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Investors' Portfolio Choice and Portfolio Theory

This chapter focuses on essential features of the modern portfolio theory, which is a central framework for studies and analyses of investors' portfolio choice. Section 2.1 introduces the foundations of the theory to provide a brief explanation how this theory effectively can assist investors in their portfolio allocation. Section 2.2 describes the mean-variance analysis approach developed and launched by Markowitz (1952). Section 2.3 reviews the part of modern portfolio theory that is concerned with economic equilibrium under the assumption that all investors optimize their portfolio in a particular manner with homogenous expectations. Section 2.4 shows how to estimate empirically the alpha and beta coefficients in this model using regression analysis. Section 2.5 concludes the chapter.

2.1 Introduction to the Modern Portfolio Theory

In a broad sense, all investment activities are about how to secure future consumption. Households, the ultimate savers and investors, maximize their utility by deciding how much of their wealth should be consumed

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today and how much should be saved or invested for consumption at a future date. Figure 2.1 shows how financial institutions and markets affect the possibilities of individual households (and others in surplus of financial means) to allocate their surplus over time into investment projects, with expected positive returns, governed by project owners (firms and other organizations) displaying a deficit of financial resources.

Financial intermediaries are at the center of the asset transformation process as shown in Fig. 2.1. The households (top) express a strong preference for saving or investing their surpluses of comparatively small amounts of funds to low risk and high liquidity. A project in the real sector can be said to have the opposite characteristics. In general, these projects are characterized by having a long duration and thereby low liquidity, a certain size/scale requiring large amounts of funds, and some degree of uncertainty characterized by exposure to high project-specific risks. Financial intermediaries operating on the money and capital markets match these households' preferences and project characteristics. While performing their tasks to transform large, risky, and illiquid assets in the form of real investment projects into small, liquid, and less risky assets in the form of financial (savings) products, these financial intermediaries increase welfare by lowering the transaction costs of capital funds transfers in terms of reduced search, contracting, and control and monitoring costs.

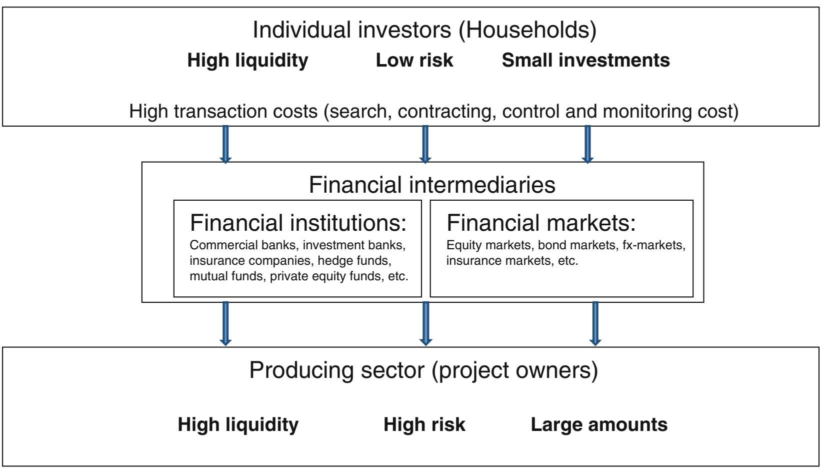

Fig. 2.1 From savings to investment: the asset transformation process

The modern portfolio theory (MPT) developed and first formalized by Markowitz (1952) is an important cornerstone of financial economic thinking. This theory explains how wealth-optimizing investors (households and their representatives) behave when making their investments on efficient financial markets. It also provides the financial market a tool for optimal portfolio diversification. The origin of the MPT involved many academics and researchers, including Nobel laureates, such as Harry M. Markowitz himself, James Tobin, William F. Sharpe, Joseph E. Stiglitz, Daniel Kahneman, Robert J. Shiller, and Eugene F. Fama, all acknowledged and famous for making important contributions to our understanding of investors' portfolio choice.

According to Markowitz (1999), who is often referred to as the father of the portfolio theory, the development of MPT can be divided into two parts. The first part is the groundbreaking work of Markowitz himself published in the *Journal of Finance* in the early 1950s. This stage focused on how a rational risk-averse investor should behave when optimizing her or his wealth. In that respect, the MPT can be thought of as being normative. The theory formalizes what investors already knew intuitively: That it is better to invest in portfolios than in single securities. What did we know or do before the MPT was launched? Markowitz (1999:5) writes, 'Diversification of investments was a well-established practice long before I published my paper on portfolio selection in 1952.' This sentence is on the same page followed by a citation of Shakespeare's *Merchant of Venice*, Act 1, Scene 1: 'My ventures are not in one bottom trusted, Nor to one place; nor is my whole estate Upon the fortune of the present year; Therefore, my merchandise makes me not sad.'

Some thousand years before Shakespeare, in ancient Athens, financial instruments and institutional arrangements were developed to deal with exposures to risk and asymmetric information, making Athens an important financial center. To spread their own risk-taking, banks were organized as partnerships instead of being owned separately. To further minimize their risk, bankers employed collateralization of valuable property. A sea loan, or *bottomry*, where the ship hull and/or its cargo served as collateral, was one such example where diversification and risk-sharing were practiced (Baskin and Miranti 1997).

Even though Markowitz was not the first to describe how to construct portfolios, he bridged the gap between theory and practice with his development of the mean-variance analysis framework. Markowitz recognized the work by Roy (1952), who presented a portfolio choice model based on covariance among returns. Markowitz (1999) generously proposes that Roy's contribution had the same value as his own. Other important contributions mentioned by Markowitz were the earlier works by Hicks (1935), Williams (1938), and Marchack (1938).

