where

$$H_{12} = \frac{S_{12}}{S_{11}}.$$

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