

# Preface

Nonlinear dynamical systems in nature, such as atmospheric flows, exhibit complex spatial patterns, e.g. cloud geometry, that lack a characteristic (single) length scale concomitant with temporal fluctuations that lack a single timescale. The mathematical concept of ‘fractals’ introduced by Mandelbrot (1977) provides powerful tools for describing and quantifying the universal symmetry of self-similarity (Schroeder 1991) underlying the seemingly irregular complex geometrical shapes and temporal fluctuations.

Spatially extended fractal objects in nature support fluctuations of dynamical processes on all timescales. The power spectra of such broadband fluctuations exhibit an inverse power law of form  $1/f^\alpha$ , where  $f$  is the frequency and  $\alpha$  the exponent. In general,  $\alpha$  decreases with  $f$  and approaches 1 for low frequencies. Such spectra, described as  $1/f$  or  $1/f$ -like spectra of temporal fluctuations, are ubiquitous to dynamical systems in nature. The frequency range over which  $\alpha$  is constant therefore exhibits self-similarity or scale invariance in temporal fluctuations, i.e. the fluctuations are fractals in time. The intensity or variance of longer and shorter period fluctuations is mutually related by a scale factor alone, independent of the nature of dynamical processes. The fluctuations exhibit long-range temporal correlations. Also, temporal fluctuations exhibit multifractal structure because  $\alpha$  varies for different ranges of frequency  $f$ .  $1/f$  power law would seem natural, and white noise (flat distribution) would be the subject of involved investigation.

Until very recently (1988), fractal geometry to the spatial pattern and fractal fluctuations in time of dynamical processes of the same extended dynamical system were treated as two disparate multidisciplinary fields of research. The long-range spatiotemporal correlations underlying spatial and temporal power law behaviour of dynamical systems were identified as a unified manifestation of self-organized criticality in 1988 (Bak et al. 1987, 1988).

The unifying concept of self-organized criticality underlying fractals, self-similar scaling, broadband frequency spectra, and inverse power law distributions offers a new and powerful means of describing certain basic aspects of spatial form and dynamical (temporal) processes of a dynamical system. The systems in which self-organized criticality is observed range from the physical to the biological to the

social. Rapid advances in applications of these new concepts have been made, particularly in the field of physiology and medicine. It is now recognized that fractal architecture to the spatial pattern serves as robust, stable structures for the regulation and maintenance of vital functions of lungs, heart, liver, kidneys, brain, etc. Self-similar fractal growth pattern in the plant kingdom gives rise to the observed beautiful phyllotactic patterns, i.e. the elaborate patterns of fruits in the sunflower capitulum, florets in the capitulum of daisy, and scales on a pineapple and on a pinecone. Phyllotactic patterns incorporate with mathematical precision the Fibonacci mathematical series, where each term is the sum of two previous terms and the ratio of each term to the previous term approaches the golden ratio  $\tau = (1 + \sqrt{5})/2 \approx 1.618$ . Such patterns, while pleasing to the eye, combine maximum packing efficiency while preserving the shape for different sizes of a particular species, such as daisy flowers of different sizes.

Atmospheric flows exhibit self-organized criticality, i.e. long-range correlations in space and time manifested as fractal geometry to the spatial pattern concomitant with inverse power law form for fluctuations of meteorological parameters such as temperature and pressure. Traditional meteorological theory cannot explain satisfactorily the observed self-similar space-time structure of atmospheric flows. A recently developed general systems theory for fractal space-time fluctuations shows that the larger-scale fluctuation can be visualized to emerge from the space-time averaging of enclosed small-scale fluctuations, thereby generating a hierarchy of self-similar fluctuations manifested as the observed eddy continuum in power spectral analyses of fractal fluctuations. The interconnected network of eddy circulations responds as a unified whole to local perturbation such as global-scale response to El Nino events. The general systems theory model predicts inverse power law form incorporating the golden ratio  $\tau$  for the distribution of space-time fluctuation pattern and also for the power (variance) spectra of the fluctuations. Since the probability distributions of amplitude and variance are the same, atmospheric flows exhibit quantum-like chaos. Long-range correlations inherent in power law distributions of fluctuations are identified as non-local connection or entanglement exhibited by quantum systems such as electron or photon. The predicted distribution is close to the Gaussian distribution for small-scale fluctuations, but exhibits *fat long tail* for large-scale fluctuations. Universal inverse power law for fractal fluctuations rules out unambiguously linear secular trends in climate parameters. Energy input into the atmospheric eddy continuum, either natural or man-made, will result in the enhancement of fluctuations of all scales manifested immediately in intensification of high-frequency fluctuations such as the Quasi-Biennial Oscillation (QBO) and El Nino-Southern Oscillation (ENSO) cycles.