The second part of the emergence of the MPT constitutes the pioneering works by Sharpe (1964), Treynor (1962), Lintner (1965), Mossin (1966), and Black (1972), eventually leading to the development and construction of the CAPM. The CAPM explains the equilibrium state of an efficient market in which investors are acting under the assumption of homogenous expectations in accordance with Markowitz's mean-variance analysis framework. Since its introduction, the CAPM has been the dominant model to determine the market prices of equities and other securities, albeit under severe scrutiny. The next two chapters address the foundation for and the main criticism of the model by bringing in behavioral aspects of human decision-making under risk and market (in)efficiency. In Chap. 4, we also present alternative capital asset pricing models, such as the arbitrage pricing model (Ross 1976) and the three-factor model launched by Fama and French (1992), among others.

Between the two stages of MPT emergence, Tobin (1958) developed and launched his separation theorem. In this work, Tobin uses individual investors' portfolio choice behavior to predict the demand and supply of capital in society. His main contribution is that he introduces a risk-free security and shows how capital asset allocation will lead to what he calls the set of super-efficient portfolios, and a capital market line on which efficient portfolio combinations outperform all other portfolios.

2.2 The Mean-Variance Analysis

There are two basic concepts that constitute the MPT: return and risk. The mean-variance analysis is about finding the optimal trade-off between return and risk in a portfolio context. The realized net return

($r_{j,t}$) of a single security j held over a time period t is the amount of cash distributed over t paid out as dividend ($\mathrm{Div}_{j,t}$) and the difference in the values (prices) of j at the end ($P_{j,t}$) and at the beginning ($P_{j,t-1}$) of t divided by $P_{j,t-1}$. This gives Eq. 2.1:

$$r_{j,t} = \frac{\mathrm{Div}_{j,t} + P_{j,t} - P_{j,t-1}}{P_{j,t-1}}. \tag{2.1}$$

Realized returns are calculated *ex post* when all risks have materialized. Investors base their portfolio choice on their beliefs or expectations of future security returns and risks. Studying investors' portfolio allocation, the focus is thus on the expectations of return and risk *ex ante*. However, for an investor, the best prediction of the future is most often found by looking at the past historical returns over a certain time horizon (T). The return of equities almost always includes some degree of uncertainty, which is why there is a need for a sagacious way to look at history and a sensible method to capture it. The first assumption is that the returns on the market, where assets (equities and/or other securities) are traded, display a normal distribution over time. Hence, the historical performance of an asset j over the time horizon T can be described using only the historical mean of j's actual return ($\widetilde{r}_{j,t}$) in each time period t and the standard deviation (σ_j) of its returns as a measure of its riskiness in terms of historical variation (volatility) of returns. In Eq. 2.2, the historical mean is measured as an arithmetic average return ($\overline{r}_j$):

$$\overline{r}_j = \frac{\sum_{t=1}^{T} \widetilde{r}_{j,t}}{T}. \tag{2.2}$$

Given normal distribution, the realized arithmetic average return ($\overline{r}_j$) in Eq. 2.2 is the net return an investor should be expected to earn during any period of the investigated time horizon. An alternative return measure is the buy-and-hold geometric mean. Equation 2.3 shows how to calculate the average geometric return $\overline{\overline{r}}_j$ for asset j over the T buy-and-hold periods:

$$\bar{\bar{r}}_j = \left[\left(1+\tilde{r}_{j,1}\right) \times \left(1+\tilde{r}_{j,2}\right) \times \cdots \times \left(1+\tilde{r}_{j,T}\right)\right]^{1/T} - 1. \tag{2.3}$$

The average geometric return ($\bar{\bar{r}}_j$) is considered to better mirror the historical performance of an equity (see, e.g., Berk and DeMarzo 2016) and is the most commonly used return measure for making comparisons over time. However, investors are in practice more likely to use the historical average arithmetic return ($\bar{r}_j$) of asset j as a predictor for the asset's future expected return when future returns are assumed to be independent events from the same underlying distribution.

The net return of a portfolio of assets is the average weighted return of all assets included in the portfolio. To determine the net return of the portfolio $\mathcal{P}$, the weight of each asset in the portfolio (w_j) is based on the current monetary value (V_j) of the investment in asset j divided by the current monetary value of the total portfolio ($V_{\mathcal{P}}$). This relationship is displayed in Eq. 2.4:

$$w_j = \frac{\text{Monetary value of portfolio share of asset } j}{\text{Monetary value of the total portfolio}} = \frac{V_j}{V_{\mathcal{P}}}. \tag{2.4}$$

The annual average portfolio return ($\bar{r}_{\mathcal{P}}$) for the N-sized portfolio, where $\sum_{j=1}^{N} w_j = 1$, is calculated with Eq. 2.5:

$$\bar{r}_{\mathcal{P}} = \sum_{j=1}^{N} w_j \bar{r}_j. \tag{2.5}$$

To complete the market, it is necessary to allow for investors to choose to 'go short.' If an investor receives new positive information about a firm, the investor can buy equities in the firm (i.e., 'go long') regardless of whether she or he currently owns any equities in the firm. New information will be incorporated in the market price when the demand for the equity increases. If short-selling is not allowed, all investors cannot react efficiently when negative information about the prospects of a firm reaches the market. Only those investors who already hold an equity stake in the specific firm can decide to sell, but other investors can decide only not to buy.

The cash flow stream related to a short position is opposite to that of going long. Firms are going short, for example, when they issue a bond. In a short position, the investor will receive cash up front that will be paid back at a later date. Going short is thus equivalent to putting a negative weight on that asset.

