

Contents

Preamble

1	Looking Backward: From Euler to Riemann	1
	Athanasios Papadopoulos	
1	Introduction	2
2	Functions	10
3	Elliptic Integrals	20
4	Abelian Functions	30
5	Hypergeometric Series	32
6	The Zeta Function	33
7	On Space	40
8	Topology	46
9	Differential Geometry	60
10	Trigonometric Series	66
11	Integration	76
12	Conclusion	78
	References	81

Part I Mathematics and Physics

2	Riemann on Geometry, Physics, and Philosophy—Some Remarks	97
	Jeremy Gray	
1	Introduction	97
2	The <i>Hypotheses</i>	98
3	Influences	101
4	Heat Diffusion and the <i>Commentatio</i>	106
	References	108

3	Some Remarks on “A Contribution to Electrodynamics” by Bernhard Riemann	111
	Hubert Goenner	
1	Introduction	111
2	Riemann’s New Result of 1858: The Retarded Potential	112
3	Gauss, Weber, and Riemann on Electrodynamic Interaction.	114
4	Riemann’s Paper.	117
5	Concluding Remarks	119
	References.	122
4	Riemann’s Memoir <i>Über das Verschwinden der ϑ-Functionen</i>	125
	Christian Houzel	
1	Jacobi’s Inversion Problem.	125
2	A Crucial Observation on Theta Functions.	128
3	The First Step of Riemann’s Proof.	129
4	The Second Step of Riemann’s Proof.	130
5	The Conclusion of the Proof.	130
6	Later Developments	133
	References.	133
5	Riemann’s Work on Minimal Surfaces	135
	Sumio Yamada	
1	Introduction	135
2	On the Surface of Least Area with a Given Boundary	136
3	Representation Formulas by Riemann and Weierstrass-Enneper.	147
4	Closing Remarks	149
	References.	150
6	Physics in Riemann’s Mathematical Papers	151
	Athanase Papadopoulos	
1	Introduction	151
2	Function Theory and Riemann Surfaces	162
3	Riemann’s Memoir on Trigonometric Series.	173
4	Riemann’s Habilitationsvortrag 1854—Space and Matter.	181
5	The <i>Commentatio</i> and the <i>Gleichgewicht der Electricität</i>	193
6	Riemann’s Other Papers.	195
7	Conclusion	199
	References.	199
7	Cauchy and Puiseux: Two Precursors of Riemann.	209
	Athanase Papadopoulos	
1	Introduction	209
2	Algebraic Functions and Uniformization.	210
3	Puiseux and Uniformization	212

4	Cauchy and His Work on Functions of a Complex Variable	220
5	Uniformization Again	230
	References.	232
8	Riemann Surfaces: Reception by the French School.	237
	Athanase Papadopoulos	
1	Introduction	238
2	Riemann Surfaces.	240
3	The Nineteenth-Century French Treatises on Analysis	244
4	Simart’s Dissertation.	271
5	Other French Dissertations and Other Works of Riemann	275
6	On the Relations Between the French and German Mathematicians.	281
7	In a Way of Conclusion	284
	References.	286
 Part II Philosophy		
9	The Origin of the Notion of Manifold: From Riemann’s Habilitationvortrag Onward	295
	Ken’ichi Ohshika	
1	Introduction	295
2	Kantian Worldview.	296
3	Riemann’s Habilitationvortrag.	298
4	Poincaré’s Analysis Situs	301
5	Definitions Using Local Charts According to Hilbert, Weyl, Kneser and Veblen-Whitehead	305
6	Conclusion: Philosophical Significance.	308
	References.	308
10	Deleuze et la Géométrie Riemannienne: Une Topologie des Multiplicités	311
	Franck Jedrzejewski	
1	Introduction	311
2	Variété et multiplicité	312
3	Espaces, mesures et multiplicités	316
4	Typologies des multiplicités	319
5	Conclusion	324
6	Extended English Abstract	325
	References.	327

11 Comprehending the Connection of Things: Bernhard Riemann and the Architecture of Mathematical Concepts 329
Arkady Plotnitsky

