Preface

This monograph is the result of many years of experience and contemplation by two octogenarian mathematical physicists, on opposite sides of the Atlantic, who, after all these years, are still endlessly fascinated by the marvelous intellectual edifice that has become quantum mechanics. And which, in the twenty-first century, continues its proliferation into entirely new avenues of human accomplishment, including experiment-based answers to metaphysical questions and the limitless potential of quantum computation.

The main purpose of the book is to make accessible to nonspecialists the still evolving fundamental concepts of QM and the terminology in which these are expressed. Hence our title: "Twenty-First Century Quantum Mechanics: Hilbert Space to Quantum Computers." Among the concepts which we emphasize are the following:

- The wavefunction of a particle: associated with a "cloud" of probability, such that the density of the cloud is greater in regions which have a higher probability of containing the particle.
- The Heisenberg uncertainty principle: conjugate pairs of observables, such as the position and the velocity of a particle, cannot both be precisely determined at the same time.
- Isotropic vectors: vectors with complex components, which are orthogonal to the rotation axis and thus remain invariant under rotation; these are used to construct *spinors*.
- The spin of elementary particles: the quantum counterpart of rotations of classical objects, described by spinors.
- The question of whether the fundamental laws of physics violate local realism. Locality means that the influence of one particle on another cannot exceed the speed of light. Realism means that quantum states have well-defined properties, independent of our knowledge of them.
- The possible existence of hidden variables: something analogous to the enormous number of microscopic details of molecular motions, which exhibit

themselves in the determinate macroscopic properties of matter, such as temperature, pressure, etc.

- Conceptual problems associated with measurement, superposition and decoherence in quantum systems: collapse of the wavefunction, Schrödinger's cat, and quantum entanglement.
- Quantum computers: if they can be made practicable, enormous enhancements in computing power, artificial intelligence and secure communication will follow.

Needless to say, the quantum theory raises very profound metaphysical and epistemological questions on the description about the "objective world." An important precept of Felix Klein's famous *Erlangen program* was that "A geometry is the study of invariants under a group of transformations." These ideas, subsequently elaborated in the contributions of Einstein, Dirac, and the philosopher Nozick, have led to, at least, a provisional understanding of what constitutes reality.

The need for an observer to "search for invariants" is of such generality that it must apply even in the lives of animals. Imagine a gazelle cautiously eying two lions. Suppose that she glances at the first lion, and then the second. In doing so, she must turn her head. But there must be some process in her brain that enables her to realize that, even after she turns her head (and thus registers a different image), the first lion is still there, surely an instance of "invariance." If we wanted to build a robot capable of distinguishing objects, then, when the robot's eyes move, its programming must include the capability of performing mathematical transformations among images viewed at varying angles at different times. Nature probably does not work in precisely the same way, but the fundamental conceptual features: (1) variability of images and (2) recognition of invariants, or common elements, must still be applicable.

These ideas certainly pertain to the classical view of Nature; what are their manifestations in quantum mechanics? The analog of the rotation of a classical observer is the evolution of the wavefunction. However there are two distinctly different modes of evolution in quantum mechanics. One is a continuous evolution, following the time-dependent Schrödinger equation; the second, called *collapse of the wave function*, is a random and instantaneous event brought about by a measurement or perturbation of the quantum system. This was, at least, the point of view of the founding fathers of quantum mechanics, mainly Bohr, Heisenberg, and Dirac. The most eminent critic of such ideas (a probabilistic interpretation of QM) was none other than Albert Einstein, as epitomized in his famous pronouncement: "God does not play dice with the Universe!"

A major aspect of the epistemological problem has been resolved by actual experiments (by Alain Aspect and others), motivated by the deep insights of John Stewart Bell. This has revealed a major incompatibility between the worldviews of classical and quantum physics. Bell's theorem states that it is impossible to explain the results of quantum physics using the causality of classical physics, thus negating the possible existence of local hidden variables. Quantum mechanics differs fundamentally from classical mechanics in that the underlying microscopic behavior is *not* determinate.

Measurement and decoherence: according to the traditional ("Copenhagen") interpretation of quantum mechanics, wave function collapse occurs when a measurement is performed. However there remains the problem of *when the collapse actually occurs*? In the past, some physicists thought that collapse is brought about when a conscious observer "takes note" of the new state of a system; but this point of view is now in the minority, since it is more reasonable to think that any inanimate apparatus can also make a measurement and produce a "quantum jump." A more realistic approach to this problem is to consider the microscopic system, the measuring apparatus and the environment as a single composite system. The wavefunction of the complete system must change during an exceedingly short interval of time from a "superposition of states," to just one of these states. This phenomenon is called *decoherence*; it can be proved mathematically rigorously in some models, although there is still much work to be done.

The superposition of two wavefunctions for a macroscopic object is also considered in the infamous Schrödinger's cat *Gedankenexperiment*. A cat is confined to a closed box with a Geiger counter, which detects randomly-occurring radioactive decays in a sample of radium. The Geiger counter is connected to a vial of cyanide, which is broken when a decay particle is detected, killing the unfortunate cat. Until the box is opened, its state can only be described as a "superposition," of a "live cat" and a "dead cat." According to the Copenhagen version of quantum mechanics, the cat "becomes" dead or alive only after an observer opens the box. As paradoxical as it seems, superposition of quantum states of macroscopic objects has now been achieved, for example, in a SQUID (superconducting quantum interference device).

Quantum computing proposes to apply uniquely quantum-mechanical phenomena, such as superposition and entanglement to operate on quantum units of information, called *qubits*. In contrast to classical bits, which can represent a variable with just two values, say 0 and 1, qubits can, in concept, contain an infinite continuum of information, in terms of superpositions of two basis qubits, such as $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$. A quantum computer could, in principal, be capable of solving problems in a matter of seconds, which might take a classical computer several centuries to accomplish. Several hypothetical quantum algorithms have already been proposed for large-integer factorization and other applications. Current realizations of quantum computers are still very far from having such capabilities. But apart from potential practical applications, quantum computing remains a profound subject of fundamental interest for both computer science and quantum mechanics.

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