Chapter 1
Introduction

Abstract Irregularity induced by chaotic dynamics is essentially different from random fluctuation based on a stochastic process, since the chaotic system can be described by a set of rigorous equations, namely deterministic equations. Lasers are essentially chaotic systems described by nonlinear differential equations with three variables and they show a rich variety of chaotic dynamics. In this chapter, we briefly discuss laser chaos in relation to ordinary nonlinear systems and present a historical perspective of chaos research in semiconductor lasers. Then, the outline of this book will be presented.

1.1 Chaos and Lasers

It was in 1963 that Lorenz (1963) investigated the behaviors of convective fluids as a model for the atmospheric flow and showed that nonlinear systems described by three variables could exhibit chaotic dynamics. Of course, many researchers were aware of the existence of complex dynamics in well-defined systems from the beginning of the early 1900s. Poincaré (1913), a prominent French mathematician, at first noted the “sensitivity to initial condition” in dynamical systems. In his book, he wrote that “it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.”

However, modern research of chaos started from the study of irregular and complex dynamics of a nonlinear system developed by Lorenz (Motter and Campbell 2013). In actual, in 1961 ahead of the study by Lorenz, Ueda showed a strange attractor in the output power from an electrical circuit of a van der Pol/Duffing oscillator, which is now known as Ueda attractor or Japanese attractor (Ueda et al. 1992). Chaos is not only a description of a different viewpoint of nonlinear phenomena but also in itself a new physics. Chaos is a phenomenon of irregular variations of systems’ outputs derived from models that are described by a set of deterministic equations. We must distinguish “chaos” from the observation of
“random” events, such as the flipping of a coin, since chaos is generated in accordance with the deterministic order, namely, chaotic dynamics refers to deterministic development with chaotic outcome (Appendix A.1). The system evolves in a deterministic way and the current state of the system depends on the previous state in a rigidly deterministic way, although the systems’ output shows random variations. This is in contrast to a random system where the present observation has no causal connection to the previous one. In spite of the deterministic models, we cannot foresee the future of the output, since chaos is very sensitive to the initial conditions, as Poincaré pointed out, and each system behaves completely different from each other even if the difference of the initial state is very small.

Chaos is always accompanied by nonlinearity. Nonlinearity in a system simply means that the measured values of the properties in the system depend in a complicated way on the conditions in the earlier state. Nonlinear property in a system does not always guarantee the occurrence of chaos, but some form of nonlinearity is required for the realization of chaotic dynamics. Chaos can be observed in various fields of engineering, physics, chemistry, biology, and economics. Even our cardiac rhythms are proved to be chaotic (Elbert et al. 1994). Though the fields are different, some of the chaotic systems can be characterized by similar differential equations. They show similar chaotic dynamics and the same mathematical tools can be applied for the analysis of their chaotic dynamics.

Nonlinear systems can be also found in optics. Many optical materials and devices show nonlinear response to the optical field and, therefore, they are candidates for nonlinear elements in chaotic systems. One such device is the laser. Since lasers themselves are nonlinear systems and are typically characterized by three variables; field, polarization of matter, and population inversion, they are also candidates for chaotic systems. Indeed, it was proved in the mid-1970s by Haken (1975) that lasers are nonlinear systems similar to the Lorenz model and show chaotic dynamics in their output powers. He assumed a ring laser model and considered two-level atoms in the laser medium. Though lasers are not always described by his model, the approximations are reasonable for most lasers. Thereafter, the laser rate equations that are described by the nonlinear equations with three variables, are called Lorenz–Haken equations after their contributor (Haken 1985). However, ordinary lasers do not exhibit chaotic behavior and only a few of the lasers with bad cavity conditions show chaotic dynamics. In the meantime, chaotic behaviors were theoretically demonstrated in a ring laser system (Ikeda 1979). Chaotic oscillations of infrared gas lasers in experiments were reported for Xe lasers, He–Ne lasers, and NH₃ lasers (Casperson 1978; Weiss and King 1982; Weiss et al. 1985). Weiss and Brock (1986) experimentally observed Lorenz–Haken-type chaos in infrared NH₃ lasers.

