# Chapter 2
# The Standard Model

The Standard Model is the theory that has been able to explain most of the phenomenology of the microscopic world and to identify its elementary constituents. Experimental evidences of its validity come from experiments of High Energy Physics of the twentieth and twenty-first centuries. This chapter briefly describes the Standard Model and the role of the Higgs boson in the theory.

## 2.1 Fundamental Particles and Forces

The Standard Model (SM) is a relativistic quantum field theory, based on the *gauge* symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ that describes three of the four fundamental interactions:

- $SU(3)_C$ is the color symmetry group, related to the strong interactions;
- $SU(2)_L$ is the weak symmetry group of isospin related to the weak interactions;
- $U(1)_Y$ is the weak symmetry group of hypercharge related to the electromagnetic interactions.

Nevertheless the SM does not describe the gravitational interaction which can be considered negligible at the collider energy scale.

The existing ordinary matter can be sorted in two main categories of fundamental building blocks according with their properties: interacting particles (fermions) and mediators of the forces (bosons) which control their interactions:

**fermions** fermions half-integer spin particle, obey the Pauli exclusion principle and governed by Fermi-Dirac statistics;
**bosons** integer spin particles that follow the Bose-Einstein statistics.

The fundamental fields which describe all the particles predicted by the SM are fermionic *matter* fields. They interact with each other through bosonic *gauge* fields. The first category can be defined by means of the chirality operator, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, and their left-handed component will transform as a $SU(2)_L$ doublet while their right-handed part will transform as $SU(2)_R$ singlet.



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The *matter* fields are composed by:

**quarks** which exist in six different flavours and are grouped in three families. For each family there is a *up* component with isospin $I = +1/2$ and a electric charge $Q = 2/3e$ and a *down* component with $I = -1/2$ and $Q = -1/3e$, where $e$ is the module of the electron charge;

**leptons** which are also six and grouped in three families of different flavour. Each family consists of one charged lepton and of one neutral charged weakly interacting particle: the neutrino. In the SM the neutrinos are treated as massless particles but the observations of neutrino oscillations prove that they have a non-vanishing value of the mass.

As just mentioned, the interactions between quarks and leptons are mediated by the exchange of gauge bosons:

**gluons** are the eight color charged mediators of the strong interaction, one for each generator of $SU(3)_C$. They are massless and electrically neutral;

**photon** is the mediator of the electromagnetic interaction, it is massless and carries no electrical charge;

$W^{\pm}$ **and** $Z$ are the three gauge bosons responsible for the weak interaction. They get mass through spontaneous symmetry breaking (see Sect. 2.5).

The main properties (particle type, generation), the masses and the quantum number of leptons, quarks and gauge bosons are summarized in Fig. 2.1.

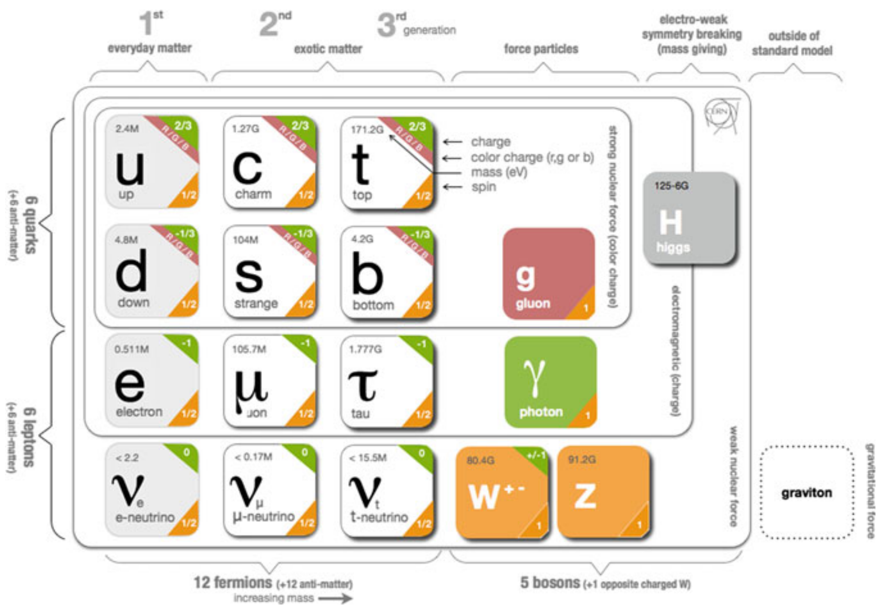


**Fig. 2.1** An overview of the standard model with the main particles and interaction mediators proprieties [1]

The SM is a local gauge theory based on the $SU(3)_C$ symmetry group that describes quantum cromodynamics and on $SU(2)_L \otimes U(1)_Y$ which defines the electroweak sector. In the next sections an explanation of the SM lagrangian from a quantum field point of view will be given.