In the Markowitz mean-variance analysis framework, investors are mean-variance optimizers. This implies that they consider not only their expected return, but also the riskiness of the investments. The investors are assumed to be risk-averse. The expected volatility of an asset's returns will therefore influence the portfolio choice of the individual investor. Based on this assumption, the essence of the MPT is to combine assets in a portfolio and thereby diversify the investment risk. The linear relationship that exists when calculating a portfolio's expected return by taking the weighted average return on the assets included in the portfolio (see Eq. 2.5) does not apply when determining the portfolio risk. To determine the riskiness of a portfolio consisting of the two assets A and B, we calculate their respective variances (σ_A^2 and σ_B^2) as well as the covariance between these two assets ($\sigma_{A,B}$). Both the risk measures are calculated from past returns and the deviation of these returns from their mean during the observed period. Equation 2.6 demonstrates how to determine the variances by showing how to calculate the variance of A measured as the expected squared deviations from the mean of its realized returns[1]:

$$\sigma_A^2 = E(\widetilde{r}_A - \overline{r}_A)^2 = \sum_{t=1}^{T} p_t \left(\widetilde{r}_{A,t} - \overline{r}_A\right)^2. \tag{2.6}$$

The most common measure of an asset's volatility is the standard deviation (here σ_A), which is the square root of the variance, i.e., $\sigma_A = \sqrt{\sigma_A^2}$. The standard deviation measure has a clear economic interpretation as it is in the same unit as the measured returns. Given normal distribution, two-thirds of the population fall within +/− the standard deviation. It is thus a measure of how dispersed the population is.

The covariance ($\sigma_{A,B}$) can be expressed as $\sigma_{A,B} = \rho_{A,B} \times \sigma_A \times \sigma_B$, where $\rho_{A,B}$ is the correlation between A and B, and σ_A and σ_B are the standard deviations for A and B, respectively. Using historical observations, Eq. 2.7 should be adopted to determine how the two assets covary:

$$\begin{aligned}\sigma_{A,B} &= E[(\tilde{r}_A - \bar{r}_A) \times (\tilde{r}_B - \bar{r}_B)] \\ &= \sum_{t=1}^{T} p_t \left(\tilde{r}_{A,t} - \bar{r}_A\right) \times \left(\tilde{r}_{B,t} - \bar{r}_B\right).\end{aligned} \tag{2.7}$$

Comparing Eqs. 2.6 and 2.7, it becomes clear that the variance is a special case of the covariance in that it measures how a single asset covaries with itself. This is a useful observation when estimating the risk of a portfolio with many equities. Let us start to describe the coupling of covariances by referring to the case of a two-asset portfolio. This gives the covariance matrix in Table 2.1.

To calculate the portfolio variance, $\sigma_{\mathcal{P}}^2$, we need to sum all covariances (including the variances) in the matrix times each weight as shown in Eq. 2.8:

$$\sigma_{\mathcal{P}}^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_{A,B}. \tag{2.8}$$

Generalizing the two-asset portfolio case by adding assets into the portfolio will make the matrix to grow exponentially. Table 2.2 displays the N-asset portfolio matrix.

The covariance matrix of N firms can be summarized as demonstrated in Eq. 2.9:

$$\sigma_{\mathcal{P}}^2 = \sum_{j=A}^{N} \sum_{k=A}^{N} w_j w_k \sigma_{j,k}. \tag{2.9}$$

Table 2.1 Covariance matrix

	Equity A	Equity B
Equity A	$\sigma_A^2 w_A^2$	$\sigma_{A,B} w_A w_B$
Equity B	$\sigma_{A,B} w_A w_B$	$\sigma_B^2 w_B^2$

Table 2.2 *N*-asset covariance matrix

	Equity A	Equity B	…	Equity N
Equity A	$\sigma_A^2 w_A^2$	$\sigma_{A,B} w_A w_B$	…	$\sigma_{A,N} w_A w_B$
Equity B	$\sigma_{B,A} w_A w_B$	$\sigma_B^2 w_B^2$	…	
…	…	…	…	
Equity N	$\sigma_{N,A} w_A w_N$	…		$\sigma_N^2 w_N^2$

To calculate the variance of a portfolio ($\sigma_{\mathcal{P}}^2$) with N assets, we may also use matrix algebra. In Eq. 2.10, the matrix $\boldsymbol{\Sigma}$ includes all the variances and covariances, and the vector, $\boldsymbol{W}$, includes all asset weights in the portfolio.

$$\sum = \left[\left[\begin{matrix} \sigma_{A,A} & \cdots & \sigma_{A,N} \\ \vdots & \ddots & \vdots \\ \sigma_{N,A} & \cdots & \sigma_{N,N} \end{matrix}\right]\right], \quad \boldsymbol{W} \ni w_j. \qquad (2.10)$$

The portfolio variance ($\sigma_{\mathcal{P}}^2$) is calculated taking the covariance matrix times the vector of portfolio weights and its transpose, i.e., $\sigma_{\mathcal{P}}^2 = \boldsymbol{W\Sigma W}'$.

The very intuition of the diversification effects (i.e., to avoid putting all one's eggs in one basket, but invest instead in a portfolio of different assets) can now be calculated. This book aims to explore in detail why investors, on average, are shown to deviate from this intuition and distort their portfolios away from an optimal portfolio by investing in the equities of a few 'favorite' firms located in their proximity. In efficient markets, the investors will not be compensated for the 'extra' risk they thereby take on. Below, we will describe and explain why adding more equities into the investor's equity portfolio will decrease his or her portfolio risk.

Using the two-asset portfolio case, Fig. 2.2 assumes that the two assets, A and B, have the following return and risk relations, respectively.