Contrary to the prediction of Haken, ordinary lasers are stable systems and only a few systems of infrared gas lasers show chaotic behaviors in their output powers. Arecchi et al. (1984) investigated laser systems from the viewpoint of the characteristic relaxation times of the three variables and categorized lasers into three classes. According to their classifications, one or two of the relaxation times are in
general very fast compared with the other timescales and most lasers are described by the rate equations with one or two variables. Therefore, they are stable systems that are categorized into class A and B lasers. Only class C lasers have the full description of the rate equations with three variables and can show chaotic dynamics. However, class A and B lasers can show chaotic dynamics when one or more degrees of freedom are introduced to the laser systems.

Class B lasers are characterized by the rate equations for field and population inversion, and they are easily destabilized by an additional degree of freedom as an external perturbation. For example, solid-state lasers, fiber lasers, and CO₂ lasers that are categorized as class B lasers, show unstable oscillations by external optical injection or modulation for accessible laser parameters. Semiconductor lasers, which are also classified into class B lasers and are the main topic of this book, are also very sensitive to self-induced optical feedback, optical injection from different lasers, optoelectronic feedback, and injection current modulation. A review of the earlier study of laser instabilities and chaos has been given by Abraham et al. (1988).

1.2 Historical Perspectives of Chaos in Semiconductor Lasers

Chaos in semiconductor lasers is of particular importance in practical applications, since chaos induced by semiconductor lasers is very fast and the main frequency of irregular oscillations is usually over giga hertz, which is much faster than those of chaos such as in electronic circuits. Indeed, chaos in semiconductor lasers is usually two digits or more faster than that attained by fast electronic circuits. Also, light is a carrier of modern communications and such chaotic oscillations match well with fast data transmissions in the existing optical network channels. Semiconductor lasers (edge-emitting and narrow-stripe types), which are the main topics of this book, are intrinsically stable lasers. However, semiconductor lasers, which are described by the field and the carrier density (equivalently the population inversion) equations, can be easily destabilized by the introduction of external perturbations such as external optical feedback, optical injection, or modulation for accessible laser parameters. Since the early 1980s, feedback-induced instabilities and chaos in semiconductor lasers have been extensively examined (Lang and Kobayashi 1980). In a semiconductor laser, the laser oscillation is affected considerably when the light reflected back from an external reflector couples with the original field in the laser cavity. A variety of dynamics can be observed in semiconductor lasers with optical feedback and they have been investigated by many researchers for the past two decades.

One of the main differences between semiconductor lasers and other lasers is the low reflectivity of the internal mirrors in the laser cavity. It ranges typically only from 10 to 30% of the intensity in edge-emitting semiconductor lasers. This makes
the self-feedback effects significant in semiconductor lasers. Another difference is a large absolute value of the linewidth enhancement factor $\alpha$ of the laser media. The value of the linewidth enhancement factor $\alpha = 2–7$ was reported depending on the laser materials, while this value is almost zero for other lasers. Then, the coupling between the phase and the carrier density is encountered in the laser dynamics. These factors lead to a variety of dynamics quite different from any other lasers. At weak to moderate external optical feedback reflectivity, the laser output shows interesting dynamical behaviors such as stable state, periodic and quasi-periodic oscillations, and chaos for the variations of the system parameters. These ranges of external reflectivity are not only interesting from the viewpoint of fundamental physics, but also very important in practical applications of semiconductor lasers, such as in optical data storage systems and optical communications. Extensive lists of the recent literature for the dynamic characteristics in semiconductor lasers with optical feedback can be found in the following references (van Tartwijk and Agrawal 1998; Otsuka 1999; Ohtsubo 1999, 2002a, 2005, 2008, 2012; Uchida 2012).

Optical injection-locking phenomena are a universal feature in lasers. Since the internal reflectivity of the facet in a semiconductor laser is very low compared with other lasers, one can easily realize injection locking from a different laser. Moreover, the effects of injection locking stand out due to the nonzero value of the linewidth enhancement factor $\alpha$ and one can observe not only stable injection locking but also various dynamics of unstable optical injection phenomena depending on the injection parameters. Semiconductor lasers usually have different laser oscillations characteristics for the same product number or even for the same wafer, but the oscillation frequency can be easily tuned on the order of GHz by changing the injection current. Therefore, a light source for injection locking to different lasers with appropriate frequency detuning is easily available. Thus, optical injection-locking phenomena have been extensively studied in semiconductor lasers. However, earlier work was limited to stable injection-locking phenomena for amplification of signals and laser stabilization. In these applications, the laser is locked to the external laser, which means that it almost copies the spectrum of the injected light. On the other hand, unstable injection locking, instabilities, and mixing of detuned frequencies occur outside the region of stable injection locking in the phase space of the frequency detuning and the injection ratio. From the viewpoint of chaos, optical injection is an addition of an extra degree of freedom to semiconductor lasers and it may induce chaotic oscillations in the laser output. It was numerically predicted (Sacher et al. 1992; Annovazzi-Lodi et al. 1994) and experimentally demonstrated (Simpson et al. 1994, 1995) that an optically injected semiconductor laser follows a period-doubling route to chaos.