## 2.2 Quantum ElectroDynamics

The first relativistic quantum field theory historically developed is Quantum ElectroDynamics (QED) [2, 3]. It is proposed to describe the electromagnetic interactions of particles.

Starting from the Dirac equation that determines the free massive fermions lagrangian

$$\mathscr{L}_{Dirac} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi \,, \tag{2.1}$$

in which $\gamma_{\mu}$ are the Dirac matrices and $m$ the mass of the fermionic field $\psi$, the invariance under the local transformation of the unitarian abelian group $U(1)$ is required

$$\psi(x) \to \psi' = e^{if(x)}\psi(x)\,, \tag{2.2}$$

where $f(x)$ is the function that defines the transformation in each point of the space. Here the fermionic field $\psi(x)$ conserves this symmetry with the introduction of the electromagnetic field $A_{\mu}(x)$, invariant as well under the same local phase transformation,

$$A_{\mu}(x) \to A'_{\mu}(x) = A_{\mu}(x) + \partial_{\mu} f(x)\,. \tag{2.3}$$

The interaction terms between the $\psi$ and $A_{\mu}$ fields ensure gauge invariance and the electrodynamic lagrangian can be obtained adding the electromagnetic kinetic term:

$$\mathscr{L}_{QED} = \bar{\psi}\left[i\gamma^{\mu}(\partial_{\mu} - ieA_{\mu}) - m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{2.4}$$

in which the tensor field is defined as $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$.

The mass term of the gauge field $A_{\mu}$ is not allowed by the local transformation invariance getting the massless photon as observed in Nature. This theory is one of the greatest achievements of particle physics. Its predictions have been verified by high precision experiments such as the measurements of the anomalous electron magnetic moment [4].

## 2.3 Quantum ChromoDynamics

Quantum ChromoDynamics (QCD) [5, 6] describes the strong interactions between quarks (carrying a color or anti-color charge) and gluons (carrying a color and an anti-color charge). It is a non-abelian-Yang-Mills theory that, using the $SU(3)_C$ symmetry group, requires the gauge fields $G^a_{\mu\nu}$ to be massless. Each of the eight generators of the non-abelian theory, $T_a$ with $a = 1, \ldots, 8$ (respecting the commutation rules $[T_a, T_b] = i f_{abc} T^c$ where $f_{abc}$ are the structure constant of the particular gauge group), introduces a mediator: the gluon.

As done in the QED case, the starting point is the Dirac lagrangian where the invariance under local transformations of the quark and gluon fields is imposed:

$$q(x) \rightarrow q'(x) = e^{i\alpha(x)_a T_a} q(x), \tag{2.5}$$

$$G^a_\mu(x) \rightarrow G^{a'}_\mu(x) = G^a_\mu(x) - \frac{1}{g}\partial_\mu \alpha(x)_a - f^{abc}\alpha_c G^c_\mu(x)\,. \tag{2.6}$$

In the formulas above $g$ represents the coupling constant of the strong interactions and $\alpha(x)_a$ with $a = 1, 2, \ldots, 8$ defines the local transformation.

The QCD lagrangian can be then formulated as

$$\mathscr{L}_{QCD} = \bar{q}(i\gamma^\mu \partial_\mu + igT^a G^a_\mu - m)q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a\,, \tag{2.7}$$

where $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu$ is not a purely kinetic term but includes also the possibility of interactions between the gluons, since the gluons themselves bring a color charge. This is one of the main differences with respect to QED and it is proper of the non-abelian groups. Finally, the local gauge invariance requires the gluons to be massless. Other important differences between QCD and QED are:

- color confinement: since the potential energy of the quarks, has an additional linear term with respect the electromagnetic one $V = \frac{a}{r} + br$, quarks cannot exist isolated therefore they cannot be directly observed;
- asymptotic freedom: the interaction between the particles becomes weaker and weaker with the increasing of the energy scale; this allows, in contrast with QED, to perform perturbative calculations only at high energy since the coupling constant $\alpha_S$ decreases with increasing energy.