The three lines in Fig. 2.2 represent different correlations between the two assets. The dotted straight line between A and B is the feasible set of portfolios that can be reached when combining A and B using different portfolio weights and their correlation is exactly one (i.e., $\rho_{A,B} = 1$). When the two are perfectly correlated, there is no diversification effect. The other extreme case occurs when A and B are perfectly negatively

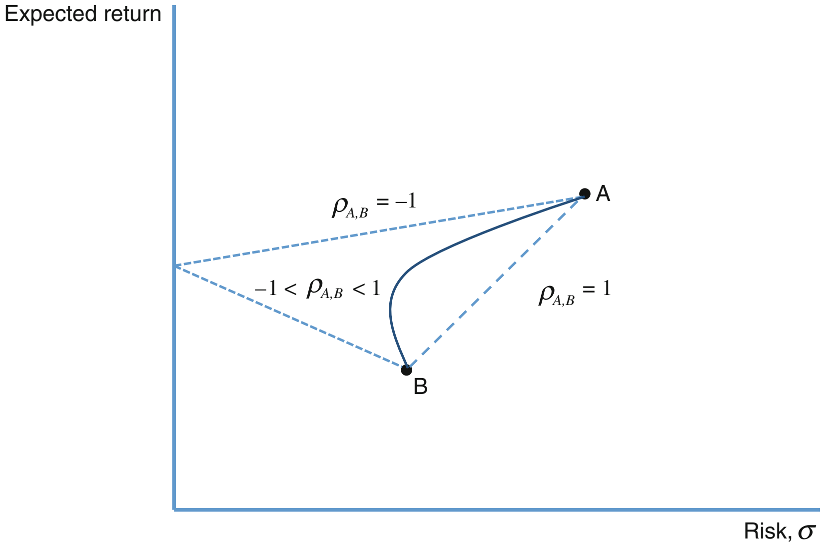

Fig. 2.2 Diversification effect between two assets

correlated, i.e., when $\rho_{A,B} = -1$. This case implies that if the price of one of the assets increases (decreases), the price of the other asset will display the exact opposite decrease (increase). This will obviously mean that the assets can be combined and weighted together to form a risk-free combination. The filled 'bended' line illustrates a possible set of feasible portfolios of the correlation between *A* and *B* when their correlations are somewhere between −1 and 1, i.e., $-1 < \rho_{A,B} < 1$. The more assets included in the portfolio, the more the curve will bend. This shows the diversification effect of the mean-variance analysis. By adding equities into the portfolio, an investor moves up and to the left in the diagram shown in Fig. 2.3. Investors will form portfolios that are as efficient as possible. The most efficient portfolios constitute the efficient frontier.

There are two basic assumptions behind the Markowitz mean-variance analysis framework: (i) Investors are risk-averse and seek to maximize their expected utility (see Chap. 3), and (ii) the capital markets are frictionless and efficient (see Chap. 4). Under these assumptions, it is most likely that non-related investors would choose different portfolios on the efficient

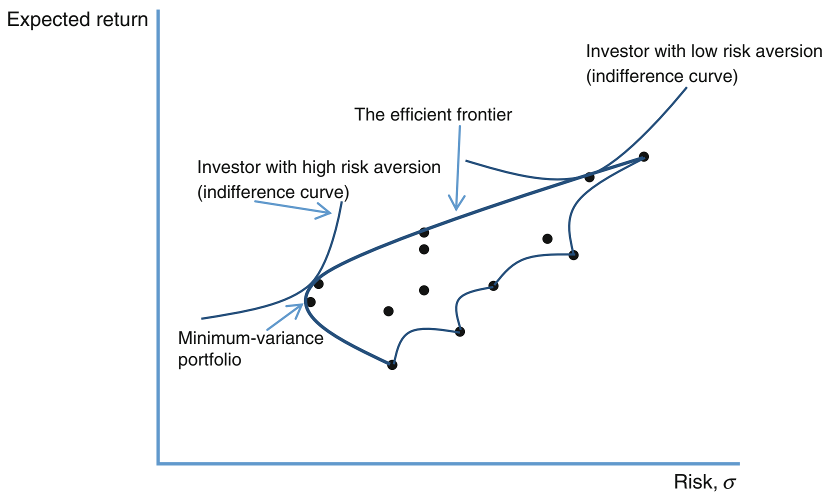

Fig. 2.3 Markowitz mean-variance portfolios and the efficient frontier

frontier depending on their level of risk aversion. A highly risk-averse investor would maximize his or her utility by investing in low-risk portfolios on the frontier. The most left portfolio on the efficient frontier is called the minimum-variance portfolio and shows the combinations of assets with the lowest variance (see Fig. 2.3).

Investors with lower risk aversion will choose optimal portfolios on the efficient set of portfolios to the right of the minimum-variance portfolio. Investors will try out different portfolios until their marginal rate of substitution, the tangent of their indifference curve, which is the investors' subjective preferences for risk and return ratio, equals the marginal rate of transformation between return and risk on the efficient set of portfolios.

Tobin (1958) expanded on Markowitz's work by adding a risk-free asset to the analysis. This led to his separation theorem contribution to the portfolio theory. Generally, whenever an asset is added to the portfolio, the efficient frontier of portfolios will expand. The same holds if we add an asset to the portfolio with a risk-free return (r_f), but the addition of such an asset will change the shape of the efficient frontier dramatically. Instead of a hyperbola, the efficient set of portfolios will be found on a straight line. This may be shown by using the covariance of the two-asset portfolio ($\sigma_{A,B}$), which is part of Eq. 2.8 ($\sigma_{\mathcal{P}}^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_B\sigma_{A,B}$). If we here assume that A is a portfolio on the efficient frontier and B is the risk-free asset, the correlation between A and B will equal zero as $\sigma_{A,B} = \rho_{A,B} \times \sigma_A \times \sigma_B = 0$, when $\rho_{A,B} = 0$. Moreover, because B is risk-free, the term $w_B^2\sigma_B^2 = 0$ suggests that the covariance for the portfolio will be $\sigma_p^2 = w_A^2\sigma_A^2$.