Chapter 1 gives a brief review of the science of nonlinear dynamics and chaos with applications in meteorology and atmospheric physics. Atmospheric flows, an example of turbulent fluid flows, exhibit fractal fluctuations of all space-time scales ranging from turbulence scale of mm-sec to climate scales of thousands of kilometres-years and may be visualized as a nested continuum of weather cycles or periodicities, the smaller cycles existing as intrinsic fine structure of the larger

cycles. The power spectra of fractal fluctuations exhibit inverse power law form signifying long-range correlations identified as self-organized criticality and are ubiquitous to dynamical systems in nature and are manifested as sensitive dependence on initial condition or ‘deterministic chaos’ in finite precision computer realizations of nonlinear mathematical models of real-world dynamical systems such as atmospheric flows. Though the self-similar nature of atmospheric flows has been widely documented and discussed during the last three to four decades, the exact physical mechanism is not yet identified. There now exists an urgent need to develop and incorporate basic physical concepts of nonlinear dynamics and chaos into classical meteorological theory for more realistic simulation and prediction of weather and climate. A historical review of nonlinear dynamics and chaos in meteorology and atmospheric physics is summarized in this chapter.

Chapter 2 gives a review of noise or random fluctuations in physical systems. ‘Noise’ or random fluctuations characterize all physical systems in nature ranging from biology, botany, physiology, meteorology, astronomy, etc. The apparently irregular or chaotic fluctuations were considered as ‘noise’ in all fields except in astronomy where the fluctuations from astronomical sources were referred to as signal. Noise and fluctuation have been a field of study since 1826 with the study of Brownian motion which indirectly confirmed the existence of atoms and molecules. The measured characteristics of noise contain recognizable patterns or signal and convey useful information about the system. Statistical data analysis techniques are used to extract the signal, i.e. recognizable patterns in the apparently random fluctuations of physical systems. The analysis of data sets and broad quantification in terms of probabilities belongs to the field of statistics. Early attempts resulted in the identification of the following two quantitative (mathematical) distributions which approximately fit data sets from a wide range of scientific and other disciplines of study. The first is the well-known statistical normal distribution, and the second is the power law distribution associated with the recently identified ‘fractals’ or self-similar characteristic of data sets in general. Abraham de Moivre, an eighteenth century statistician and consultant to gamblers, made the first-recorded discovery of the normal curve of error (or the bell curve because of its shape) in 1733. The importance of the normal curve stems primarily from the fact that the distributions of many natural phenomena are at least approximately normally distributed. This normal distribution concept underlies how we analyse experimental data over the last two hundred years. Most quantitative research involves the use of statistical methods presuming *independence* among data points and Gaussian ‘normal’ distributions. The Gaussian distribution is reliably characterized by its stable mean and finite variance. Normal distributions place a trivial amount of probability far from the mean, and hence, the mean is representative of most observations. Even the largest deviations, which are exceptionally rare, are still only about a factor of two from the mean in either direction and are well characterized by quoting a simple standard deviation. However, apparently rare real-life catastrophic events such as major earth quakes, stock market crashes, and heavy rainfall events, occur more frequently than indicated by the normal curve; i.e., they exhibit a probability distribution with a *fat tail*. Fat tails indicate a power law pattern and interdependence.

The ‘tails’ of a power law curve—the regions to either side that correspond to large fluctuations—fall off very slowly in comparison with those of the bell curve. The normal distribution is therefore an inadequate model for extreme departures from the mean. For well over a century, evidence had been mounting that real-world behaviour, in particular, behaviour of *systems*, whether natural, social, economic, or financial does not follow normal distribution characteristics. There is increased evidence for non-normality in real-world settings and in its place an alternative distribution, namely the power law distribution, is shown to be exhibited by real-world systems in all fields of science and other areas of human interest. In this chapter, the following are discussed: (i) a brief history of the two chief quantitative methods of statistical data analysis, namely the statistical normal distribution and the power law distribution; (ii) the association of power law distributions with complex systems, scale invariance, self-similarity, fractals,  $1/f$  noise, long-term memory, phase transitions, critical phenomena, and self-organized criticality; (iii) current status of power law distributions; (iv) power law relations (bivariate) and power law (probability) distributions; (v) allometric scaling and fractals; (vi) fractals and the golden section in plant growth; (vii) turbulent fluid flow structure, fractals, and the golden ratio ( $\approx 1.618$ ); (viii) fractal space-time and the golden ratio; (ix) power law (probability) distributions in the meteorological parameter precipitation, temperature, quaternary ice volume fluctuations, and atmospheric pollution; and (x) general systems theory model for self-organized criticality in atmospheric flows with universal quantification for power law distribution in terms of the golden ratio.

In Chap. 3, a general systems theory model for atmospheric flows is presented and is shown that self-organized criticality (SOC) is a signature of quantum-like chaos.