Direct modulation for accessible parameters is not an easy task in most lasers, however, the output power of a semiconductor laser is easily controlled through the injection current and, at the same time, the laser frequency can be changed by the injection current modulation. Small amplitude injection current modulation or even large modulation under appropriate conditions for laser oscillations may produce faithful copies of the modulation amplitude for the output power in a semiconductor
laser. However, modulation for the injection current is a perturbation to the laser and also the introduction of an extra degree of freedom to it. Indeed, instabilities and chaotic oscillations have been observed by the injection current modulation in semiconductor lasers under the conditions of high frequency modulation with a large modulation index close to the relaxation oscillation frequency of the laser (Hori et al. 1988).

Optoelectronic feedback systems in which the emitted light from a semiconductor laser is once detected and fed back through the injection current are also studied to stabilize the laser oscillations. For a certain range of optoelectronic feedback, the laser may indeed be stabilized and the method is applied to obtain an ultra-high coherent light source. However, optoelectronic feedback has a similar effect to the above perturbations on the dynamics in semiconductor lasers. We can also observe unstable pulsation oscillations in the output of a semiconductor laser for certain conditions (Olesen et al. 1986). With the availability of high-speed electronic circuits, optoelectronic feedback systems having a time response of the same order as the relaxation oscillation frequency (on the order of GHz) have been studied and useful applications of chaos dynamics have been proposed based on high-speed optoelectronic feedback (Tang and Liu 2001a, b).

A variety of novel semiconductor laser devices with different structures has been proposed and fabricated beside narrow-stripe edge-emitting semiconductor lasers, for example, self-pulsating semiconductor lasers, vertical-cavity surface-emitting semiconductor lasers (VSCELs), broad-area semiconductor lasers, quantum-dot semiconductor lasers, quantum-cascade semiconductor lasers, and so on. These lasers themselves have extra degrees of freedom in addition to the characteristics of ordinary narrow-stripe edge-emitting semiconductor lasers. For example, space-dependent differential terms due to a wide stripe width are introduced in the rate equations for broad-area lasers and these terms play an important role in the laser dynamics. Therefore, these newly developed lasers themselves are unstable and exhibit chaotic dynamics without any external perturbations (Yamada 1993; Law et al. 1997; Gehrig and Hess 2000). The studies of chaotic dynamics in semiconductor lasers including new structure devices are excellent models for nonlinear chaotic systems and are very interesting from the viewpoint of basic chaotic research. Instabilities and chaotic behaviors are also greatly enhanced by additional external perturbations to these lasers in the same manner as narrow-stripe edge-emitting semiconductor lasers.

In the case of the vertical-cavity surface-emitting lasers (VCSELs), the reflectivity of the internal mirrors is very high at more than 99%, however, they are also sensitive to external optical feedback due to a small number of photons in the internal cavity. Therefore, semiconductor lasers of all types are essentially very sensitive to external optical feedback. In spite of the differences in device structures, the dynamics of semiconductor lasers are the same as long as the laser rate equations are written in the same or similar forms. The dynamics of narrow-stripe edge-emitting single-mode semiconductor lasers have been extensively studied for a long time and a lot of fruitful results have been obtained. However, they are still important issues for the fundamental physics of optical chaos and also for practical
applications. On the other hand, a little investigation into the dynamics of newly developed laser structures has been carried out.