The straight line of efficient portfolios between the risk-free asset and the risky portfolio on the efficient frontier is called the capital allocation line (CAL). The CAL shows the average portfolio returns ($\bar{r}_{\mathcal{P}}$) that an investor can obtain from an optimal combination of the risk-free asset and the efficient portfolio set given the return on the efficient portfolio ($\bar{r}_E$), the risk-free return (r_f), and the standard deviations, σ_p and σ_E, of the combined portfolio and efficient portfolio, respectively. This gives Eq. 2.11:

$$\bar{r}_{\mathcal{P}} = r_{rf} + \sigma_{\mathcal{P}}\frac{\bar{r}_{E-}r_{rf}}{\sigma_E}. \tag{2.11}$$

Technically, there are infinitely many CALs (one for each risky asset). However, in the Markowitz mean-variance analysis framework, each investor will hold the portfolio on the efficient frontier that maximizes her or his utility. From the minimum variance portfolio (M), it is possible to construct any portfolio on the CAL_{M} by combining the M-portfolio with the risk-free asset. This is illustrated in Fig. 2.4. All portfolio combinations to the right of the M-portfolio include positive weights in both the risk-free asset and M. This implies that investors are partly lenders by investing some proportion of their capital in the risk-free asset. Investing in portfolio combinations on CAL_{M} to the right of M, the investor becomes a borrower who invests not only her or his own capital in M, but also the capital borrowed to the risk-free rate. This leveraging implies additional risk-taking.

By choosing another portfolio on the efficient set of portfolios, say $\breve{M}$, we can increase our utility. The slope $\frac{r_{E-}r_f}{\sigma_E}$ in Eq. 2.11, often referred to as the Sharpe ratio, is higher for CAL_{M} compared to CAL_{M}. The combination of the risk-free asset and the efficient portfolio that is tangent to the efficient frontier has the highest ratio. The specific CAL that

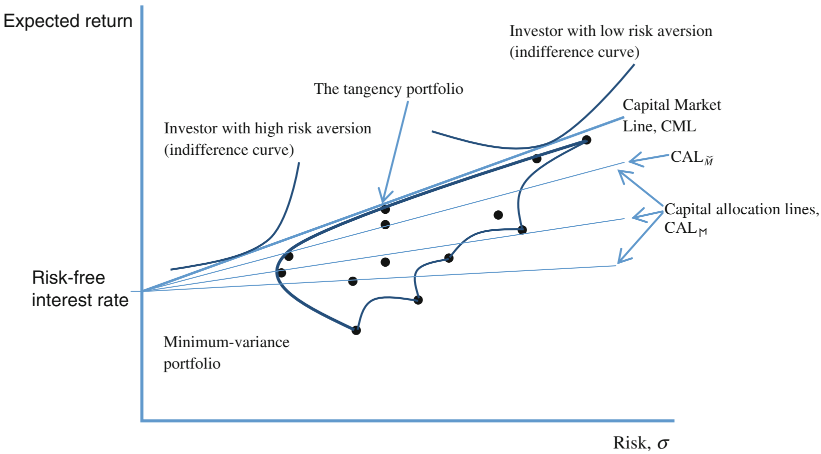

Fig. 2.4 Capital market line, tangency portfolio, and lending and borrowing to the risk-free return

consists of a combination of the tangency portfolio and the risk-free asset is called the capital market line (CML). Investors maximizing their utility will choose a combination along the CML. As shown in Fig. 2.4, a highly risk-averse individual investor, instead of choosing the portfolio on the efficient frontier, close to the minimum-variance portfolio, will now combine the tangency portfolio with the risk-free asset and thereby receive a much higher reward in terms of the trade-off between return and risk. The investor will choose the portfolio combination tangent to her or his marginal rate of substitution. This is where the tangent of the investor's utility function equals the slope of the CML, the Sharpe ratio. An individual investor with moderate levels of risk aversion may choose portfolios to the left of the tangency portfolio by borrowing capital and making use of the leverage effect.

2.3 The Capital Asset Pricing Model (CAPM)

How useful then is the mean-variance analysis framework for finding efficient portfolios or being a model for explaining individual investors' behavior? One observation we can make so far is that we should combine many assets to find the tangency portfolio. Today, there are software programs that solve the linear equation optimal solution problem. However, the number of calculations and collection of data needed seem to be impossible to perform even with modern computer power. If it were possible to compute, it still would have to be based on historical values and, thus, one candidate only among many others for being the future tangency portfolio.

By introducing one additional assumption, it is possible to calculate efficient portfolios based on historical covariances and returns deduced directly from market observations. This was one of the basic tricks made by Treynor (1962) and Sharpe (1964), which were further developed by Lintner (1965), Mossin (1966), and Black (1972). Their work extended the mean-variance analysis framework of Markowitz and Tobin by the creation of the CAPM. In addition to the aforementioned assumptions, including that investors are rational mean-variance maximizers (risk-averse) and operating on a frictionless efficient market, the CAPM assumes that all investors have homogenous expectations. This means that all investors reach the same conclusion about possible future states of means and variances.

If markets are frictionless, the investor can borrow and lend at the same rate. This assumption is necessary for the CML to be a straight line. If all investors, besides being risk-averse, have homogenous expectations, no individual can fool another when having the same information. Hence, the model is free from any arbitrage possibilities. Every investor is assumed to be rational and, therefore, seeking to maximize his or her return by searching for the tangency portfolio. If markets are frictionless, all information is available to every investor at no cost; in turn, if the investors have homogenous expectations, they will all find and hold the same tangency portfolio. Under the CAPM assumptions, this tangency portfolio is called the market portfolio; in theory, it contains market-value-weighted proportions of all assets in the market.

