Information, Noise, and Energy Dissipation: Laws, Limits, and Applications

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Abstract This chapter addresses various subjects, including some open questions related to energy dissipation, information, and noise, that are relevant for nano- and molecular electronics. The object is to give a brief and coherent presentation of the results of a number of recent studies of ours.

1 Energy Dissipation and Miniaturization

It has been observed, in the context of Moore's law, that the power density dissipation of microprocessors keeps growing with increasing miniaturization [1–4], and quantum computing schemes are not principally different [5, 6] for general-purpose computing applications. However, as we point out in Sect. 2 and seemingly in contrast with the above statements, *the fundamental lower limit of energy dissipation of a single-bit-flip event (or switching event) is independent of the size of the system*. Therefore, the increasing power dissipation may stem from the following practical facts [1–4]:

- *A larger number of transistors on the chip*, contributing to a higher number of switching events per second;
- *lower relaxation time constants with smaller elements*, allowing higher clock frequency and the resulting increased number of switching events per second;

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- *increasing electrical field and current density*, because the power supply voltage is not scaled back to the same extent as the device size; and
- *enhanced leakage current and related excess power dissipation*, caused by an exponentially increasing tunneling effect associated with decreased insulator thickness and increased electrical field.

It is clearly up to future technology to approach the fundamental limits of energy dissipation as much as possible.

It is our goal in this chapter to address some of the basic, yet often controversial, aspects of the fundamental limits for nano- and molecular electronics. Specifically, we deal with the following issues:

- *The fundamental limit of energy dissipation for writing a bit of information*. This energy is always positive and characterized by Brillouin's negentropy formula and our refinement for longer bit operations [7–10].
- *The fundamental limits of energy dissipation for erasing a bit of information* [7–12]. This energy can be zero or negative; we also present a simple proof of the non-validity of Landauer's principle of erasure dissipation [11, 12].
- *Thermal noise in the low-temperature and/or high-frequency limit*, i.e., in the quantum regime (referred to as "zero-point noise"). It is easy to show that both the quantum theory of the fluctuation–dissipation theorem and Nyquist's seminal formula are incorrect and dependent on the experimental situation [13, 14], which implies that further studies are needed to clarify the properties of zero-point fluctuations in resistors in electronics-based information processors operating in the quantum limit.

2 Fundamental Lower Limits of Energy Dissipation for Writing an Information Bit [7–10]

Szilard [15] (in 1929, in an incorrect way) and Brillouin [16] (in 1953, correctly) concluded that the minimum energy dissipation H_1 due to changing a bit of information in a system at absolute temperature T is given as

$$H_1 \approx kT \ln 2, \tag{1}$$

where k is Boltzmann's constant. Later Brillouin [17], Kish [7–10], and Alicki [18] independently refined this equation, for arbitrary bit flips (writing or erasure), to read

$$H_1 \approx kT \ln\left(\frac{1}{\varepsilon}\right), \tag{2}$$

where $\varepsilon < 0.5$ is the bit-error probability of the operation. A physical representation of these equations is given in Fig. 1. We note in this context that von Neumann [19]
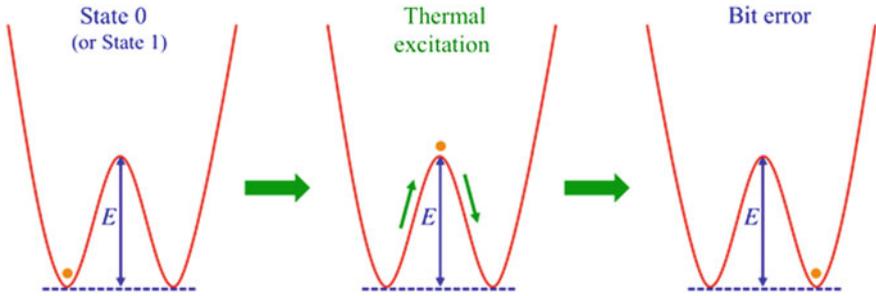
Fig. 1 Model example for a non-volatile memory: particle in a double-well-potential (simulating, e.g., a magnetic memory bit). Bit value 0: particle in the *left* well; bit value 1: particle in the *right* well (without loss of generality). When thermal fluctuations generate a spontaneous bit flip, a bit error emerges. H_1 in Eqs. (2) and (3) is implied by moving the particle over the energy barrier E and is equal to this quantity

also mentions, as an aside, this functional form in his last book about the brain. Equation (1) leads to Eq. (2) in the limit $\varepsilon = 0.5$, where the efficiency of the operation is zero. Equations (1) and (2) are valid when the measurement time window is short compared to the correlation time τ of thermal fluctuations and, for longer time windows $t_w \geq \tau$, Kish and Granqvist [7–10] presented a correction in the low-error limit as

$$H_1 \approx kT \left[\ln\left(\frac{1}{\varepsilon}\right) + \ln\left(\frac{t_w}{\tau}\right) \right]. \tag{3}$$

It is essential to realize that any operation that requires at least a yes/no decision [10], or any similar "two-alternative action," has to obey this constraint.

