

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Modeling Languages for Optimization . . . . .	1
1.2	Modeling with Pyomo . . . . .	3
1.2.1	Simple Examples . . . . .	3
1.2.2	Graph Coloring Example . . . . .	5
1.2.3	Key Pyomo Features . . . . .	6
1.3	Getting Started . . . . .	9
1.4	Book Summary . . . . .	10
1.5	Discussion . . . . .	11
 <b>Part I An Introduction to Pyomo</b>		<b>13</b>
<b>2</b>	<b>Mathematical Modeling and Optimization</b>	<b>15</b>
2.1	Mathematical Modeling . . . . .	15
2.1.1	Overview . . . . .	15
2.1.2	A Modeling Example . . . . .	16
2.2	Optimization . . . . .	18
2.3	Linear and Nonlinear Optimization Models . . . . .	20
2.3.1	Definition . . . . .	20
2.3.2	A Linear Approximation . . . . .	20
2.4	Modeling with Pyomo . . . . .	22
2.4.1	An Abstract Formulation . . . . .	22
2.4.2	A Concrete Formulation . . . . .	23
2.4.3	Linear Version . . . . .	24
2.5	Solving the Pyomo Model . . . . .	26
2.5.1	Solvers . . . . .	26
2.5.2	The <code>pyomo</code> Command . . . . .	26
2.5.3	Python Scripts . . . . .	27
<b>3</b>	<b>Pyomo Overview</b>	<b>29</b>
3.1	Introduction . . . . .	29
3.2	The Warehouse Location Problem . . . . .	30
3.3	Pyomo Models . . . . .	31
3.3.1	Components for Variables, Objectives and Constraints . . . . .	31

- 3.3.2 Indexed Components . . . . . 32
- 3.3.3 Construction Rules . . . . . 34
- 3.3.4 Abstract and Concrete Models . . . . . 35
- 3.3.5 A Concrete Model for the Warehouse Location Problem . 37
- 3.3.6 Modeling Components for Sets and Parameters . . . . . 40
- 3.3.7 An Abstract Model for the Warehouse Location Problem . 41
- 3.4 Solving the Pyomo Model . . . . . 43
  - 3.4.1 Using the `pyomo` Command . . . . . 43
  - 3.4.2 Scripting the Solution Process . . . . . 44
- 4 Pyomo Models and Components: An Introduction 47**
  - 4.1 An Object-Oriented AML . . . . . 47
  - 4.2 Common Component Paradigms . . . . . 48
    - 4.2.1 Indexed Components . . . . . 49
  - 4.3 Variables . . . . . 50
    - 4.3.1 Var Declarations . . . . . 50
    - 4.3.2 Working with Var Objects . . . . . 53
  - 4.4 Objectives . . . . . 53
    - 4.4.1 Objective Declarations . . . . . 54
    - 4.4.2 Working with Objective Objects . . . . . 55
  - 4.5 Constraints . . . . . 55
    - 4.5.1 Constraint Declarations . . . . . 56
    - 4.5.2 Working with Constraint Objects . . . . . 58
  - 4.6 Set Data . . . . . 59
    - 4.6.1 Set Declarations . . . . . 59
    - 4.6.2 Working with Set Objects . . . . . 63
  - 4.7 Parameter Data . . . . . 64
    - 4.7.1 Param Declarations . . . . . 65
    - 4.7.2 Working with Param Objects . . . . . 68
  - 4.8 Named Expressions . . . . . 69
    - 4.8.1 Expression Declarations . . . . . 69
    - 4.8.2 Working with Expression Objects . . . . . 70
  - 4.9 Suffix Components . . . . . 71
    - 4.9.1 Suffix Declarations . . . . . 71
    - 4.9.2 Working with Suffixes . . . . . 73
  - 4.10 Build Components . . . . . 74
  - 4.11 Other Modeling Components . . . . . 76
- 5 The Pyomo Command 79**
  - 5.1 Overview . . . . . 79
  - 5.2 The `check` Subcommand . . . . . 80
  - 5.3 The `convert` Subcommand . . . . . 81
  - 5.4 The `help` Subcommand . . . . . 82
  - 5.5 The `solve` Subcommand . . . . . 83
    - 5.5.1 Specifying the Model Object . . . . . 84