## 2.4 Electroweak Interactions

The first theory of weak interactions was introduced by Fermi [7]: it gave an explanation to the $\beta$-decay under the hypothesis of the existence of a contact interaction, the associated interaction lagrangian is:

$$\mathscr{L}_{Fermi} = -\frac{G_F}{\sqrt{2}}\bar{\psi}_u\gamma^\mu(1-\gamma_5)\psi_d\bar{\psi}_\nu\gamma_\mu(1-\gamma_5)\psi_e\,. \tag{2.8}$$

Nevertheless, this theory violates unitarity and is not renormalizable, being an effective field theory that describes weak processes at low energy.

The electroweak theory proposed by Weinberg [8] and Salam [9] was born with the purpose of proving that QED and weak interactions are different manifestations of the same interaction.

The $SU(2)_L \otimes U(1)_Y$ symmetry group, whose generators are the weak isospin $\vec{\tau} = \frac{1}{2}\vec{\sigma}$ (where $\vec{\sigma}$ are Pauli matrices) and $Y$ the hypercharge operator, defines a chiral theory. The fermions' *left-handed* and *right-handed* components are transformed in different ways under local gauge transformations,

$$\psi(x)_L \to \chi'(x)_L = e^{i\vec{\alpha}(x)\vec{\tau}+i\beta(x)Y}\psi(x)_L\,, \tag{2.9}$$
$$\psi(x)_R \to \psi'(x)_R = e^{i\beta(x)Y}\psi(x)_R\,. \tag{2.10}$$

Here $\alpha(x)$ and $\beta(x)$ represent the phases of the local gauge transformation, $\chi_L$ the weak isospin doublet (for instance $\binom{\nu_e}{e}$) and describes the *left-handed* fermions, while $\psi_R$ is the isospin singlet which depicts the *right-handed* fermions. The *gauge* invariance of this theory, under the transformations in Eqs. (2.9) and (2.10) brings to the electroweak Lagrangian:

$$\begin{aligned}\mathscr{L}_{EWK} = \sum_{\ell=e,\mu,\tau} & \bar{\psi}^\ell_L\gamma^\mu\big[i\partial_\mu + ig\vec{\tau}\cdot\vec{W}_\mu - \frac{g'}{2}B_\mu\big]\psi^\ell_L + \\ & + \bar{\psi}^\ell_R\gamma^\mu\big[i\partial_\mu + g'B_\mu\big]\psi^\ell_R + \\ & -\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}\cdot B^{\mu\nu}\,,\end{aligned} \tag{2.11}$$

with $\vec{W}_{\mu\nu} = \partial_\nu\vec{W}_\mu - \partial_\mu\vec{W}_\nu$ (same as $B_{\mu\nu}$), where the fields $\vec{W}_\mu$ and $B_\mu$ are introduced for the $SU(2)_L$ and $U(1)_Y$ symmetry group respectively, and the coupling constants $g$ and $g'$ are added for the respective interactions.

In order to re-obtain the photon field it is possible to apply a transformation of these fields:

$$A_\mu = W^3_\mu\sin\theta_W + B_\mu\cos\theta_W\,, \tag{2.12}$$
$$Z_\mu = W^3_\mu\cos\theta_W - B_\mu\sin\theta_W\,, \tag{2.13}$$
$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp W^2_\mu)\,, \tag{2.14}$$

where $\theta_W$ is the Weinberg angle defined in terms of the coupling constants $g$ and $g'$ through:

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}\,, \tag{2.15}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}\,. \tag{2.16}$$

The electric charge can hence be written as a function of $g$ and $\theta_W$:

$$e = g\sin\theta_W = g'\cos\theta_W\,. \tag{2.17}$$

However this theory is not complete because the gauge invariance does not allow massive terms for the $W$ and $Z$ bosons ($m^2 W_\mu W^\mu$ and $m^2 B_\mu B^\mu$) and the fermions ($m^2 f\bar{f}$). This contradicts the experimental results that prove only the photon as massless gauge boson.

