

Contents

1	Probability	1
1.1	The Elements of Probability: $(\Omega, \mathcal{B}, \mu)$	1
1.1.1	Events and Sample Space: (Ω)	1
1.1.2	σ -algebras (\mathcal{B}_Ω) and Measurable Spaces $(\Omega, \mathcal{B}_\Omega)$	3
1.1.3	Set Functions and Measure Space: $(\Omega, \mathcal{B}_\Omega, \mu)$	6
1.1.4	Random Quantities	10
1.2	Conditional Probability and Bayes Theorem	14
1.2.1	Statistically Independent Events	15
1.2.2	Theorem of Total Probability	18
1.2.3	Bayes Theorem.	19
1.3	Distribution Function	23
1.3.1	Discrete and Continuous Distribution Functions	24
1.3.2	Distributions in More Dimensions	28
1.4	Stochastic Characteristics	35
1.4.1	Mathematical Expectation	35
1.4.2	Moments of a Distribution	36
1.4.3	The “Error Propagation Expression”.	44
1.5	Integral Transforms	45
1.5.1	The Fourier Transform	45
1.5.2	The Mellin Transform.	53
1.6	Ordered Samples	63
1.7	Limit Theorems and Convergence	67
1.7.1	Chebyshev’s Theorem.	68
1.7.2	Convergence in Probability.	69
1.7.3	Almost Sure Convergence	70
1.7.4	Convergence in Distribution	71
1.7.5	Convergence in L_p Norm	76
1.7.6	Uniform Convergence.	77

Appendices	81
References	85
2 Bayesian Inference	87
2.1 Elements of Parametric Inference	88
2.2 Exchangeable Sequences	89
2.3 Predictive Inference	91
2.4 Sufficient Statistics	92
2.5 Exponential Family	94
2.6 Prior Functions	95
2.6.1 Principle of Insufficient Reason	96
2.6.2 Parameters of Position and Scale	97
2.6.3 Covariance Under Reparameterizations	103
2.6.4 Invariance Under a Group of Transformations	109
2.6.5 Conjugated Distributions	115
2.6.6 Probability Matching Priors	119
2.6.7 Reference Analysis	125
2.7 Hierarchical Structures	133
2.8 Priors for Discrete Parameters	135
2.9 Constrains on Parameters and Priors	136
2.10 Decision Problems	137
2.10.1 Hypothesis Testing	139
2.10.2 Point Estimation	145
2.11 Credible Regions	147
2.12 Bayesian (\mathcal{B}) Versus Classical (\mathcal{F}) Philosophy	148
2.13 Some Worked Examples	154
2.13.1 Regression	154
2.13.2 Characterization of a Possible Source of Events	158
2.13.3 Anisotropies of Cosmic Rays	161
References	166
3 Monte Carlo Methods	169
3.1 Pseudo-Random Sequences	170
3.2 Basic Algorithms	171
3.2.1 Inverse Transform	171
3.2.2 Acceptance-Rejection (Hit-Miss; J. Von Neumann 1951)	178
3.2.3 Importance Sampling	183
3.2.4 Decomposition of the Probability Density	185
3.3 Everything at Work	186
3.3.1 The Compton Scattering	186
3.3.2 An Incoming Flux of Particles	192
3.4 Markov Chain Monte Carlo	199
3.4.1 Sampling from Conditionals and Gibbs Sampling	214

- 3.5 Evaluation of Definite Integrals 218
- References 219
- 4 Information Theory 221**
 - 4.1 Quantification of Information 221
 - 4.2 Expected Information and Entropy 223
 - 4.3 Conditional and Mutual Information 226
 - 4.4 Generalization for Absolute Continuous Random Quantities 228
 - 4.5 Kullback–Leibler Discrepancy and Fisher’s Matrix 229
 - 4.5.1 Fisher’s Matrix 230
 - 4.5.2 Asymptotic Behaviour of the Likelihood Function 232
 - 4.6 Some Properties of Information 234
 - 4.7 Geometry and Information 238
 - References 244



<http://www.springer.com/978-3-319-55737-3>

Probability and Statistics for Particle Physics

Maña, C.

2017, X, 244 p. 27 illus., 11 illus. in color., Hardcover

ISBN: 978-3-319-55737-3