- 5.5.2 Selecting Data with Namespaces . . . . . 86
- 5.5.3 Customizing Pyomo’s Workflow . . . . . 89
- 5.5.4 Customizing Solver Behavior . . . . . 93
- 5.5.5 Analyze Solver Results . . . . . 94
- 5.5.6 Managing Diagnostic Output . . . . . 94
- 5.6 Discussion . . . . . 96
- 6 Data Command Files . . . . . 97**
- 6.1 Model Data . . . . . 97
- 6.2 The `set` Command . . . . . 98
  - 6.2.1 Simple Sets . . . . . 98
  - 6.2.2 Sets of Tuple Data . . . . . 99
  - 6.2.3 Set Arrays . . . . . 100
- 6.3 The `param` Command . . . . . 101
  - 6.3.1 One-dimensional Parameter Data . . . . . 101
  - 6.3.2 Multi-Dimensional Parameter Data . . . . . 103
- 6.4 The `table` Command . . . . . 105
- 6.5 The `load` Command . . . . . 108
  - 6.5.1 Simple Load Examples . . . . . 109
  - 6.5.2 Load Syntax Options . . . . . 110
  - 6.5.3 Interpreting Tabular Data . . . . . 112
  - 6.5.4 Loading from Spreadsheets and Relational Databases . . . 114
- 6.6 The `include` Command . . . . . 117
- 6.7 Data Namespaces . . . . . 117
- 6.8 Discussion . . . . . 118

**Part II Advanced Features and Extensions . . . . . 119**

- 7 Nonlinear Programming with Pyomo . . . . . 121**
- 7.1 Introduction . . . . . 121
- 7.2 Building Nonlinear Programming Formulations . . . . . 122
  - 7.2.1 Nonlinear Expressions . . . . . 122
  - 7.2.2 The Rosenbrock Problem . . . . . 123
- 7.3 Solving Nonlinear Programming Formulations . . . . . 126
  - 7.3.1 Nonlinear Solvers . . . . . 126
  - 7.3.2 Additional Tips for Nonlinear Programming . . . . . 127
- 7.4 Nonlinear Programming Examples . . . . . 128
  - 7.4.1 Variable Initialization for a Multimodal Function . . . . . 129
  - 7.4.2 Optimal Quotas for Sustainable Harvesting of Deer . . . . . 130
  - 7.4.3 Estimation of Infectious Disease Models . . . . . 135
  - 7.4.4 Reactor Design . . . . . 138
- 8 Structured Modeling with Blocks . . . . . 145**
- 8.1 Introduction . . . . . 145
- 8.2 Block structures . . . . . 147

8.3	Blocks as Indexed Components . . . . .	148
8.4	Construction Rules within Blocks . . . . .	149
8.5	Extracting values from hierarchical models . . . . .	150
8.6	Blocks Example: Optimal Multi-Period Lot-Sizing . . . . .	150
8.6.1	A Formulation Without Blocks . . . . .	152
8.6.2	A Formulation With Blocks . . . . .	153
<b>9</b>	<b>Generalized Disjunctive Programming</b>	<b>157</b>
9.1	Introduction . . . . .	157
9.2	Modeling GDP in Pyomo . . . . .	159
9.3	Solving GDP models . . . . .	161
9.3.1	Big-M transformation . . . . .	162
9.3.2	Convex hull transformation . . . . .	162
9.4	A mixing problem with semi-continuous variables . . . . .	163
<b>10</b>	<b>Stochastic Programming Extensions</b>	<b>165</b>
10.1	Introduction . . . . .	165
10.2	Stochastic Programming: Definition and Notation . . . . .	166
10.3	Modeling in PySP . . . . .	167
10.3.1	The Deterministic Reference Model . . . . .	168
10.3.2	The Scenario Tree . . . . .	171
10.3.3	Scenario Parameter Specification . . . . .	174
10.4	Generating and Solving the Extensive Form . . . . .	176
10.5	Progressive Hedging: A Generic Decomposition Strategy . . . . .	180
10.5.1	The <code>runph</code> Script . . . . .	182
10.6	Progressive Hedging Extensions: Advanced Configuration . . . . .	187
10.6.1	Bundling . . . . .	187
10.6.2	Watson and Woodruff Extensions . . . . .	188
10.6.3	Solving a Constrained Extensive Form . . . . .	194
10.6.4	Alternative Convergence Criteria . . . . .	195
10.6.5	User-Defined Extensions . . . . .	196
10.7	Solving PH Scenario Sub-Problems in Parallel . . . . .	197
10.8	Bounds . . . . .	198
<b>11</b>	<b>Differential Algebraic Equations</b>	<b>201</b>
11.1	Introduction . . . . .	201
11.2	Pyomo DAE Modeling Components . . . . .	202
11.3	Solving Pyomo Models with DAEs . . . . .	204
11.3.1	Finite Difference Transformation . . . . .	205
11.3.2	Collocation Transformation . . . . .	206
11.4	Additional Features . . . . .	207
11.4.1	Applying Multiple Discretizations . . . . .	207
11.4.2	Restricting Control Input Profiles . . . . .	208
11.4.3	Plotting . . . . .	208
<b>12</b>	<b>Mathematical Programs with Equilibrium Constraints</b>	<b>211</b>