## 2.5 The Higgs–Brout–Englert Mechanism

In 1964, Higgs [10], Brout and Englert [11] provided a model, today known as Higgs–Brout–Englert mechanism, to solve the mass problem for fermions and boson preserving the gauge invariance. This model introduces a new scalar boson through a mechanism of spontaneous symmetry breaking: the Higgs boson. The symmetry breaking can occur when the lagrangian of the system shows a symmetry in its ground state, which is degenerate. In such case, there is actually no clear choice to describe the state of minimum energy therefore the symmetry is broken by choosing one of the degenerate eigenstates.

In the electroweak theory the $SU(2)_L \otimes U(1)_Y$ symmetry group is spontaneously broken in $U(1)_{em}$ (charged related group), which has to be conserved. For the Goldstone theorem, three massless Goldstone bosons appear, then absorbed by three of the four gauge bosons, giving mass to the vector bosons and keeping the photon massless. The easiest way to break the symmetry under the transformation of the $SU(2) \otimes U(1)$ group consists of introducing a complex scalar field in form of a isospin doublet:

$$\Phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix}, \tag{2.18}$$

where $\phi_i$ are real fields.

The easiest scalar Higgs lagrangian can be written in terms of a kinematic and a potential term:

$$\mathscr{L}_{Higgs} = (D^\mu\Phi)^\dagger D^\mu\Phi - V_{Higgs}\,, \tag{2.19}$$

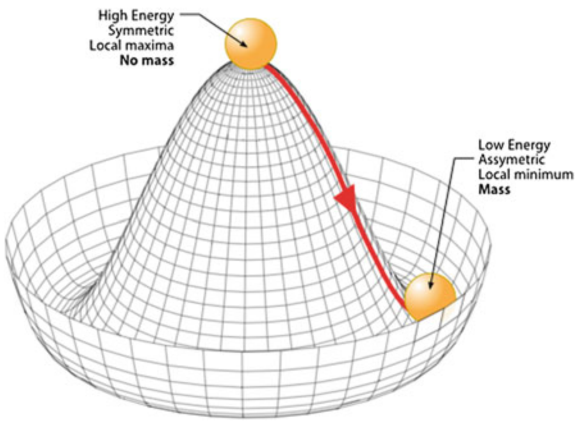


**Fig. 2.2** Higgs potential $V_{Higgs} = \mu^2\Phi\Phi^\dagger + \lambda(\Phi\Phi^\dagger)^2$ in the $(\Re(\Phi), \Im(\Phi))$ plane

where

$$D^\mu = \partial^\mu + \frac{i}{2}g\sigma_j W_j^\mu + ig'YB^\mu \,, \tag{2.20}$$

$$V_{Higgs} = \mu^2\Phi\Phi^\dagger + \lambda(\Phi\Phi^\dagger)^2 \,, \tag{2.21}$$

with $\mu$ and $\lambda$ which are free parameters.

In order to have a stable theory, the potential has to be inferiorly bounded which corresponds to impose $\lambda > 0$; nevertheless the sign of $\mu^2$ is not determined hence for $\mu^2 < 0$ the minimum is degenerate and it does not coincide with the origin. It indeed belongs to a circumference as shown in Fig. 2.2. Among the various minimum states on the surface of a four-dimensional hypersphere, choosing a particular vacuum expectation value with $\phi_1 = \phi_2 = \phi_4 = 0$ and $\phi_3 = v$:

$$\Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \text{with} \qquad v = \sqrt{\frac{-\mu}{\lambda}} \,, \tag{2.22}$$

the electroweak symmetry breaking, with the consequent generation of massive vector bosons and the symmetry invariance of $U(1)_{em}$, is ensured.

Applying a perturbative expansion around the vacuum state, four scalar fields $\theta_1$, $\theta_2$, $\theta_3$, $h(x)$ are introduced and the $\Phi(x)$ field can be defined as:

$$\Phi(x) = e^{\frac{i\vec{\tau}\cdot\theta(x)}{v}}\begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \approx \frac{1}{\sqrt{2}}\begin{pmatrix} \theta_2 + i\theta_1 \\ v + h - i\theta_3 \end{pmatrix} . \tag{2.23}$$

The four fields are independent and parametrize correctly the fluctuations around the origin $\Phi(0)$. $\theta_1$, $\theta_2$ e $\theta_3$ are the massless Goldstone bosons generated by the spontaneous symmetry breaking of the electroweak group. The lagrangian is still locally gauge invariant for $SU(2)$ and the Goldstone bosons can be deleted by exploiting the gauge freedom. The resulting field can be finally written as:

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}, \tag{2.24}$$

where $h(x)$ is the Higgs scalar field.