The CAPM is an equilibrium model that can be derived from the mean-variance analysis. It can be used to determine the expected return ($\overline{r}_j$) for any risky asset j given the risk-free rate (r_f), the expected return on the market portfolio ($\bar{r}_M$), and the assets' relative risk to the market portfolio (β_j). The basic CAPM formula is expressed in Eq. 2.12:

$$\bar{r}_j = r_f + \beta_j(\bar{r}_M - r_f). \tag{2.12}$$

The CAPM formula shows the relation between the risk, measured as beta (β_j), and the single asset's expected return ($\bar{r}_j$). The difference between the market return ($\bar{r}_M$) and the return on the risk-free asset (r_f) is the risk premium investors on the market expect for entering the specific risky asset market (most often the equity market).

In the Markowitz mean-variance analysis, there is no direct relationship between an asset's risk measured as the standard deviation (or the variance) and the pricing of the asset. Each asset is priced in accordance with all assets' contributions to the risk in the market portfolio. To maximize their utility, the rational investors invest in the market portfolio, implying that they thereby hold a fraction of all assets available on the market. Each individual investor can choose his or her preferred risk level by combining the market portfolio with the risk-free asset accordingly. The beta (β_j) of the risky asset j discloses how much risk this asset contributes to the market portfolio. Hence, the CAPM formula can be used to price a single asset when it is included in a diversified market portfolio.

Formally, the β_j equals the standard deviation (σ_j) of the risky asset j times the correlation between the market portfolio and the asset ($\rho_{j,M}$) divided by the standard deviation of the market portfolio (σ_M). It can also be expressed as the covariance between the single asset return and the market return ($\sigma_{j,M}$) divided by the variance of the market portfolio (σ_M^2). This gives Eq. 2.13, which demonstrates how to determine the beta of a risky asset j:

$$\beta_j = \rho_{j,M}\frac{\sigma_j}{\sigma_M} = \frac{\sigma_{j,M}}{\sigma_M^2}, \tag{2.13}$$

where the covariance between the single asset return and the market return is normalized with the variance of the market. If asset j follows the market exactly, then the equation becomes as if dividing the variance of the market (j has then the market risk) with the variance of the market itself, the resulting market beta (β_M) obviously equals one. Normalizing any risky asset's covariance with the market ($\sigma_{j,M}$) with the variance of the market (σ_M^2), the beta describes how much of the volatility can be explained by market volatility. We call this the risky asset's market risk. This risk cannot be diversified away by combining the asset with other risky assets. Therefore, this is the risk the single asset contributes to the market portfolio. Any specific portfolio beta can be calculated by summarizing the individual beta asset. This is shown in Eq. 2.14:

$$\beta_{\mathcal{P}} = \sum_{j=1}^{N} w_j \times \beta_j. \tag{2.14}$$

There is obviously a difference in the price of a single equity when it is included in a well-diversified equity portfolio and when it is priced by an un-diversified investor. When the un-diversified investor compares the risk–reward return for a single equity, she or he will pay the price that corresponds to the total risk of that equity determined by its standard deviation. The diversified owner will be willing to pay a higher price as the single equity does not contribute with the same amount of risk to the investor's portfolio because the risk–reward return will be determined by the beta of the equity. A well-diversified investor will therefore always be able to outbid an un-diversified investor on a single equity.

The major advantage of the CAPM, compared to the mean-variance analysis framework, is that it is a tool for pricing single equities. In Fig. 2.5, we have drawn the security market line (SML) with beta (β_j) and the expected return ($\bar{r}_j$) of equity j on each axis, respectively.

Using the CAPM formula, the market portfolio beta (β_M) corresponds to an expected return on the market (r_M) of 14% in the example displayed in Fig. 2.5. The risk-free rate (r_f) is here set to 6%, which implies a market premium ($\bar{r}_M - r_f$) of 8%. Hence, if an equity j has a beta (β_j) of 1.2, its expected return ($\bar{r}_j$) would equal 15.6%.

A single equity may deviate from the SML for only a short period, and the mispricing will disappear with changes in the demand and supply of that equity. The equity q in Fig. 2.5 is placed below the SML. An equity

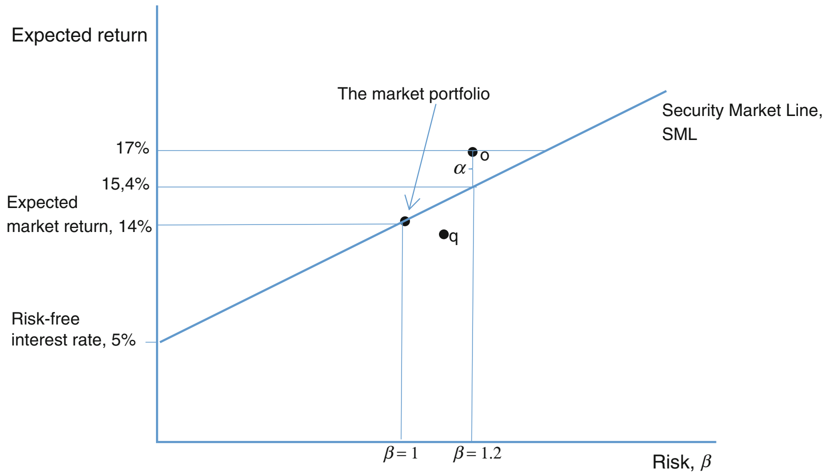

Fig. 2.5 Capital asset pricing model, security market line, and beta

with an expected return below the SML is overpriced. This is a signal to rational investors to sell and/or, if possible, go short in equity *q*. Then, the price of *q* will decrease; accordingly, its expected return will increase. For an equity displaying an expected return above the SML, like equity *o*, rational investors will expect the opposite, causing the demand for *o* to increase. If we can buy an equity that is expected to generate 17% in return with a beta of 1.2, this equity is underpriced. The extra return of 1.4% units is the excess return over the expected risk-adjusted return from the CAPM computation.