3 On Energy Dissipation During Information Erasure

3.1 Types of Erasure of Data in Memories [12]

Erasure can be accomplished in several different ways, as mentioned next:

(a) *Secure erasure by resetting the bits to zero* (which is the type of erasure assumed in the original version of Landauer's principle [20]; see Eq. 5) This type of erasure is used only for security applications in computers because it is extremely slow and very energy-intensive.

(b) *"Erasure" by writing-over* [7, 8]. Here the memory bits are not reset but instead the blocks of the memory to be erased are designated as "free" but otherwise left alone to be written over by new data. The number of address bits, and the "erasure"-related dissipation, scales as $\log_2 N$, where N is the size of the whole

memory. This type of "erasure" is used in computers; it is the fastest option and requires minimal energy dissipation. The logarithmic scaling is in direct contradiction with Landauer's principle as elaborated below.

(c) *Information-theoretic erasure* (ITE) [11, 12]. This erasure is a randomization/ thermalization process with minimum energy dissipation, which can be as low as zero when erasure is done in a passive way as discussed below. For the case of ITE, bit errors are generated by thermal noise, which results in 50% chance for the values 0 and 1 after erasure and hence no information about the original memory content. On the other hand, it is well known that Shannon's information entropy S_I under these conditions attains its absolute maximum and is given, for the case of N bits, by

$$S_I = \sum_{j=1}^{N} \sum_{m=0}^{1} p_{j,m} \ln\left(\frac{1}{p_{j,m}}\right) = \sum_{j=1}^{N} \sum_{m=0}^{1} 0.5 \ln\left(\frac{1}{0.5}\right) = N \qquad (4)$$

so that the information entropy during ITE can only increase or remain constant. Here $p_{j,m}$ stands for the probability of the j-th bit storing the value $m \in \{0, 1\}$. In order to check that this erasure principle is both physical and physically realizable, we introduced and analyzed two device concepts with regard to ITE: one with double-wells-potential and another with capacitors [11, 12]. We found that, when the working conditions of such a capacitor-based memory involved less than $kT/2$ stored energy (and an equivalent negative thermal entropy S_{th}) for holding the originally stored information, ITE (i.e., thermalization) entailed that the system of capacitors absorbed heat from the environment, thus yielding negative energy dissipation. However, we emphasize that the energy dissipation of the shown ITE schemes is always positive when the control of the switches [9, 10] for arranging the erasure is also accounted for.

3.2 Landauer's Principle

The "classical" version of Landauer's principle (see, e.g., in [20]) asserts that

$$\Delta Q_{th} = T \Delta S_{th} \geq -kT \ln(2) \Delta S_I, \qquad (5)$$

where ΔQ_{th} and ΔS_{th} are produced heat and thermal entropy, and ΔS_I is the change of S_I during erasure.

3.3 Non-validity of Landauer's Principle

(a) Equation 5 states that the change of information entropy can be "converted" into the lower limit of energy dissipation during the erasure of a memory. It should be emphasized that the greater-than-or-equal-to sign—rather than a greater-than sign—is very important because the equality must represent a physical possibility, at least at the conceptual level. One of the most straightforward and relevant objections [9, 10] to Eq. 5 is that, in the case of equality, the memory cell's error probability is 50% even in the short-time limit, i.e., the memory does not function and $p_{j,1} = 1$ it is practically useless for the case of even larger energy dissipation; the limit of equality is unphysical in Eq. 5.

(b) ITE and Eq. 4 are also in direct contradiction with Landauer's principle (Eq. 5) because, even in the absence of any available negative thermal entropy, the minimum energy dissipation would be allowed to be negative during erasure, which obviously is not correct. This impossibility is illustrated by the example of passive ITE for double-well-potential-based memories below [11, 12]. A single memory cell is shown in Fig. 2.

