Chapter 2
Noise or Random Fluctuations in Physical Systems: A Review

Abstract 'Noise' or random fluctuations characterize all physical systems in nature ranging from biology, botany, physiology, meteorology, astronomy, etc. The apparently irregular or chaotic fluctuations were considered as 'noise' in all fields except in astronomy, where the fluctuations from astronomical sources were referred to as signal. Noise and fluctuation has been a field of study since 1826 with the study of Brownian motion which indirectly confirmed the existence of atoms and molecules. The measured characteristics of noise contain recognizable patterns or signal and convey useful information about the system. Statistical data analysis techniques are used to extract the signal, i.e. recognizable patterns in the apparently random fluctuations of physical systems. The analysis of data sets and broad quantification in terms of probabilities belongs to the field of statistics. Early attempts resulted in identification of the following two quantitative (mathematical) distributions which approximately fit data sets from a wide range of scientific and other disciplines of study. The first is the well-known statistical normal distribution and the second is the power-law distribution associated with the recently identified 'fractals' or self-similar characteristic of data sets in general. Abraham de Moivre, an eighteenth-century statistician and consultant to gamblers made the first recorded discovery of the normal curve of error (or the bell curve because of its shape) in 1733. The importance of the normal curve stems primarily from the fact that the distributions of many natural phenomena are at least approximately normally distributed. This normal distribution concept underlies how we analyse experimental data over the last 200 years. Most quantitative research involves the use of statistical methods presuming *independence* among data points and Gaussian 'normal' distributions. The Gaussian distribution is reliably characterized by its stable mean and finite variance. Normal distributions place a trivial amount of probability far from the mean and hence the mean is representative of most observations. Even the largest deviations, which are exceptionally rare, are still only about a factor of two from the mean in either direction and are well characterized by quoting a simple standard deviation. However, apparently rare real-life catastrophic events such as major earth quakes, stock market crashes, heavy rainfall events, etc., occur more frequently than indicated by the normal curve, i.e. they exhibit a probability distribution with a *fat tail*. Fat tails indicate a power-law pattern and interdependence.

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The 'tails' of a power-law curve—the regions to either side that correspond to large fluctuations—fall off very slowly in comparison with those of the bell curve. The normal distribution is therefore an inadequate model for extreme departures from the mean. For well over a century evidence had been mounting that real-world behaviour in particular, behaviour of *systems*, whether natural, social, economic, or financial does not follow normal distribution characteristics. There is increased evidence for non-normality in real-world settings and in its place an alternative distribution, namely the power-law distribution is shown to be exhibited by real-world systems in all fields of science and other areas of human interest. In this chapter, the following are discussed. (i) A brief history of the two chief quantitative methods of statistical data analysis, namely the statistical normal distribution and the power-law distribution. (ii) The association of power-law distributions with complex systems, scale invariance, self-similarity, fractals, 1/*f* noise, long-term memory, phase transitions, critical phenomena, and self-organized criticality. (iii) Current status of power-law distributions. (iv) Power-law relations (bivariate) and power-law (probability) distributions. (v) Allometric scaling and fractals. (vi) Fractals and the golden section in plant growth. (vii) Turbulent fluid flow structure, fractals, and the golden ratio (≈1.618). (viii) Fractal space-time and the golden ratio. (ix) Power-law (probability) distributions in the meteorological parameters precipitation, temperature, quaternary ice volume fluctuations and atmospheric pollution. (x) General systems theory model for self-organized criticality (SOC) in atmospheric flows with universal quantification for power-law distribution in terms of the golden ratio.

Keywords Noise and fluctuations · Power law · 1/*f* noise · Self-organized criticality (SOC) · General systems theory

2.1 Introduction

The random fluctuations in space and time are ubiquitous to physical systems in nature and are commonly known as 'noise'. Such noise or fluctuations from astronomical sources however have always been referred to as signal. The apparent 'noise' exhibits some form of regularity seen in (i) size (or duration) distribution in space (or time) and (ii) power (variance) spectrum which exhibits inverse power-law form namely 'flicker noise or 1/*f* noise' (Press 1978).

The roots of noise research trace back to the Scottish botanist Robert Brown who carried out his famous experiments in 1827, observing fluctuating pollen on the surface of a film of water. The first unsolved noise problem was to find out the origin of Brownian motion. It took over 80 years to fully solve it. Around 1905, through the work of both Smoluchowski (1906) and Einstein (1905), the problem was finally settled and presented it as a way to indirectly confirm the existence of atoms and molecules. The random fluctuations of Brownian motion are a visible manifestation of the ceaseless molecular bombardment of suspended particulates in a liquid or gas (Abbott 2001).

Noise and fluctuation has been a field of study since 1826 and now covers all fields of science and other areas of human interest. Noise is generally associated with degradation in performance, particularly in linear systems. However, it is now recognized that noise can play a constructive role in nonlinear systems, whose performance can be optimized at nonzero noise levels. Also, the measured characteristics of noise contain recognizable patterns or signal and convey useful information about the system. In recent years, noise is an intensive field of study in physical, biological, and other systems since it is recognized that depending on circumstances, noise plays a constructive or destructive role and can be utilized for enhancing required system performance (Abbott 2001).