2.4 Using Regression to Determine the Alpha and Beta Coefficients

To estimate the covariance and correlation between an equity and the market portfolio, the historical observed returns over a specific period are often used. The single equity's observed returns are regressed against the observed returns for the market portfolio in accordance with the formula shown in Eq. 2.15, where the intercept alpha (α_j) indirectly reveals if the equity is over- or underpriced and the last term ($\varepsilon_{j,t}$) is the error term:

$$r_{j,t} - r_{f,t} = \alpha_j + \beta_j \left(r_{M,t} - r_{f,t} \right) + \varepsilon_{j,t}. \tag{2.15}$$

The intercept alpha (α_j) and beta coefficients (β_j) are estimated using the ordinary least squares (OLS) estimation technique. The β_j is the slope of the best fitted line and indicates the extent to which movements in the single equity j are associated with movements in the overall market. This is displayed in Fig. 2.6.

Once again, a firm with a high equity beta is more sensitive to market movements than a firm with a low beta. The intercept alpha measures the risk-adjusted return that the investors holding equity j receive over the observed period. The alpha equals zero if there is no excess return and becomes negative if the investor takes a risk-adjusted loss.

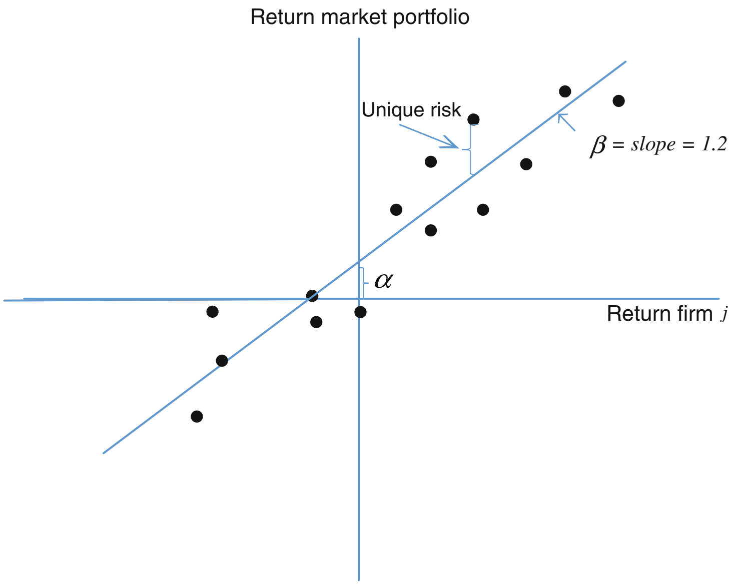

Fig. 2.6 Estimating beta using regression model

The volatility of an equity contains two components: the systematic and the unsystematic risk. Unsystematic risk describes the fluctuations in the equity's returns that are not related to market fluctuations. The error term ($\varepsilon_{j,t}$) is expected, on average, to be zero when the underlying distribution of the data is normally distributed. Each single observed error term, however, corresponds to the unique, unsystematic, diversifiable risk. Depending on the goodness of fit for the regression (measured by the R-square), the market risk (β_j) of j explains the total volatility to a higher or lower degree. A high goodness of fit suggests that an equity's volatility is highly explained by the systematic market risk.

Equation 2.15 is widely used in practice to estimate CAPM parameters. The historical return values are used to predict (future) expected returns. This approach may be appropriate if the intercept alpha (α_j) and beta coefficients (β_j) are stable over time. A significant number of studies assume that the 'true' betas are constant over time; for some economic or behavioral reasons, the underlying betas may tend to regress toward the overall mean over time. However, it has been discussed that betas estimated in a subgroup of periods may be more extreme than those estimated in other periods and hence may vary over time, a bias known as 'order' or 'selection' bias (see Blume 1975).

One approach to address these concerns is to estimate time-varying CAPM parameters using, for instance, some overlapping periods. In Table 2.3, we show an output from a regression model which estimates 36-month rolling alpha (α_j) and beta coefficients (β_j) for each fund using US mutual funds data between 1963 and 2014 from Center for Research in Security Prices (CRSP). We use the return on S&P500 as a proxy for the return on the market portfolio and the return on the three months US treasury bills as a proxy for the risk-free rate to compute the excess returns. In Appendix, we provide SAS codes that produce these rolling regression outputs so that the reader may simply replicate these regressions.

In Table 2.3, we observe that the average alpha is about 0.2% and the beta is about 0.83 for US mutual funds during 1963 and 2014. The low average beta is due to the fact that the US mutual funds also invest in other markets than the USA and thus are exposed to other risks than the US market. This raises other issues whether return on S&P500 is a good

Table 2.3 Results from the rolling (36 months) CAPM regressions on US mutual funds between 1963 and 2014

Variable	*N*	*N* Miss	Minimum	Mean	Median	Maximum	Std Dev
MRET	2,265,723	22,286	−0.252	0.006	0.007	0.216	0.041
RF	2,288,009	0	0.000	0.001	0.000	0.014	0.002
SPRTRN	2,288,009	0	−0.218	0.007	0.012	0.163	0.042
Ri[a]	2,265,723	22,286	−0.258	0.005	0.005	0.213	0.041
Rm[b]	2,288,009	0	−0.224	0.005	0.009	0.158	0.043
Alpha	13,834	0	−0.016	0.002	0.002	0.021	0.004
Beta	13,834	0	−0.357	0.825	0.925	1.736	0.357
RMSE	13,834	0	0.001	0.018	0.014	0.069	0.011

[a] $Ri = \text{MRET} - \text{RF}$
[b] $Ri = \text{SPRTRN} - \text{RF}$

proxy for the return on the market in using CAPM regressions. We leave this question to studies focusing on mutual fund performance. In Fig. 2.7, we show an example of the regression line for a particular fund in a particular period.