Through external perturbations, semiconductor lasers are either stabilized or destabilized. The effects of such perturbations on laser dynamics, stability and instability, are two sides of the same coin. Examples include optical feedback, optical injection, and optoelectronic feedback. To stabilize laser oscillations, the disturbances may be weak or strong. The lasers can then be strongly stabilized under appropriate conditions of external parameters and operating conditions of the lasers. Stabilization of semiconductor lasers is very important with regard to their application. For example, frequency stabilization, linewidth narrowing, power stabilization, polarization fixing, and beam shaping are very important in optical communications, optical data storage systems, and optical measurements. In particular, ultra-stabilized semiconductor lasers are expected in broadband optical communications, high precision optical measurements, and standard light sources. Semiconductor lasers are rather unstable compared with other lasers, and their stabilization has been an important issue from the beginning of their development. For newly developed semiconductor lasers, such as VCSELs and broad-area semiconductor lasers, we can apply the same techniques of laser stabilization as those for narrow-stripe edge-emitting semiconductor lasers. However, these lasers themselves demonstrate instabilities in their solitary oscillations. In VCSELs, controls for polarization and spatial mode instabilities are essential in applications. In broad-area semiconductor lasers, filament suppression of the oscillation pattern can greatly improve beam quality, producing a high-density beam. Such unstable oscillations are also stabilized using similar techniques through external controls, as discussed above. Semiconductor lasers are still developing, and stabilization both by the device structure and through external controls is currently an important research area.

In the meantime, important breakthroughs for applications of chaos were made in the early 1990s. The ideas of chaos control and chaos synchronization were proposed and developed in this decade as common interests in various fields of nonlinear and chaos research. The ideas of chaos control (Ott et al. 1990) and chaos synchronization (Pecora and Carroll 1990) were proposed and developed in this decade. Noise suppression of feedback-induced chaotic oscillations in semiconductor lasers has been proposed based on chaos control (Liu et al. 1995). Also, fixed point or periodic oscillations, precursor to the onset of chaos in semiconductor lasers with optical feedback, can be used for laser control and optical measurements (Donati et al. 1995). In optical feedback interferometry, which is called self-mixing interferometry, the feedback light from a target to be measured is usually detected by a photodetector installed within a package of a semiconductor laser and a very compact sensor for the interferometric measurements can be constructed. However, in some types of semiconductor lasers, it is not easy to install a photodetector within a laser package. For example, it is difficult to fabricate a photodetector in quantum-cascade semiconductor lasers, which emit THz optical light. More than that, a stand-alone and fast response photodetector is not available in THz wavelength. Instead of using a photodetector in self-mixing interferometry, the voltage
variations across the laser diode induced by self-mixing effects can be employed for the detection of mixing signals. This provides an effective tool for interferometric measurements based on self-mixing interferometry in THz optical wave region, which cannot be obtained by conventional technology (Lim et al. 2011).

As another application, the possibility of chaotic communications has been discussed based on chaos synchronization in two chaotic solid-state laser systems (Colet and Roy 1994). After their pioneering work, the study of secure data transmissions and communications has also been discussed based on synchronization in two chaotic semiconductor laser systems (Chen and Wornell 2001; Feature Section IEEE J Quantum Electron 2002; Ohtsubo 2002b; Uchida 2012). Further, chaos synchronization occurs among many coupled semiconductor lasers. Zero-lag or cluster synchronization among semiconductor lasers appears depending on the topological structure of coupling networks (Nixon et al. 2011; Ohtsubo et al. 2015). The phenomena are strongly related to crowd synchrony in many coupled nonlinear elements (Strogatz et al. 2005). The keys of the dynamics in those networks are synchronization among nonlinear elements and consistent responses as drive-response systems, which are a typical feature appeared in neural networks, such as in neurons in the human brain. Depending on the phenomenon, bio-inspired information processing called reservoir computing is proposed, in which a single semiconductor laser subjected to optical feedback is used as a reservoir in the neural networks (Appeltant et al. 2011). Irregular chaotic oscillations in semiconductor lasers have also provided a new method of ultrafast physical random number generations, which is the key technology in modern cryptographic applications (Uchida et al. 2008; Uchida 2012). Also for those applications, photonic integrated circuits to generate chaotic light have been proposed (Argyris et al. 2008). Harnessing chaotic lasers is very attractive from the viewpoint of applications, since optics is very fast and contains parallelism as a nature of light. Applications of chaotic lasers are still growing and developing. Thus, chaotic lasers are not only important for basic research but also for engineering applications.