- 12.1 Introduction . . . . . 211
- 12.2 Modeling Equilibrium Conditions . . . . . 212
  - 12.2.1 Complementarity Conditions . . . . . 212
  - 12.2.2 Complementarity Expressions . . . . . 212
  - 12.2.3 Modeling Mixed-Complementarity Conditions . . . . . 213
- 12.3 MPEC Transformations . . . . . 216
  - 12.3.1 Standard Form . . . . . 217
  - 12.3.2 Simple Nonlinear . . . . . 217
  - 12.3.3 Simple Disjunction . . . . . 218
  - 12.3.4 AMPL Solver Interface . . . . . 219
- 12.4 Solver Interfaces and Meta-Solvers . . . . . 219
  - 12.4.1 Nonlinear Reformulations . . . . . 220
  - 12.4.2 Disjunctive Reformulations . . . . . 220
  - 12.4.3 PATH and the ASL Solver Interface . . . . . 221
- 12.5 Discussion . . . . . 222
- 13 Bilevel Programming . . . . . 223**
  - 13.1 Introduction . . . . . 223
  - 13.2 Motivating Problems . . . . . 224
    - 13.2.1 Linear Bilevel Programs with Continuous Variables . . . . . 225
    - 13.2.2 Quadratic Min/Max . . . . . 225
  - 13.3 Modeling Bilevel Programs . . . . . 225
  - 13.4 Solving Linear Bilevel Programs . . . . . 227
    - 13.4.1 Global Optimization . . . . . 228
    - 13.4.2 Local Optimization . . . . . 229
  - 13.5 Solving Quadratic Min-Max Bilevel Programs . . . . . 229
  - 13.6 Discussion . . . . . 232
- 14 Scripting . . . . . 235**
  - 14.1 Introduction . . . . . 235
  - 14.2 A Basic Optimization Script . . . . . 236
  - 14.3 Creating and Modifying Pyomo Models . . . . . 237
    - 14.3.1 Modifying Model Parameters . . . . . 239
    - 14.3.2 Modifying Model Structure . . . . . 240
  - 14.4 Using Solvers . . . . . 242
  - 14.5 Investigating the Solution . . . . . 243
    - 14.5.1 Solver Results . . . . . 244
    - 14.5.2 Retrieving Variable Values . . . . . 245
  - 14.6 Scripting Examples . . . . . 246
    - 14.6.1 Warehouse Location Loop and Plotting . . . . . 246
    - 14.6.2 A Sudoku Solver . . . . . 247
- A A Brief Python Tutorial . . . . . 255**
  - A.1 Overview . . . . . 255
  - A.2 Installing and Running Python . . . . . 256

- A.3 Python Line Format . . . . . 257
- A.4 Variables and Data Types . . . . . 258
- A.5 Data Structures . . . . . 260
  - A.5.1 Strings . . . . . 260
  - A.5.2 Lists . . . . . 260
  - A.5.3 Tuples . . . . . 261
  - A.5.4 Sets . . . . . 261
  - A.5.5 Dictionaries . . . . . 262
- A.6 Conditionals . . . . . 262
- A.7 Iterations and Looping . . . . . 263
- A.8 Functions . . . . . 264
- A.9 Objects and Classes . . . . . 265
- A.10 Modules . . . . . 266
- A.11 Python Resources . . . . . 266
  
- Bibliography** . . . . . **267**
  
- Index** . . . . . **273**



<http://www.springer.com/978-3-319-58819-3>

Pyomo — Optimization Modeling in Python  
Hart, W.E.; Laird, C.D.; Watson, J.-P.; Woodruff, D.L.;  
Hackebeil, G.A.; Nicholson, B.L.; Sirola, J.D.  
2017, XVIII, 277 p. 13 illus., 8 illus. in color., Hardcover  
ISBN: 978-3-319-58819-3