Figure 2.7 shows that this particular fund has a negative alpha of 0.17% and a beta of one in this particular month (the 93rd period after its inception date). Since some of the return observations lie on or outside the 95% confidence interval, the estimated alpha is not

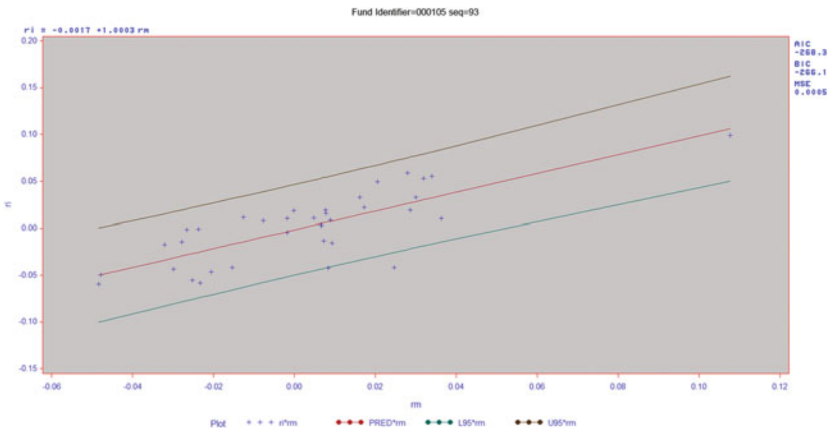

Fig. 2.7 Rolling CAPM regression line for a particular fund at a particular time, an example

significantly different from zero at the 5% significance level. Thus, this fund in this particular period does not outperform the market, which is in line with what the CAPM would suggest.

2.5 Concluding Remarks

The MPT gives a theoretical foundation for why deviations from an optimal investment strategy should lead to underperforming portfolios. The theory, including the mean-variance analysis framework and the CAPM, is based on some crucial assumptions, namely investors' homogenous expectations, rational behavior, and utility maximization as well as efficient markets. We have assumed throughout this chapter that markets are efficient. This does not mean that the expected returns are the actual returns *ex post*. The expected returns are one of many possible candidates. As mentioned at the beginning of this chapter, the CAPM has been the subject of numerous empirical tests. In Chap. 4, we present a summary of the most important findings and discuss how the theory has evolved into different directions. Before that, we focus on theories of rational decision-making in Chap. 3.

Notes

1. Where p_t = the probability distribution. When using realized returns to estimate the variance and covariance of our observations, we use the following two formulas instead, as they take into consideration the loss of degrees of freedom when using $\sigma_A^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(\tilde{r}_{A,t} - \bar{r}_A\right)^2$ for the variance and $\sigma_{A,B} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\tilde{r}_{A,t} - \bar{r}_A\right)\left(\tilde{r}_{B,t} - \bar{r}_B\right)$ for the covariance, respectively.

Appendix

SAS codes for producing results from 36-month rolling CAPM regressions in Table 2.3.

```
libname dat 'D:\Mutual Funds\CRSP'; /*Assign the lib reference where the data are
located*/
data allt; set dat.crsp;
year=year(CALDT);
 month=month(CALDT);
proc means data=allt n nmiss min mean median max std nolabels; var mret rf SPRTRN;
run;
/*mret: total month-end return on the mutual fund from CRSP, rf: risk-free rate
from CRSP, SPRTRN: return on S&P 500 obtained from CRSP*/
proc sort data=allt; by CRSP_FUNDNO CALDT; /*Assigning sequence for each fund*/
data allt; set allt; by CRSP_FUNDNO CALDT;
retain seq;
if first.CRSP_FUNDNO then seq=1; else
seq+1;
proc freq; tables seq; run;
data allt; set allt; /*Excess returns*/
ri=MRET-RF;
rm=SPRTRN-RF;
proc means data=allt n nmiss min mean median max std nolabels; var ri rm;
run;
/*36 months rolling CAPM regressions for each fund month. You may change the time
period to your own preference*/
proc sort data=allt; by CRSP_FUNDNO seq;
DATA rwin / view=rwin;
array _Y {36} _temporary_ ;
array _X1 {36} _temporary_ ;
array _X2 {36} _temporary_ ;
array _X3 {36} _temporary_ ;
array _X4 {36} _temporary_ ;
set allt;
by CRSP_FUNDNO; retain N 0;
N = ifn(first.CRSP_FUNDNO,1,N+1); I=mod(N-1,36)+1;
_Y{I}=ri;
_X1{I}=rm;
_X2{I}=SMB; /*Fama-Frenhch, and Carhart Factors which may be included in the
regression. We exclude them in these analyses.*/
_X3{I}=HML;
_X4{I}=UMD;
if N>=36 then do I= 1 to 36;
ri=_Y{I};
rm=_X1{I};
SMB=_X2{I};
HML=_X3{I};
UMD=_X4{I};
output;
end;
run; proc reg data=rwin noprint outest=myests;
by CRSP_FUNDNO seq; /*For each fund month we run a 36 months rolling regression*/
model ri=rm; /*CAPM regressions: Fama-French, and Carhart Factors are excluded*/
plot ri*rm / pred nostat mse aic bic /*Plots CAPM regression line for each fund
month (in case you have a small number of fund months)*/
        caxis=red ctext=blue cframe=ligr
        legend=legend1 modellab=' ';
run; quit;
proc means data=myests n nmiss min mean median max std nolabels; var Intercept rm
_RMSE_; run; /*Intercept=Alpha, rm=Beta*/
```

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