1.3 Outline of This Book

In this book, we focus on the dynamics and applications in semiconductor lasers subjected to external perturbations. In Chap. 2, we first introduce general forms of laser rate equations, which are equivalent to the Lorenz equations, and the classifications of lasers are given. The instabilities intrinsically involved in the rate equations are studied. Next, semiconductor lasers as class B lasers are described. The possibility of unstable oscillations in semiconductor lasers by the introduction of external perturbations is discussed. A solitary semiconductor laser is characterized by two equations for the field and the carrier density (population inversion). We then derive the forms of the rate equations for edge-emitting semiconductor lasers with a narrow stripe width in Chap. 3. Linear stability analysis used as a common tool for investigating the stability and instability conditions of nonlinear
chaotic systems is introduced and the laser relaxation oscillation frequency, which plays an important role in the chaotic dynamics, is derived. Several fundamental characteristics of semiconductor lasers are also introduced.

In Chap. 4, the theory of optical feedback in semiconductor lasers is presented. The effects of feedback in various external reflectors, including grating mirrors and phase-conjugate mirrors, are taken into account and the formulations of their systems are presented by introducing rate equations with optical feedback effects. In Chap. 5, substantial feedback effects and chaotic dynamics in semiconductor lasers are discussed and both numerical and experimental results are given under variations of the system parameters. Feedback-induced chaos depending on the external cavity length and the feedback fraction is investigated. Chaos induced by external optical injection with frequency detuning is also an important issue in semiconductor laser systems. The theory of optical injection and their instabilities are discussed in Chap. 6. Unstable and chaotic oscillations are observed in the region outside the stable injection locking in the phase space of the frequency detuning and the injection fraction. The coexistence state of chaotic attractors, which is known as one of the characteristics in nonlinear systems, is demonstrated in the injection-locking systems. The effects for the modulation bandwidth in optically injection-locked semiconductor lasers are discussed in this chapter. Also, enhancement of chaotic frequency by strong optical injection locking is demonstrated. In Chap. 7, dynamic characteristics of optoelectronic feedback and injection current modulations are presented. Unstable chaotic pulsations induced by feedback and modulation to the injection current are investigated in relation to the characteristics of optoelectronic feedback circuits.

The rate equations of semiconductor lasers with various laser structures are introduced in Chap. 8. We assume a single mode oscillation for a semiconductor laser in the preceding chapters, however, the effects of the multimode oscillations in narrow-stripe edge-emitting semiconductor lasers are considered and dynamic properties of multimode lasers are discussed in this chapter. Stable and unstable periodic oscillations, and chaotic pulsations of self-pulsating semiconductor lasers, which are developed as light sources for optical data storage systems, are studied. VCSELs have quite different structure from other semiconductor lasers, since the laser has a circular aperture with very short cavity length compatible with the optical wavelength. Such lasers show spatial and polarization dynamics and their control is an important issue for applications. Broad-area lasers and laser arrays are also interesting future devices in engineering applications as high power lasers. They show unique spatio-temporal dynamics and the controls of instabilities are important in applications. Quantum-dot semiconductor lasers have been developed for stable and highly coherent light sources in optical communications. However, they still exhibit instabilities for external perturbations. Quantum-cascade semiconductor lasers are expected as new THz light sources. They have a mechanism of light emission different from other semiconductor lasers, namely, the carriers are only electrons and the laser oscillations are based on inter-subband optical transition through multi-stage quantum-cascade band structures. They also show interesting dynamics. These new types of semiconductor lasers themselves contain instability
arising from their structures and show chaotic behaviors even in the absence of external perturbations.

We cannot foresee the future of chaotic oscillations for time evolution, since chaos has a strong dependence on the initial condition of a system. However, chaos can be controlled. In Chap. 9, methods of chaos control are introduced. Control of chaotic oscillations in semiconductor lasers with optical feedback is discussed and the reduction of the feedback-induced relative intensity noise (RIN) is demonstrated based on the method of chaos control. These methods can also be applied not only to ordinary narrow-stripe edge-emitting semiconductor lasers but also to other semiconductor lasers with newly developed structures. Either stabilities or instabilities are enhanced by external perturbations in semiconductor lasers, and their dynamics are discussed in Chap. 10. In this chapter, methods of stabilization and control of semiconductor lasers are presented. Some of these are closely related to chaos control, as discussed in Chap. 9, and others involve forced control of stable oscillations. Stabilization for laser oscillations such as linewidth, frequency, spatial modes, polarization, and so on, are introduced in narrow-stripe edge-emitting semiconductor lasers. Similar control techniques are also applied to newly developed semiconductor lasers. In semiconductor lasers with optical and optoelectronic feedback systems, periodic oscillations of the outputs preceding the onset of chaos are observed for the variations of chaotic parameters. These properties can be applied to various measurements, such as interferometric displacement and vibration measurements. In Chap. 11, applications of self-mixing interferometers and active interferometers are discussed, in which bistable states of the systems before the onset of chaotic bifurcations are used. In particular, self-mixing interferometry using quantum-cascade semiconductor lasers provides an effective interferometric measurement in THz optical wave region, which cannot be obtain by conventional technology. Other interferometric and correlation techniques of chaotic semiconductor lasers for various measurements are discussed in this chapter.