Replacing Eqs. (2.24) in (2.19) and writing $W^3_\mu$ and $B_\mu$ in terms of $Z_\mu$ and $A_\mu$, the gauge boson masses can be expressed as a function of the coupling and of the vacuum expectation value:

$$M_W = \frac{1}{2}vg, \ M_Z = \frac{1}{2}v\sqrt{g^2 + g'^2}\,. \tag{2.25}$$

A direct relationship between the vector boson masses and Weinberg angle can be also found:

$$\frac{M_W}{M_Z} = \cos\theta_W\,. \tag{2.26}$$

Three of the four degrees of freedom introduced in the theory are thus absorbed in the $W^\pm$ and $Z$ fields giving them mass and leaving the photon massless.

Furthermore the value of $v$ can be determined using the empirical value of the Fermi constant $G_F$ evaluated from the muon decay:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \tag{2.27}$$

and, using Eq. (2.25), the vacuum expectation value is

$$v = \sqrt{\frac{1}{\sqrt{2}G_F}} \sim 246\ \text{GeV}\,. \tag{2.28}$$

Applying this formalism in the mass term $V_{Higgs}$ in Eq. (2.21), a direct relationship between the Higgs boson mass and the vacuum expectation value can be obtained as

$$m_h = \sqrt{2v^2\lambda}\,. \tag{2.29}$$

Nevertheless, the Higgs boson mass $m_h$ is not predicted by the theory because $\lambda$ is a free parameter.

In the SM, the Higgs boson doublet can be used also to generate the quark and lepton masses. It can be achieved by adding to $\mathscr{L}_{EWK}$ in Eq. 2.12 a term that is invariant under $SU(2)_L \otimes U(1)_Y$:

$$\mathscr{L}_{leptons} = -\lambda_l\left[(\bar{\nu}, \bar{l})_L\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} l_R + \text{h.c.}\right], \tag{2.30}$$

where $\lambda_l$ is the Yukawa coupling, which defines the coupling of the interaction between the Higgs and fermion fields. Substituting the Higgs field formulated in Eq. (2.24), the lepton mass term and the interaction between the lepton $l$ and the Higgs field can be obtained:

$$\mathscr{L}_{leptons} = -m_l \bar{l} l - \frac{m_l}{v} \bar{l} l h \,. \tag{2.31}$$

This procedure can be also used in the case of quarks but, since both the components of the doublet are massive, in order to build the upper component in the doublet the following parametrization is used:

$$\Phi_C = -i\sigma_2 \Phi^* = \begin{pmatrix} -\bar{\phi^0} \\ \phi^- \end{pmatrix} \rightarrow \sqrt{\frac{1}{2}} \begin{pmatrix} v + h \\ 0 \end{pmatrix}. \tag{2.32}$$

The resulting gauge invariant lagrangian is

$$\mathscr{L}_{quark} = -\lambda_d^{ij} (\bar{u_i}, \bar{d_i'})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_{jR} - \lambda_u^{ij} (\bar{u_i}, \bar{d_i'})_L \begin{pmatrix} -\bar{\phi^0} \\ \phi^- \end{pmatrix} u_{jR} + \text{h.c.}\,, \tag{2.33}$$

with $i, j, n = 1, 2, 3$ and $d_i' = \sum_{n=1}^{3} V_{in} d_n$, where $V_{in}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix and $d_n$ are the $d$, $s$ and $b$ quarks. Finally the lagrangian can be also expressed in diagonal form:

$$\mathscr{L}_{quark} = -m_d^i \bar{d}_i d_i \left(1 + \frac{h}{v}\right) - m_u^i \bar{u}_i u_i \left(1 + \frac{h}{v}\right). \tag{2.34}$$

The choice of the Higgs field is hence sufficient to generate the masses of the gauge boson and of the fermions, but the fermions masses remain free parameters of the theory which not predicted by the SM.

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