

Preface

Quantum mechanics is usually studied for systems where the position and momentum take all real values. Weyl and later Schwinger studied quantum systems where the position and momentum take a finite number of values. The emergence of the subject of quantum information created a lot of interest in such systems, which became known as qudits. This research is interdisciplinary and combines ideas from mathematics, physics, computer science, chemistry, and materials science. Within this general area, the so-called problem of mutually unbiased bases led to the study of quantum systems with variables in Galois fields. The monograph discusses various aspects of these ‘finite quantum systems.’

It also considers rigorously the limit where the dimension of the system becomes very large using ideas from the timely area of profinite groups. These ‘profinite quantum systems’ have links to a different area of research, known as p-adic physics. The latter studies quantum systems with variables which are p-adic numbers and combines algebraic number theory with quantum physics. It has applications in condensed matter, particle physics, string theory, etc. The monograph approaches this area from a different angle, using inverse and direct limits and profinite groups.

I have a strong interest in all these areas, and I have written three review articles on finite quantum systems (in 2004), on systems with variables in Galois fields (in 2007), and on systems with variables which are p-adic numbers (in 2013). They are the background for this monograph, but the material is completely rewritten, some gaps have been filled, and other topics such as finite geometries, mutually unbiased bases, weak mutually unbiased bases, and quantum logic have been added.

Overall, the aim of this monograph is to present the material by adding a novel flavor to it (as discussed in section 1.4 below). The presentation is concise but informative, and the general theory is complemented with examples. The level of rigor is appropriate for a mathematical physics monograph. The proofs describe the important steps, so that the reader can easily fill the gaps.

The monograph is suitable for Ph.D. students and other researchers in quantum physics, quantum optics, quantum information, p-adic physics, mathematical

physics, applied mathematics, and computer science. The reader is expected to have some knowledge of certain mathematical areas, as follows:

- In Chap. 5, about finite geometries.
- In Chap. 6, about lattice theory.
- In Chaps. 8 and 9 about Galois theory.
- In Chaps. 10, 11, and 12 about p-adic numbers, profinite groups, and inverse and direct limits.

There is a brief introduction to these concepts in the monograph (with references to the literature), but the aim is to establish the notation and explain how to do practical calculations. So some prior knowledge of these topics will be helpful.

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Apostolos Vourdas



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Vourdas, A.

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