Synchronization of two chaotic nonlinear systems is interesting not only from the viewpoint of fundamental physics but also from applications. It is not self-evident that two chaotic nonlinear systems show synchronization and the theoretical background for chaos synchronization has not yet been fully established. However, synchronization of chaotic oscillations has been observed by experiments and numerical simulations. Chaos synchronization can be also observed in systems of chaotic semiconductor lasers. The systems and conditions for synchronization in chaotic semiconductor lasers are discussed in Chap. 12. Chapter 13 follows the applications of chaos synchronization. Since strict conditions must be satisfied for chaos synchronization in nonlinear systems, one can construct a secure communication channel in the sense of hardware levels. The possibility of chaos communications is presented based on chaos synchronization in semiconductor laser systems. Also, chaotic communications through the existing public communication channel is demonstrated. Chaos communications in Chap. 13 are based on analog techniques of hiding messages behind irregular chaotic undulations. Laser chaos, especially chaos induced by semiconductor lasers is fast and is suitable for data transmissions in current optical communication channels.
Chaos synchronization is not only appeared in two coupled semiconductor lasers but also more than two coupled systems. Chapter 14 discusses phenomena of zero-lag and cluster synchronization in many coupled semiconductor lasers with different topological structures. The dynamics of these systems have an analogy with those of neuron networks in the human brain. In this chapter, we also present a new type of information process and its application based on reservoir computing, in which a single semiconductor laser subjected to optical feedback is used as a reservoir in the neural network. The keys for common dynamics of such systems are the consistency in drive-response nonlinear systems and the synchronization properties between distant nonlinear elements. On the other hand, digital techniques of chaotic semiconductor lasers for cryptographic applications have been developed. For distributions of random keys in cryptography, the method of ultrafast physical random number generations over the rate of giga-bit-per-second has been proposed. In Chap. 15, the principle and practice of fast physical random number generations using chaotic semiconductor lasers are discussed. For the purpose of communication applications of chaotic semiconductor lasers, compact and stable light sources as chaotic generators are expected. Monolithic and optically integrated circuits for chaotic light generators are presented in this chapter.

The origins of chaos are unique for each nonlinear system, but there are common tools for the analyses of chaotic dynamics. For detailed descriptions of chaos and their analyses, the reader is referred to appropriate books. However, for readers not familiar with chaos and its analyses, I finally attached an appendix on the origins of chaos in nonlinear systems and some of their common tools for chaotic data analyses. In this book, I treat the main topics of chaos dynamics and applications in semiconductor lasers, however, they are not the entirety of the research. Other related topics are dynamics in new types of semiconductor lasers, such as micro-cavity semiconductor lasers (Lee et al. 2002; Cao and Wiersig 2015), random lasers (Cao 2003), flared broad-area lasers (Levy and Hardy 1997), and others. Also, optoelectronic chaotic oscillator, in which chaos is not generated from the nonlinearity of a semiconductor laser itself but from the nonlinear delayed response of light due to delayed optoelectronic hybrid feedback through EO modulator (an integrated EO Mach–Zehnder interferometer), is an attractive system, and widely used as a useful chaotic source for various applications (Argyris et al. 2005; Larger and Dudley 2010). The systems are also important and interesting with relation to the study of stabilities, instabilities, and chaos in nonlinear systems. But, we here limit the discussion for the fundamentals and applications of the dynamics intrinsically induced by the nonlinearity in semiconductor lasers themselves. Research into chaos in semiconductor lasers is still ongoing, and we can expect fruitful results both for basic physics and practical applications.
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Semiconductor Lasers
Stability, Instability and Chaos
Ohtsubo, J.
2017, XXV, 666 p. 331 illus., 97 illus. in color., Hardcover
ISBN: 978-3-319-56137-0