Continuum mechanics and its respective subtopics such as strength of materials, theory of elasticity, and plasticity are of utmost importance for mechanical and civil engineers. Tensors of different types, such as vectors and forms, appear most naturally in this context. Ultimately, when it comes to large inelastic deformations, operations like push-forward, pull-back, covariant derivative, and Lie derivative become inevitable. The latter form a part of modern differential geometry, also known as tensor calculus on differentiable manifolds.

Unfortunately, in many academic institutions, an engineering education still relies on conventional vector calculus and concepts like dual vector space, and exterior algebra are successfully ignored. The expression “manifold” arises more or less as a fancy but rather diffuse technical term. Analysis on manifolds is only mastered and applied by a very limited number of engineers. However, the manifold concept has been established now for decades, not only in physics but, at least in parts, also in certain disciplines of structural mechanics like theory of shells. Over the years, this has caused a large gap between the knowledge provided to engineering students and the knowledge required to master the challenges, continuum mechanics faces today.

The objective of this book is to decrease this gap. But, as the title already indicates, it does not aim to give a comprehensive introduction to smooth manifolds. On the contrary, at most it opens the door by presenting fundamental concepts of analysis in Euclidean space in a way which makes the transition to smooth manifolds as natural as possible.

The book is based on the lecture notes of an elective course on tensor calculus taught at TU-Bergakademie Freiberg. The audience consisted of master students in Mechanical Engineering and Computational Materials Science, as well as doctoral students of the Faculty of Mechanical, Process and Energy Engineering. This introductory text has a special focus on those aspects of which a thorough understanding is crucial for applying tensor calculus safely in Euclidean space, particularly for understanding the very essence of the manifold concept. Mathematical proofs are omitted not only because they are beyond the scope of the book but also because the author is an engineer and not a mathematician. Only in some
particular cases are proofs sketched in order to raise awareness of the effort made by mathematicians to work out the tools we are using today. In most cases, however, the interested reader is referred to corresponding literature. Furthermore, invariants, isotropic tensor functions, etc., are not discussed, since these subjects can be found in many standard textbooks on continuum mechanics or tensor calculus.

Prior knowledge in real analysis, i.e., analysis in $\mathbb{R}$, is assumed. Furthermore, students should have a prior education in undergraduate engineering mechanics, including statics and strength of materials. The latter is surely helpful for understanding the differences between the traditional and modern version of tensor calculus.

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