For the sake of simplicity, we suppose that originally all bits are in the 1 state with $p_{j,1} = 1$ and $p_{j,0} = 0$, which means that $S_I = 0$. We now wait for a time $t_w \gg \tau_0 \exp(E/kT)$ until the double-wells are "thermalized" and $p_{j,1} = p_{j,0} = 0.5$ implying that $S_I = N$ (bits) so that the information entropy of the memory has increased to $\Delta S_I = N$ without any energy dissipation or energy investment or control. Landauer's principle does not specify any restriction on the duration of erasure, and hence, it applies here and yields that

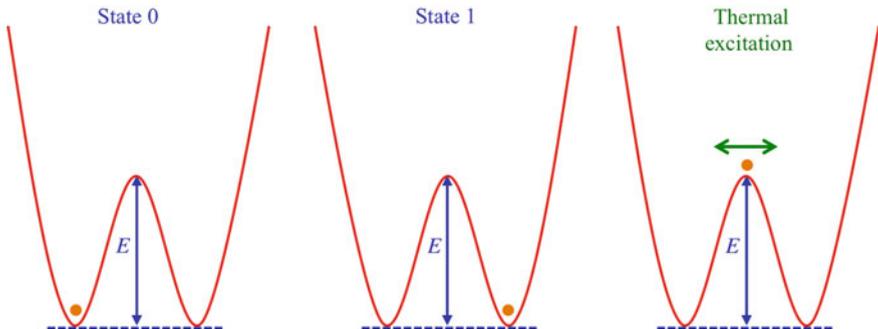$$\Delta Q_{th} = T\Delta S_{th} \geq -kT \ln(2)\Delta S_I = -NkT \ln(2) \gg 0, \tag{6}$$

Fig. 2 Passive information-theoretic erasure in a zero-energy-dissipation fashion by passively waiting during times that are much longer than the thermalization time constant at ambient temperature [11, 12]. Erasure then happens without energy dissipation. Alternative ways of reaching information-theoretic erasure with no or negative energy dissipation are described in [11]

which is incorrect because the energy dissipation during this erasure is always exactly zero.

The examples above indicate how important is to check any proposed general mathematical principle in physics with Gedanken experiments and physical conceptual models embracing all of the essential details. Mathematics is infinitely richer than physics, and ultimately, the Laws of Physics will select those few mathematical principles, models, and solutions that are physical, i.e., realistic. It is obvious from our considerations that Landauer's principle is unphysical.

(c) In contrast to the results above, Bennet's formulation [21, 22] of Landauer's principle claims that only erasure is dissipative—i.e., the $1 \rightarrow 0$ bit flip— because such an operation involves shrinking of the space-state from two possible bit-values (0, 1) into a determined single-bit value (0) and a concomitant reduction of the entropy in the memory, which must be compensated by a corresponding entropy production (i.e., heat) in the rest of the system (5).

One of the reasons for the dichotomous opinions is that statistical information measures are insufficient to describe dissipation in memories during erasure. A simple example is given below for erasure by resetting the bits to zero.

The erasure of a classical-physical memory cannot in all cases depend on whether we know the data in the memory or not. Suppose first that we know the data bits. This means that the Shannon information entropy, or any other information measure, is zero even if the memory is full of data because the probabilities of the bit being 0 or 1 in each memory cell are either 0 or 1, and this leads to zero information entropy. However, even after erasure, the system would be in a known deterministic state with all bits now being 0 and still leading to zero information entropy. Thus, there was no information entropy loss because there was no information to begin with. This fact highlights that statistical information measures are irrelevant.

(d) When we do not know the data in the memory, perhaps the most fundamental reason for not using information entropy to describe memory dissipation is Alfred Renyi's arguments about deterministic systems such as error-free computers [23]. The information entropy of deterministically generated data is less than or equal to the information entropy of the given algorithm and its initialization parameters. For example, let us suppose that we generate π with a simple deterministic algorithm and record its bits into a memory. All elements of this algorithm are known and deterministic, and therefore, its information entropy is zero! Even when we enter more and more bits of π into the memory and the data size N, and corresponding erasure dissipation, approaches infinity the information entropy of this infinitely large random data sequence will be $\log_2 N$, i.e., the address required to identify the last digit. While the erasure dissipation scales with N, the information before erasure scales logarithmically. No physical mechanism exists to compensate for this nonexistent, logarithmically scaling dissipation. *Reductio ad absurdum.*

Landauer's principle has also been intensely debated by Porod et al. [24–26], Norton [e.g., 27, 28], and Gyftopoulos and von Spakovsky [29].

