# Contents

Part I  Existence and Regularity Results, Quantitative Methods and Their Convergence

1  Qualitative Methods for Classes of Nonlinear Systems:
   Constructive Existence Results ........................................ 3
   1.1  First Order Differential-Operator Equations and Inclusions .... 3
       1.1.1  Setting of the Problem .................................... 3
       1.1.2  Main Assumptions ....................................... 5
       1.1.3  Special Basis and Approximations for Multi-valued Mappings .................................. 8
       1.1.4  Results ........................................ 10
   1.2  Second Order Operator Differential Equations and Inclusions 30
   1.3  Evolitional Variational Inequalities: Penalty Method and Strong Solutions ................................... 33
   1.4  Nonlinear Parabolic Equations of Divergent Form ............. 42
   References .................................................................. 43

2  Regularity of Solutions for Nonlinear Systems .................. 47
   2.1  Regularity of All Weak Solutions for a Parabolic Feedback Control Problem ........................................ 47
   2.2  Artificial Control Method for Nonlinear Partial Differential Equations and Inclusions: Regularity of All Weak Solutions .... 50
   2.3  Regularity of All Weak Solutions for Nonlinear Reaction-Diffusion Systems with Nonlinear Growth ............. 52
       2.3.1  Reaction-Diffusion Equations ............................ 52
       2.3.2  Systems of Reaction-Diffusion Equations .............. 55
   2.4  Examples of Applications ........................................ 60
       2.4.1  A Parabolic Feedback Control Problem ............... 60
       2.4.2  A Model of Conduction of Electrical Impulses in Nerve Axons ........................................ 61
       2.4.3  Climate Energy Balance Model ............................. 61
3 Advances in the 3D Navier-Stokes Equations ......................... 69
  3.1 Weak, Leray-Hopf and Strong Solutions .......................... 69
  3.2 Leray-Hopf Property for a Weak Solution of the 3D
      Navier-Stokes System: Method of Artificial Control ............ 72
  3.3 The Existence of Strong Solutions and 1-Dimensional
      Dynamical Systems .................................... 74
  3.4 Extremal Solutions: Existence and Continuity Results
      in Strongest Topologies .................................. 79
References ................................................ 86

Part II Convergence Results in Strongest Topologies

4 Strongest Convergence Results for Weak Solutions
of Non-autonomous Reaction-Diffusion Equations
with Carathéodory’s Nonlinearity .................................... 89
  4.1 Translation-Compact, Translation-Bounded and Translation
      Uniform Integrable Functions ................................ 89
  4.2 Setting of the Problem ...................................... 90
  4.3 Preliminary Properties of Weak Solutions .................... 91
  4.4 Strongest Convergence Results in $C^{\text{Loc}}(\mathbb{R}_+; H)$ ...... 95
  4.5 Strongest Convergence Results for Solutions in the Natural
      Extended Phase Space .................................... 98
  4.6 Examples of Applications .................................. 106
      4.6.1 Non-autonomous Complex Ginzburg–Landau Equation ..... 106
      4.6.2 Non-autonomous Lotka–Volterra System
            with Diffusion ..................................... 107
References ................................................ 108

5 Strongest Convergence Results for Weak Solutions
of Feedback Control Problems ..................................... 111
  5.1 Setting of the Problem ...................................... 111
  5.2 Regularity of All Weak Solutions and Their Additional
      Properties ............................................... 112
  5.3 Convergence of Weak Solutions in the Strongest Topologies ... 113
  5.4 Examples of Applications .................................. 116
      5.4.1 A Model of Combustion in Porous Media .................. 116
      5.4.2 A Model of Conduction of Electrical Impulses
            in Nerve Axons ..................................... 117
      5.4.3 Climate Energy Balance Model .......................... 118
References ................................................ 122
### 6 Strongest Convergence Results for Weak Solutions of Differential-Operator Equations and Inclusions

- **6.1 First Order Differential-Operator Equations and Inclusions**
  - 6.1.1 Convergence Results for Autonomous Evolution Equations
  - 6.1.2 Convergence Results for Nonautonomous Evolution Inclusions

- **6.2 Second Order Operator Differential Equations and Inclusions**

- **6.3 Examples of Applications**
  - 6.3.1 Nonlinear Parabolic Equations of Divergent Form
  - 6.3.2 Nonlinear Non-autonomous Problems on Manifolds with and Without Boundary: A Climate Energy Balance Model
  - 6.3.3 A Model of Conduction of Electrical Impulses in Nerve Axons
  - 6.3.4 Viscoelastic Problems with Nonlinear “Reaction-Displacement” Law

### References

---

### Part III Uniform Global Behavior of Solutions: Uniform Attractors, Flattening and Entropy

- **7 Uniform Global Attractors for Non-autonomous Dissipative Dynamical Systems**
  - 7.1 General Methodology
  - 7.2 Main Constructions and Results
  - 7.3 Proof of Theorem 7.1
  - 7.4 Example of Applications
  - 7.4.1 Autonomous Evolution Problem
  - 7.4.2 Non-autonomous Evolution Problem
  - 7.4.3 Non-autonomous Differential-Operator Inclusion

### References

---

### 8 Uniform Trajectory Attractors for Non-autonomous Nonlinear Systems

- 8.1 Uniform Trajectory Attractor for Non-autonomous Reaction-Diffusion Equations with Carathéodory’s Nonlinearity

- 8.2 Structure of Uniform Global Attractor for Non-autonomous Reaction-Diffusion Equations

- 8.3 Uniform Trajectory Attractors for Nonautonomous Dissipative Dynamical Systems

- 8.4 Notes on Applications

### References

---
9 **Indirect Lyapunov Method for Autonomous Dynamical Systems** . . . 211

9.1 First Order Autonomous Differential-Operator Equations and Inclusions. 211

9.2 Second Order Autonomous Operator Differential Equations and Inclusions. 215

9.3 Examples of Applications. 219

9.3.1 A Model of Combustion in Porous Media. 220

9.3.2 A Model of Conduction of Electrical Impulses in Nerve Axons. 220

9.3.3 Viscoelastic Problems with Nonlinear “Reaction-Displacement” Law. 221

References. 232

Index. 239
Qualitative and Quantitative Analysis of Nonlinear Systems
Theory and Applications
Zgurovsky, M.Z.; Kasyanov, P.O.
2018, XXXIII, 240 p. 43 illus., 23 illus. in color., Hardcover
ISBN: 978-3-319-59839-0