1 Introduction 329

2 Philosophy: Planes of Thought and the Architecture of Concepts 336

3 Mathematics: Space, Geometry, and the Concept of Manifold 343

4 Physics: “The Reality Underlying Space” 353

5 Conclusion 361

References 362

Part III Some Recent Developments

12 The Riemann Mapping Theorem and Its Discrete Counterparts 367
Feng Luo

1 Introduction 367

2 Koebe–Andreev–Thurston’s Circle Packing Theorem 368

3 A Discrete Uniformization Theorem 375

References 387

13 The Riemann–Roch Theorem 389
Norbert A’Campo, Vincent Alberge and Elena Frenkel

1 Introduction 389

2 Line Bundles 391

3 Sheaf Cohomology 396

4 Further Preparations 400

5 The Riemann–Roch Theorem 405

6 Divisors and the Riemann–Roch Theorem 407

7 The Use of the Riemann–Roch Theorem in Teichmüller’s Work 409

References 410

14 Metric Geometries in an Axiomatic Perspective 413
Victor Pambuccian, Horst Struve and Rolf Struve

1 Introduction 414

2 Metric Planes 422

3 Higher-Dimensional Metric Spaces 433

4 The Dimension-Free Case 436

5 Projective-Metric Geometry 437

6 Cayley-Klein Geometries 442

References 451

15	Generalized Riemann Sums	457
	Toshikazu Sunada	
1	Introduction	457
2	Generalized Riemann Sums	460
3	Classical Example 1	462
4	Classical Example 2	464
5	The Inclusion-Exclusion Principle	468
6	Generalized Poisson Summation Formulas	472
7	Is $\mathbb{Z}_{\text{prim}}^d$ a Quasicrystal?	475
	References.	478
16	From Riemannian to Relativistic Diffusions	481
	Jacques Franchi	
1	Introduction	481
2	Euclidean Brownian Motion	484
3	Riemannian Brownian Motion	486
4	The Relativistic Dudley Diffusion in Minkowski Space	487
5	The Lorentzian Frame Bundle $G(\mathcal{M})$ over (\mathcal{M}, g)	490
6	The Basic Relativistic Diffusion	493
7	Covariant Ξ -relativistic Diffusions	500
8	Example of Robertson-Walker (R-W) Manifolds	503
9	Sectional Relativistic Diffusion.	509
	References.	510
 Part IV Relativity		
17	On the Positive Mass Theorem for Closed Riemannian Manifolds	515
	Andreas Hermann and Emmanuel Humbert	
1	Introduction	515
2	ADM Mass in General Relativity	517
3	The Mass of a Closed Manifold	527
4	Equivalence of the Two Positive Mass Conjectures	532
5	Some Recent Results on the Positive Mass Conjecture.	533
6	An Idea of the Proof of Theorem 5.2	535
7	Preservation of Mass by Surgery	538
	References.	539
18	On Local Characterization Results in Geometry and Gravitation	541
	Marc Mars	
1	Introduction	541
2	Classical Characterizations	542

3	Local Characterizations of the Schwarzschild and Kruskal Spacetimes	546
4	Local Characterization of pp-Waves and Related Spacetimes	555
5	Local Characterizations of the Kerr, Kerr–Newman and Kerr–De Sitter Metrics	559
	References.	567
19	The Conformal Approach to Asymptotic Analysis	571
	Jean-Philippe Nicolas	
1	Introduction	571
2	Conformal Compactification	574
3	Peeling	581
4	Conformal Scattering	593
5	Concluding Remarks	607
	References.	607
 Part V Concluding Chapter		
20	Bernhard Riemann and His Work	613
	Lizhen Ji	
1	Introduction	613
2	Riemann’s Work I: His Best Known Works.	615
3	Riemann’s Work II: Some Little Known or Even Unknown Works.	619
4	Riemann’s Publications and his Impact	621
5	How Riemann Developed.	629
6	People Who Influenced Riemann	632
	References.	634
Index	637



<http://www.springer.com/978-3-319-60038-3>

From Riemann to Differential Geometry and Relativity

Ji, L.; Papadopoulos, A.; Yamada, S. (Eds.)

2017, XXXIV, 647 p. 24 illus., Hardcover

ISBN: 978-3-319-60038-3