3.4   Erasure Dissipation in Practical Computing [7, 8]

Finally, we address the problem of non-secure bit erasure in large memories in computers. Do computers execute erasure when they discard information? The answer is usually "no" since, in practice, they do not reset the memory bits but just change the address of the boundary of the free part of the memory as illustrated in Fig. 3. The number of bits in the address scales as $\log_2(N)$, where N is the size of the whole memory, and hence (in accordance with Eq. 3), the energy dissipation is of the order of [7, 8]

$$H_N \approx kT \left[\ln\left(\frac{1}{\varepsilon}\right) + \ln\left(\frac{t_w}{\tau}\right) \right] \log_2(N). \tag{7}$$

At fixed error rate and observation time window, the energy dissipation of erasure scales as

$$H_N \propto \log_2(N), \tag{8}$$

which is a much more optimal situation than for the Landauer's limit of $kT \ln(2)$ energy dissipation per bit. Thus, even if Landauer's principle were valid, it would still be of limited practical importance.
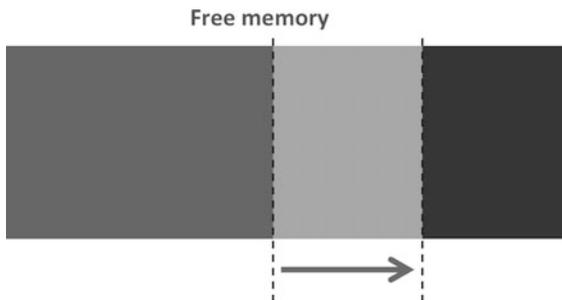
Free memory

Fig. 3 Simplified one-dimensional illustration of discarding information in an idealized computer memory [7, 8]. *Gray* tones signify space free for writing, and *black* denotes occupied space. The address of the boundary of the free memory is moved along the *arrow* to discard information and increase the free-memory part

3.5 Conclusion About the Non-validity of Landauer's Principle

Our main conclusion is that statistical information measures are irrelevant for treating the energy dissipation during memory erasure. Dissipation due to erasure is negligible in practical applications, and theoretically, it can be made even negative, though that is usually not practical [11, 12].

4 Thermal Noise in the Quantum Regime [13, 14]

The considerations leading to the results above were based on classical statistical physics. In the quantum domain, which we turn to presently, the situation is different. In that domain, there is an ongoing debate [13, 14] about Johnson noise. Here we point out that neither the celebrated fluctuation–dissipation theorem nor Nyquist's theory for the quantum regime can be valid under general experimental conditions [13].

Thermal noise (Johnson noise) in resistors was discovered by Johnson [30] and explained by Nyquist [31] in 1927, one year after the foundations of quantum physics were completed. The Johnson–Nyquist formula states that

$$S_u(f) = 4R(f)hfN(f,T), \qquad (9)$$

where $S_u(f)$ is the one-sided power density spectrum of the voltage noise on the open-ended complex impedance $Z(f)$ with real part $\mathrm{Re}[Z(f)] = R(f)$, and h is Planck's constant. The Planck number $N(f,T)$ is the mean number of hf energy quanta in a linear harmonic oscillator with resonance frequency f at temperature T and is given by

$$N(f,T) = [\exp(hf/kT) - 1]^{-1}. \qquad (10)$$

Hence, we have the well-known $N(f,T) = kT/(hf)$ case for the classical-physical range with $kT \gg hf$. Equation 10 results in an exponential cutoff for the Johnson noise in the quantum range with $f > f_P = kT/h$, in accordance with Planck's thermal radiation formula. In the deeply classical (low-frequency) limit, with $f \ll f_P = kT/h$, Eqs. 9 and 10 yield the familiar form used at low frequencies, i.e.,

$$S_{u,l}(f) = 4kTR(f), \qquad (11)$$

where the Planck cutoff frequency f_P is about 6000 GHz at room temperature. This is well beyond the reach of today's electronics.

A quantum-theoretical treatment of the one-sided power density spectrum of the Johnson noise was given 24 years after Johnson's and Nyquist's work by Callen and Welton [32] (often referred to as the fluctuation–dissipation theorem, FDT). The quantum version [32] of the Johnson–Nyquist formula has a number 0.5 added to the Planck number, corresponding to the zero-point (ZP) energy of linear harmonic oscillators, so that

$$S_{u,q}(f) = 4R(f)hf[N(f,T) + 0.5]. \tag{12}$$

Thus, the quantum correction of Eq. 9 is a temperature-independent additive term in Callen–Welton's one-sided power density spectrum (Eq. 10) according to

$$S_{u,ZP}(f) = 2hfR(f), \tag{13}$$

which depends linearly on frequency and exists for any $f > 0$, even in the deeply classical frequency regime and at zero temperature. The zero-point term described by Eq. 13 has gained widespread theoretical support over the years [33–37].

