Preface

The Mathematics Olympiad for students of the Mechanics and Mathematics Faculty has been organized at Taras Shevchenko National University of Kyiv since 1974. After a while the competition opened up to qualified students from any higher school of Kyiv and beginning in 2004, it became a nice tradition to invite the strongest mathematics students of leading Kyiv high schools to participate. Since then representatives of Ukrainian Physics and Mathematics Lyceum, Liceum No. 171 "Leader", Liceum "Naukova Zmina", Liceum No. 208, and Rusanivky Liceum have repeatedly become prize winners of the Olympiad.

Most of the Olympiad winners are students of the Mechanics and Mathematics Faculty, but students from the following departments or institutions have also performed successfully: Institute of Physics and Technology and Institute of Applied System Analysis of National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Faculty of Cybernetics and Faculty of Physics of Taras Shevchenko National University of Kyiv, National Pedagogical Dragomanov University, and National University of Kyiv-Mohyla Academy.

Results of the Olympiad are taken into account when forming teams of All-Ukrainian students' Mathematics Olympiad, International Mathematics Competition for University Students (IMC) and other student competitions. Materials and results of many mathematics competitions in which Ukrainian students take part can be found on the students' page of this website of Mechanics and Mathematics Faculty http://www.mechmat.univ.kiev.ua.

As a rule, first- and second-year undergraduates and third- and fourth-year undergraduate students compete separately. Along the history of the Olympiad, the number of problems distributed has changed several times. Most recently, the jury of Olympiad composed two sets of problems—one for first- and second-year undergraduates and the second set for senior undergraduate students. Each set contained 7–10 problems. For first-and second-year undergraduates, problems were included for fields such as calculus, algebra, number theory, geometry, and discrete mathematics. Problem sets for third and fourth year undergraduates included additional topics in measure theory, functional analysis, probability theory, complex analysis, differential equations, etc. Solutions to all the problems do not rely on

statements out of curriculum of obligatory courses studied at Mechanics and Mathematics Faculty, but the solutions demand creative usage of obtained knowledge. Most of the problems are not technical and admit a short and elegant solution. A few complicated problems, which demand general mathematical culture and remarkable inventiveness, are included in both versions of the assignment, and this helps to compare the results of all the participants.

In 1997–1999 some of the problems were borrowed from Putnam Competitions [1, 3, 4]. Almost all the problems of the last 17 years are original. Their authors are lecturers, Ph.D. students, senior students, and graduating students of the Mechanics and Mathematics Faculty, as well as colleagues from Belgium, Canada, Great Britain, Hungary, and the USA. Since 2003 participants obtain an assignment, where the author's name is indicated beside the corresponding problem.

The competition lasts for 3 hours. Of course, this time interval is not enough to solve all the problems, and therefore, a participant can focus first of all on the problems, which are the most interesting for him/her. Typically, almost all the problems are solved by some of participants; a winner solves more than half of problems, and all who solve at least 2–3 problems become prize winners or get the letter of commendation. The jury of olympiad checks the works and gives a preliminary evaluation. Approximately one week later, an analysis of problems is held, appeal, and winners are awarded.

For many years, until 1995, the jury leader was also the head of Mathematical Analysis Department, Prof. Anatoliy Yakovych Dorogovtsev (1935–2004), a famous expert in mathematical statistics and the theory of stochastic equations. For a long time he led a circle in calculus for first- or second-year undergraduate students (until now such circles work at Faculty of Mechanics and Mathematics and at Institute of Mathematics of the National Academy of Sciences of Ukraine). Anatoliy Yakovych proposed numerous witty problems in calculus, measure theory, and functional analysis. For a few years a jury leader was also the head of the Probability Theory and Mathematical Statistics Department as well as a Corresponding Member of the NAS of Ukraine, Myhailo Yosypovych Yadrenko (1932–2004). Myhailo Yosypovych was an outstanding expert in the theory of random fields and had authored many clever problems in probability theory and discrete mathematics. In particular years, the organizers of Olympiad were a Corresponding Member of the NAS of Ukraine Volodymyr Vladyslavovych Anisimov, lecturers Oleksiy Yuriyovych Konstantinov, Volodymyr Stepanovych Mazorchuk, and Volodymyr Volodymyrovych Nekrashevych. From 1999 until now, the permanent jury leader has also been the head of Mathematical Analysis Department, Prof. Igor Oleksandrovych Shevchuk, a famous expert in approximation theory. Members of jury for the last Olympiads were Andriy Bondarenko, Volodymyr Brayman, Alexander Kukush, Yevgen Makedonskyi, Dmytro Mitin, Oleksiy Nesterenko, Vadym Radchenko, Oleksiy Rudenko, Vitaliy Senin, Sergiy Shklyar, Sergiy Slobodyanyuk, and Yaroslav Zhurba.

There are several famous mathematicians among the former winners of the Olympiad of Mechanics and Mathematics Faculty. In particular, Prof. O.G. Reznikov (1960–2003) used powerful methods of calculus in problems of modern geometry and was a member of London Mathematical Society. In 2016 Dr. M.S. Viazovska was awarded the Salem Prize for a conceptual breakthrough in the sphere packing problem. In 2013 Dr. A.V. Bondarenko was awarded the Vasil Popov International Prize for outstanding achievements in approximation theory. State prizes of Ukraine were awarded: to Prof. A.Ya. Dorogovtsev for a monograph in stochastic analysis; D.Sc. in Physics and Mathematics V.V. Lyubashenko for a cycle of papers in algebra; D.Sc. in Physics and Mathematics O.Yu. Teplinskyi for papers in theory of dynamical systems. Candidate of Sciences in physics and mathematics A.V. Knyazyuk (1960–2013) was a famous teacher of the Kyiv Natural Science Luceum No. 145. We mention also Professors I.M. Burban, O.Yu. Daletskyi, P.I. Etingof, M.V. Kartashov, Yu. G. Kondratyev, K.A. Kopotun, A.G. Kukush, O.M. Kulik, V.S. Mazorchuk, Yu. S. Mishura, V.V. Nekrashevych, A.Yu. Pylypenko, V.M. Radchenko, V.G. Samoylenko, G.M. Shevchenko, and B.L. Tsyagan. We apologize if we have forgotten anybody.

The first part of the book contains all the problems of Olympiads dated 1995–2016. We hope that you will enjoy both self-reliant problem solving and an acquaintance with the solutions presented in the second part of the book. Some problems from earlier Olympiads can be found in the articles [2, 5, 6].

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