Atmospheric flows exhibit long-range spatiotemporal correlations manifested as the fractal geometry to the global cloud cover pattern concomitant with inverse power law form for power spectra of temporal fluctuations on all space-time scales ranging from turbulence (centimetres-seconds) to climate (kilometres-years). Long-range spatiotemporal correlations are ubiquitous to dynamical systems in nature and are identified as signatures of *self-organized criticality*. Standard models in meteorological theory cannot explain satisfactorily the observed self-organized criticality in atmospheric flows. Mathematical models for simulation and prediction of atmospheric flows are nonlinear and do not possess analytical solutions. Finite precision computer realizations of nonlinear models give unrealistic solutions because of *deterministic chaos*, a direct consequence of round-off error growth in iterative numerical computations. Recent studies show that round-off error doubles on an average for each iteration of iterative computations. Round-off error propagates to the mainstream computation and gives unrealistic solutions in numerical weather prediction (NWP) and climate models, which incorporate thousands of iterative computations in long-term numerical integration schemes. A general systems theory model for atmospheric flows developed by the author predicts the observed *self-organized criticality* as intrinsic to quantum-like chaos in flow dynamics. The model provides universal quantification for *self-organized criticality*

in terms of the golden ratio  $\tau$  ( $\approx 1.618$ ). Model predictions are in agreement with a majority of observed spectra of time series of several standard meteorological and climatological data sets representative of disparate climatic regimes. Universal spectrum for natural climate variability rules out linear trends. Man-made greenhouse gas-related atmospheric warming would result in intensification of natural climate variability, seen immediately in high-frequency fluctuations such as QBO and ENSO and even shorter timescales. Model concepts and results of analyses are discussed with reference to possible prediction of climate change. Model concepts, if correct, rule out unambiguously linear trends in climate. Climate change will only be manifested as increase or decrease in the natural variability. However, more stringent tests of model concepts and predictions are required before applications to such an important issue as *climate change*. The cell dynamical system model for atmospheric flows is a general systems theory applicable, in general, to all dynamical systems in other fields of science, such as *number theory*, *biology*, *physics*, and *botany*.

In Chap. 4, it is shown that the distribution for month-wise temperature and rainfall in the UK region follows the universal inverse power law distribution predicted by the general systems theory model discussed in Chap. 2. S. Lovejoy and his group at the McGill University, Canada, have done pioneering work during the last three decades to show conclusively that meteorological parameters, such as temperature, rainfall, and pressure, exhibit self-similar space-time fractal fluctuations generic to dynamical systems in nature such as fluid flows, spread of forest fires, and earthquakes. The power spectra of fractal fluctuations display inverse power law form signifying long-range correlations. A general systems theory model (Chap. 2) predicts universal inverse power law form incorporating the golden ratio for the fractal fluctuations. The model-predicted distribution was compared with observed distribution of fractal fluctuations of all size scales (small, large, and extreme values) in the historic month-wise temperature (maximum and minimum) and total rainfall for the four stations Oxford, Armagh, Durham, and Stornoway in the UK region, for data periods ranging from 92 years to 160 years. For each parameter, the two cumulative probability distributions, namely *cmax* and *cmin*, starting from, respectively, maximum and minimum data value were used. The results of the study show that (i) temperature distributions (maximum and minimum) follow model-predicted distribution except for Stornoway, minimum temperature *cmin*; (ii) rainfall distribution for *cmin* follows model-predicted distribution for all the four stations; and (iii) rainfall distribution for *cmax* follows model-predicted distribution for the two stations Armagh and Stornoway. The present study suggests that fractal fluctuations result from the superimposition of eddy continuum fluctuations.

In Chap. 5, it is shown that signatures of universal characteristics of fractal fluctuations are seen in global mean monthly temperature anomalies. Self-similar space-time fractal fluctuations are generic to dynamical systems in nature such as atmospheric flows, heartbeat patterns, and population dynamics. The physics of the long-range correlations intrinsic to fractal fluctuations is not completely understood. It is important to quantify the physics underlying the irregular fractal fluctuations

for prediction of space-time evolution of dynamical systems. A general systems theory model (Chap. 2) for fractals visualizes the emergence of successively larger-scale fluctuations resulting from the space-time integration of enclosed small-scale fluctuations. The theoretical model predictions are as follows: (i) the probability distribution and the power spectrum for fractal fluctuations are the same inverse power law function incorporating the *golden ratio*. (ii) The predicted distribution is close to the Gaussian distribution for small-scale fluctuations but exhibits *fat long tail* for large-scale fluctuations with higher probability of occurrence than predicted by Gaussian distribution. (iii) Since the power spectrum (variance, i.e., square of eddy amplitude) also represents the probability densities as in the case of quantum systems such as the electron or photon, fractal fluctuations exhibit quantum-like chaos. (iv) The *fine structure constant* for spectrum of fractal fluctuations is a function of the golden ratio and is analogous to atomic spectra equal to about  $1/137$ . Global gridded time series data sets of monthly mean temperatures for the period 1880–2007/2008 were analysed. The data sets and the corresponding power spectra exhibit distributions close to the model-predicted inverse power law distribution. The model-predicted and observed universal spectrum for interannual variability rules out linear secular trends in global monthly mean temperatures. Global warming either man-made or natural results in intensification of fluctuations of all scales and manifests immediately in high-frequency fluctuations.

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