We note that absolute zero temperature cannot be reached in a physical system which means that, when discussing the zero-temperature limit, we always assume a nonzero temperature that is close-enough to zero so that $N(f,T) \ll 0.5$ holds at the measurement frequency.

4.1 A New Approach to Assess Zero-Point Johnson Noise: Energy and Force in a Capacitor [13]

For the sake of simplicity, we assume that the resistors and capacitors discussed below are macroscopic with sufficiently large density of defects that yield strong-enough defect scattering so that the phase breaking length [38] of charge transport is always much less than the smallest characteristic size of the resistors and capacitor. Thus, the resistance does not converge to zero but saturates at a nonzero, low-temperature residual value (an effect used, e.g., in low-temperature noise-thermometry). This assumption does not reduce the significance of our results and claims because the second law of thermodynamics must be valid at arbitrary conditions under thermal equilibrium.

For our present considerations of the zero-point term in the Johnson noise, the main conclusion is that the actual measurement scheme has a crucial role in the outcome of the observation. Thus, the natural question emerges: Can we use other types of measurements and check whether or not the implications of Eqs. 12 and 13 are apparent in those experiments?

Here we design two new measurement schemes utilizing the energy and force in a capacitor shunting a resistor, where the time-energy uncertainty principle is irrelevant so that we are free from an artifact pointed out by Kleen [39].

(a) **Energy in a shunting capacitor** [13]

We first consider the mean energy in a capacitor shunting a resistor. Figure 4 illustrates this system, which is a first-order low-pass filter with a single pole at a frequency $f_L = (2\pi RC)^{-1}$. Here R and C denote resistance and capacitance, respectively.

The real part of the impedance is given as $\mathrm{Re}[Z(f)] = R(1 + f^2 f_L^{-2})^{-1}$ and thus, in accordance with Callen–Welton [32] and Eq. 12, the one-sided power density spectrum $S_{u,C}(f)$ of the voltage on the impedance (and on the capacitor) is

$$S_{u,C}(f) = \frac{4RhfN(f,T)}{1 + f^2 f_L^{-2}} + \frac{2Rhf}{1 + f^2 f_L^{-2}}, \tag{14}$$

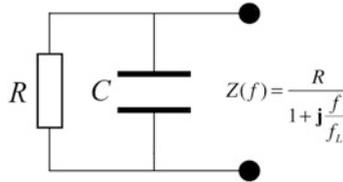where the first term is classical-physical while the second one is its quantum (zero-point) correction; see Fig. 5.

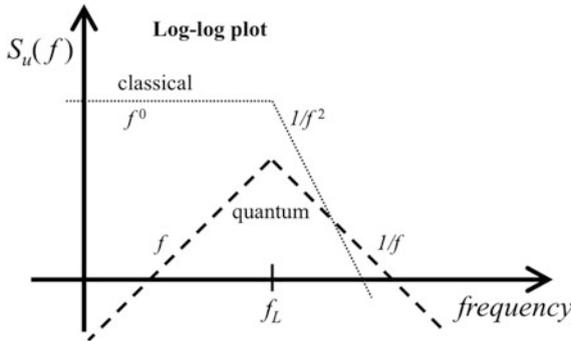**Fig. 4** Resistor R shunted by a capacitor C. $Z(f)$ is frequency-dependent impedance

Fig. 5 Bode plot, with low- and high-frequency asymptotes, of the classical and quantum (zero-point) component of the power density spectrum of the voltage on the capacitor at finite temperature. The classical Lorentzian spectrum has white and $1/f^2$ spectral regimes. At zero temperature, only the quantum term exists; it is an f-noise at low frequencies and converges to $1/f$ at $f > f_L$

