# Contents

Foreword (by A. Eamonn Kelly) ................................................................. v  
Contents ............................................................................................................. vii  
List of Figures .................................................................................................. xi  
List of Tables .................................................................................................... xiii  
Overall Book Abstract, and Individual Chapter Abstracts ............................... xv  
Preface to the Series ........................................................................................ xxi  
Preface to the Book ........................................................................................ xxiii

1. **Identifying a Problem with School Algebra** ............................................. 1  
   A Fundamental Problem with School Algebra ........................................... 1  
   Some Performance Data, and Associated Critiques of Practices in School Algebra .... 2  
   The Background to How and Why this Book was Written ......................... 5  
   Overview of this Book ............................................................................... 6

2. **Historical Reflections on How Algebra Became a Vital Component of Middle- and Secondary-School Curricula** .................................................. 11  
   Providing Historical Frameworks for Mathematics Education Research ........ 11  
   Algebra in Secondary School Mathematics: The Debate Over Purpose .......... 14  
   Is School Algebra a Unidimensional Trait? ................................................ 44  
   The History of Mathematics and the History of School Mathematics .......... 47

3. **Framing a Classroom Intervention Study in a Middle-School Algebra Environment** ................................................................. 59  
   Applying Principles of Design Research in Mathematics Education ............ 59  
   The Role of Theory in the Middle-School Algebra Investigation .................. 62

4. **Document Analysis: The Intended CCSSM Elementary- and Middle-School Algebra Curriculum** .......................................................... 71  
   Approaches to Introducing Algebra: Structure and Modeling ....................... 71  
   Overview of the Main Study ....................................................................... 72  
   Concepts of Algebraic Structure and Modeling, as Presented in the Common-Core State Standards for Mathematics ................................................. 73  
   Structural and Functional Approaches, as Presented in the Mathematics Textbook Used by Participating Students ................................................. 80  
   The Problem as it Appeared to be at the Beginning of the Study .................. 83

5. **Review of Pertinent Literature** ............................................................... 87  
   The Design Research Foundation for the Study ......................................... 88  
   Theoretical Bases for the Study .................................................................. 90  
   Signifiers, Objects, Interpretants, and Charles Sanders Peirce ..................... 91  
   Cognitive Structures and Individual Learners ............................................. 102
Contents

6. Research Design and Methodology .................................................................115
   Setting up the Intervention ........................................................................116
   Professional Development in a Theoretical Context ..............................121
   Intervention Setting and Theoretical Base .............................................121
   Procedures ................................................................................................123
   Instrumentation .......................................................................................124
   Research Hypotheses, and Issues Related to the Quantitative Analysis ....128
   Issues Related to the Qualitative Analyses ............................................136
   Concluding Comments ............................................................................138

7. Quantitative Analyses of Data .................................................................141
   Overview of Quantitative Data ...............................................................141
   Analyses of Pre-Teaching Data ..............................................................144
   Calculation of Effect Sizes ...................................................................152
   Summary and Concluding Comments on the Quantitative Analyses .......153

8. Qualitative Analyses of Data .................................................................155
   Intended, Implemented and Received Curricula ....................................155
   Peirce’s Triadic Semiotic Position and Herbart’s Theory of Apperception ..156
   Analyses of Qualitative Data Generated by the “Structure” Intervention ..158
   Analyzing Samples of Interview Data Relating to “Structure” ..............160
   Analyses of Qualitative Data Generated by the Modeling Intervention .....166
   Cognitive Growth in Modeling Related to the Subscript Notation for Sequences 170
   Cognitive Growth in Modeling Related to Generalizing for the $n$th Term .171
   Analyses of Student Responses to the Six Interview Tasks ....................174
   Qualitative Analyses of the Implemented Curriculum .........................186
   Concluding Comments with Respect to the Qualitative Analyses ..........189

9. Answers to Research Questions, and Discussion ......................................193
   Answer to Research Question 1 .............................................................193
   Answer to Research Question 2 .............................................................195
   Answer to Research Question 3 .............................................................197
   Answer to Research Question 4 .............................................................199
   Answer to Research Question 5 .............................................................201
   Answer to Research Question 6 .............................................................203
   Possibilities for Future Related Research ............................................205
   Final Comments on the Workshop “Lessons” .......................................206
   Comments on the Study by the Two Participating Teachers .................207

10. Postscript: Framing Research Aimed at Improving School Algebra ..........211
    Sometimes the Most Appropriate Theories for Mathematics Education
    Research will Come from the Past .......................................................211

Literature Which Helped Frame the Design of the Teaching Intervention ..........106
Research Questions ..................................................................................108
Giving Precedence to Peirce’s, Herbart’s, and Del Campo and Clements’ Theories ......212
Did the Intervention Improve the Participating Students’ Knowledge of, and Ability to Generalize, and Apply, School Algebra? ................................. 216
Moving Forward .............................................................................................................. 217

Author Biographies ............................................................................................................ 223
List of Appendices ............................................................................................................ 225
Appendix A: Protocol for Algebra Interviews with Seventh-Graders .......................... 227
Appendix B: Algebra Test (Three Parallel Versions Are Reproduced) ....................... 229
Appendix C: “Questionnaire” Completed by Seventh-Grade Students at School W at the Beginning of the Algebra Workshops on “Structure” ................. 241
Appendix D: Statement of Instructional Aims for the Structure Workshops with the Seventh-Grade Students at School W ............................................ 243
Appendix E: Detailed Lesson Plans for Four Workshops on “Structure” for Seventh-Grade Students at School W, Including Homework Challenges for Each Workshop ......................................................... 245
Appendix F: Detailed Plans for Group Tasks in the Modeling Workshops: Finding Recursive and Explicit Rules for Patterns ........................................ 261
Appendix G: Classroom Observation Schedule ............................................................. 287
Appendix H: Pre-Teaching to Post-Teaching “Growth” with Respect to the Five Basic Cognitive Structure Components ...................................................... 289
Appendix I: Generalization Categories (After Radford, 2006) .................................. 295

Composite Reference List .................................................................................................. 299
Author Index .................................................................................................................... 317
Subject Index .................................................................................................................... 323
Using Design Research and History to Tackle a Fundamental Problem with School Algebra
Kanbir, S.; Clements, M.A.; Ellerton, N.F.
2018, XXIV, 327 p. 55 illus., 14 illus. in color., Hardcover
ISBN: 978-3-319-59203-9