The mean energy in the capacitor is given by

$$\langle E_C \rangle = 0.5C \langle U_C^2(t) \rangle = 0.5C \int\limits_0^{f_c} S_{u,C}(f) df, \tag{15}$$

where $f_c \gg f_L$ is the cutoff frequency of the transport in the resistor. At near-zero temperature, the classical component $\langle U_{C,c}^2(t) \rangle$ of $\langle U_C^2(t) \rangle$ vanishes, i.e.,

$$\lim_{T \to 0} \langle U_{C,c}^2(t) \rangle = \lim_{T \to 0} \left\{ 4Rh \int\limits_0^{f_c} \frac{f[\exp(hf/kT) - 1]^{-1}}{1 + f^2 f_L^{-2}} df \right\} = 0, \tag{16}$$

but the quantum (zero-point) term remains and is

$$\langle U_{C,q}^2(t) \rangle = \int\limits_0^{f_c} \frac{2hfR}{1 + f^2 f_L^{-2}} df = hR f_L^2 \ln\left(1 + \frac{f_c^2}{f_L^2}\right). \tag{17}$$

Thus, the energy in the capacitor, in the zero-temperature approximation, is

$$\langle E_C \rangle = \frac{h}{8\pi^2 RC} \ln\left(1 + 4\pi^2 R^2 C^2 f_c^2\right). \tag{18}$$

Equation 18 implies that, by choosing different resistance values, the capacitor is charged up to different mean-energy levels. This energy can be measured by, for example, switching the capacitor between two resistors with different resistance values and evaluating the dissipated heat as discussed below.

(b) **Force in a capacitor** [13]

In a plane circular capacitor, where the distance x between the planes is much smaller than the smallest diameter d of the planes, the attractive force between the planes [21] is given by

$$F = \frac{E_C}{x}. \tag{19}$$

Equations 18 and 19 imply that the mean force in the capacitor shunting a resistor (see Fig. 4) is

$$\langle F(x) \rangle = \frac{\langle E_C \rangle}{x} = \frac{1}{x} \frac{h}{8\pi^2 RC(x)} \ln\left[1 + 4\pi^2 R^2 C^2(x) f_c^2\right], \tag{20}$$

where the x-dependence of the capacitance is expressed by $C(x) = \varepsilon\varepsilon_0 A/x$. Here A is the surface of the planes and ε is dielectric permeability. Consequently, Eq. 20

indicates that, at a given distance x, different resistance values result in different forces.

4.2   A New Approach to Assess Zero-Point Johnson Noise: Two "Perpetual Motion Machines" [13]

The above effects on energy and force in a capacitor could be used to build two different "perpetual motion machines," *provided the zero-point term is available for these kinds of measurements*, as further discussed below. This fact proves that the fluctuation–dissipation theorem (see Eq. 12) cannot be correct under general conditions.

(a)   Zero-point noise-based "perpetual heat-generator" [13]

Figure 6 delineates a "heat-generator" and comprises an ensemble of N units, each containing two different resistors and one capacitor. The capacitors in the units are periodically alternated between the two resistors by centrally controlled switches in a synchronized fashion that makes the relative control energy negligible [40]. The duration τ_h of the period is chosen to be long enough that the capacitors are sufficiently "thermalized" by the zero-point noise, i.e., $\tau_h \gg \max\{R_1 C, R_2 C\}$. We now suppose that $R_1 < R_2$ and that the parameters satisfy $\max\left\{(4\pi R_i C)^{-1}\right\} \ll f_c$. Whenever the switch makes the $1 \Rightarrow 2$ transition, the energy difference will then dissipate in the system of R_2 resistors as

$$0 < E_h = N \frac{h}{8\pi^2 C} \left[\frac{\ln\left(1 + 4\pi^2 R_1^2 C^2 f_c^2\right)}{R_1} - \frac{\ln\left(1 + 4\pi^2 R_2^2 C^2 f_c^2\right)}{R_2} \right]. \tag{21}$$

After the reverse $2 \Rightarrow 1$ transition, the capacitors will be recharged by the system of R_1 resistors to their higher mean-energy level.
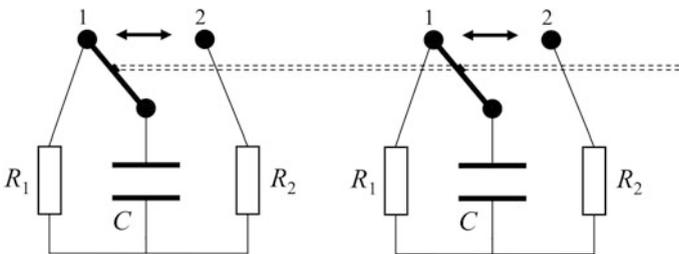

Fig. 6 Heat-generator-based "perpetual motion machine." The coupled switch is periodically alternated between the two states

Hence, the "heat-generator" system pumps energy from the system of R_1-resistors to the system of R_2-resistors, where this energy is dissipated as heat. The heat can be utilized to drive the switches of this "perpetual motion machine." Such a result violates not only the second law of thermodynamics by its negentropy production under thermal equilibrium, but it also violates the energy conservation law.

(b) **Zero-point noise-based "perpetual motion engine"** [13]

The second perpetual motion machine is a two-stroke engine illustrated in Fig. 7. This is the zero-point energy version of the two-stroke Johnson noise engine described earlier [40]. The engine has N parallel cylinders with elements and parameters identical to those in Fig. 6. The only difference is that the capacitors have a moving plate that acts as a piston. The plates are coupled to a device which moves them in a periodic and synchronized fashion. When the plate separation reaches its nearest and farthest distance limits denoted x_{min} and x_{max}—where the corresponding capacitance values are C_{max} and C_{min}, respectively—the switch alternates the driving resistor; see Fig. 8. During contraction, the attractive force between the capacitor plates should be higher than during expansion. Since the force is higher when the capacitor is connected to R_1, the driver is R_1 and R_2 (with $R_1 < R_2$) during contraction and expansion, respectively. At a given distance x, the difference in the attractive force between the cases of the capacitor being attached to R_1 and R_2 is [13, 40]

$$\langle \Delta F(x) \rangle = \frac{1}{x} \frac{h}{8\pi^2 C(x)}$$
$$* \left\{ \frac{1}{R_1} \ln \left[1 + 4\pi^2 R_1^2 C^2(x) f_c^2 \right] - \frac{1}{R_2} \ln \left[1 + 4\pi^2 R_2^2 C^2(x) f_c^2 \right] \right\}. \tag{22}$$

At a given value of x, the total force difference in N cylinders is

$$\Delta F_N(x) = N \langle \Delta F(x) \rangle. \tag{23}$$
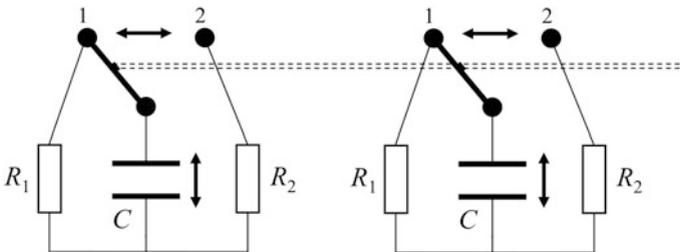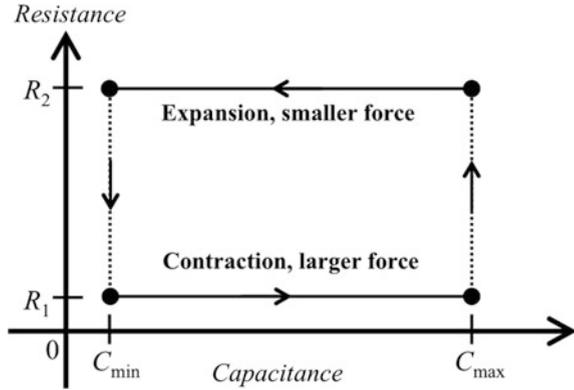

Fig. 7 A moving-plate capacitor piston-based "perpetual motion machine." The coupled switch is periodically alternated between the two states. See also Fig. 8

The distance changes during contraction and expansion, and therefore the force difference, must be integrated over x. With $R_1 < R_2$ and at any given plate distance (and corresponding capacitance), the force $N\langle F(x)\rangle$ is stronger during contraction than during expansion; see Fig. 8. During a full cycle, net positive work is executed by the engine according to

$$W = \oint_{x_{\min}, x_{\max}} N\langle F(x)\rangle dx = \int_{x_{\max}}^{x_{\min}} \Delta F_N(x)dx > 0. \tag{24}$$

While this two-stroke engine produces positive work during its whole cycle, a heat-generation effect also sets in for switching at $C_{\max}$, i.e., heat is generated in R_2 similarly to what happens in the first perpetual motion machine (see Sect. 4.2).

It should be noted that the Casimir effect also implies an attractive force between the capacitor plates. However, the Casimir pressure decays [41] as x^{-4} which implies that the Casimir force, at a fixed capacitance, falls off as x^{-3}. At the same time, the force due to the zero-point noise decays as x^{-1}. Thus, the Casimir effect can always be made negligible in the "perpetual motion machines" via a proper choice of the actual range of x values between the plates during operation.

The two "perpetual motion machines" discussed above explicitly violate not only the second law of thermodynamics but also the energy conservation law. Thus, the key assumption underlying their creation—i.e., the fluctuation–dissipation theorem (Eqs. 12 and 13) for the Johnson noise of resistors and impedances—cannot be valid under general conditions.

4.3 Is the Johnson–Nyquist Formula Valid? [13]

Can one conclude that the zero-point term must be omitted and that the remaining original Johnson–Nyquist formula (Eq. 9) is valid for the capacitor-based measurement scheme? We first suppose that Eq. 9 correctly describes the Johnson noise at arbitrary conditions. It is well known that Eq. 11, in the classical limit of

$$\frac{1}{2\pi RC} = f_L \ll \frac{kT}{h},$$

(25)

yields $\langle U_{C,c}^2(t) \rangle = kT/C$ and an ensuing mean energy of $kT/2$ in the capacitor. This is in accordance with Boltzmann's energy equipartition theorem and implies that the second law of thermodynamics is satisfied. But the situation is different in the quantum limit, with

$$\frac{kT}{h} \ll f_L,$$

(26)

because in the narrow noise-bandwidth caused by the exponential high-frequency cutoff of $N(f, T)$ the voltage noise spectrum of the capacitor is proportional to R so that $\langle U_{C,c}^2(t) \rangle \propto R$, which is evident also from Eqs. 16 and 26. Thus, in the quantum regime, according to Nyquist's old result (Eq. 1), the mean energy in the capacitor varies as

$$\langle E_C \rangle \propto RC,$$

(27)

which is an inverse scaling compared to the one in the zero-point noise limit; see Eq. 18. The result implies that, in the quantum limit (Eq. 26), the old Johnson–Nyquist formula (Eq. 9) also leads to the "perpetual motion machines" outlined in Sect. 4.2, except that the direction of the energy flow is opposite. It is also clear that the two energies encapsulated in Eqs. 18 and 27 cannot compensate each other except at a single temperature, which is unimportant when the second law of thermodynamics is violated at other temperatures.

We conclude that not only does the zero-point Johnson noise depend on the external (measurement) circuitry connected to the resistor but, in the quantum limit, Nyquist's old result (Eq. 8) also suffers from the same problem.

4.4 Conclusions and Observations About the Fluctuation– Dissipation Theorem [13, 14]

Both Nyquist and Callen–Welton were mistaken in their expectation of a general, system-independent formula for a single noise source in the resistor. We strongly

believe that the problem does not originate from the lumped (discrete) versus distributed circuit elements situation in the external circuitry, and we observe that we are accompanied in that view by, for example, Nyquist in his classical derivation [31] with a waveguide, Ginsburg–Pitaevskii in their quantum derivation [35] with classical discrete linear circuit elements, and Koch–van Harlingen–Clarke whose experimental analysis [42] employed classical discrete linear and nonlinear circuit elements.

A clarification should be made here: *We do not claim that zero-point energy does exist!* The present question is totally different: *What is the actual shape of the Johnson noise spectrum in the quantum limit of different types of measurements?*

Taking into account the experimental facts, as well as the old and new considerations, leads us to the conclusion that, in the quantum limit, it is impossible to propose a Johnson noise formula that identifies a single, measurement-system-independent (external-circuit-independent) noise source in the resistor to account for the measured noise and its effects. We surmise that this fact is in accordance with the principles of quantum physics, namely that the measurement device interferes with the recorded effect.

5 Summary and Comments

This chapter has addressed a number of basic—though often controversial—aspects of some fundamental limits for nano- and molecular electronics. In particular, we discussed the fundamental limit of energy dissipation for writing a bit of information and found that this energy is always positive and characterized by Brillouin's negentropy formula. We then turned to the fundamental limits of energy dissipation for erasing a bit of information and argued that this energy can be zero or negative, and we furthermore put forward a simple proof that Landauer's principle of erasure dissipation is invalid. Finally, we covered thermal noise in the low-temperature and/or high-frequency limit, i.e., "zero-point noise," in the quantum regime and found from simple arguments that both the quantum theory of the fluctuation–dissipation theorem and Nyquist's seminal formula are incorrect and dependent on the experimental situation, which points at the need of further studies to clarify the properties of zero-point fluctuations in resistors in electronics-based information processors operating in the quantum limit.

Our expositions are sometimes simplified and somewhat abridged versions of the original papers referred to in the headings of the various sections and subsections above, and the reader is referred to these papers for more complete discussions.

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