2.2 Statistical Methods for Data Analysis

Statistical data analysis techniques are used to extract the signal, i.e. recognizable patterns in the apparently random fluctuations of physical systems.

Selvam (2009) summarized the current status of statistical methods for data analysis as follows. Dynamical systems such as atmospheric flows, stock markets, heartbeat patterns, population growth, traffic flows, etc., exhibit irregular space-time fluctuation patterns. Quantification of the space-time fluctuation pattern will help predictability studies, in particular, for events which affect day-to-day human life such as extreme weather events, stock market crashes, traffic jams, etc. The analysis of data sets and broad quantification in terms of probabilities belongs to the field of statistics. Early attempts resulted in identification of the following two quantitative (mathematical) distributions which approximately fit data sets from a wide range of scientific and other disciplines of study. The first is the well-known statistical normal distribution and the second is the power-law distribution associated with the recently identified ‘fractals’ or self-similar characteristic of data sets in general. In the following, a summary is given of the history and merits of the two distributions.

2.3 Statistical Normal Distribution

Historically, our present-day methods of handling experimental data have their roots about four hundred years ago. At that time scientists began to calculate the odds in gambling games. From those studies emerged the theory of probability and subsequently the theory of statistics. These new statistical ideas suggested a different and more powerful experimental approach. The basic idea was that in some experiments, random errors would make the value measured a bit higher and in other experiments random errors would make the value measured a bit lower. Combining these values by computing the average of the different experimental results would make the errors cancel and the average would be closer to the 'right' value than the result of any one experiment (Liebovitch and Scheurle 2000).

Abraham de Moivre, an eighteenth-century statistician and consultant to gamblers made the first recorded discovery of the normal curve of error (or the bell curve because of its shape) in 1733. The normal distribution is the limiting case of the binomial distribution resulting from random operations such as flipping coins or rolling dice. Serious interest in the distribution of errors on the part of mathematicians such as Laplace and Gauss awaited the early nineteenth century when astronomers found the bell curve to be a useful tool to take into consideration the errors they made in their observations of the orbits of the planets (Goertzel and Fashing 1981).

The bell curve was noticed by Gauss in the distribution of estimates of geographical measurements in the Bavarian hills. He used the curvature of Earth to improve accuracy of measurement. The distribution of estimates was found to cluster around the mean with symmetry on either side. Gauss is credited with developing the 'least squares' method for minimizing random errors in statistical inference. Along with Gauss' studies, Laplace in 1810 showed mathematically that the normal distribution follows from the central limit theorem, namely, the sum of a large number of mutually independent, identically distributed random variables is approximately normally distributed (Haldane 2012).

In probability theory, the standard Gaussian distribution arises as the limiting distribution of a large class of distributions of random variables (with suitable centering and normalization) characterized by a finite variance, which is nothing but the statement of the central limit theorem (Sornette 2012).

The importance of the normal curve stems primarily from the fact that the distributions of many natural phenomena are at least approximately normally distributed. This normal distribution concept has molded how we analyse experimental data over the last 200 years. We have come to think of data as having values most of which are near an average value, with a few values that are smaller, and a few that are larger. The probability density function, PDF(x), is the probability that any measurement has a value between x and $x + \mathrm{d}x$. We suppose that the PDF of the data has a normal distribution. Most quantitative research involves the use of statistical methods presuming *independence* among data points and Gaussian 'normal' distributions (Andriani and McKelvey 2007). The Gaussian distribution is reliably characterized by its stable mean and finite variance (Greene 2002). Normal distributions place a trivial amount of probability far from the mean and hence the mean is representative of most observations. Even the largest deviations, which are exceptionally rare, are still only about a factor of two from the mean in either direction and are well characterized by quoting a simple standard deviation (Clauset et al. 2009). However, apparently rare real-life catastrophic events such as major earth quakes, stock market crashes, heavy rainfall events, etc., occur more frequently than indicated by the normal curve, i.e. they exhibit a probability distribution with a *fat tail*. Fat tails indicate a power-law pattern and interdependence. The 'tails' of a power-law curve—the regions to either side that correspond to large fluctuations—fall off very slowly in comparison with those of the bell curve

(Buchanan 2004). The normal distribution is therefore an inadequate model for extreme departures from the mean (Selvam 2009).

The following references are cited by Goertzel and Fashing (1981) to show that the bell curve is an empirical model without supporting theoretical basis: (i) Modern texts usually recognize that there is no theoretical justification for the use of the normal curve, but justify using it as a convenience (Cronbach 1970). (ii) The bell curve came to be generally accepted, as M. Lippmnan remarked to Poincare (Bradley 1968), because '…*the experimenters fancy that it is a theorem in mathematics and the mathematicians that it is an experimental fact*'. (iii) Karl Pearson (best known today for the invention of the product-moment correlation coefficient) used his newly developed chi-square test to check how closely a number of empirical distributions of supposedly random errors fitted the bell curve. He found that many of the distributions that had been cited in the literature as fitting the normal curve were actually significantly different from it, and concluded that 'the normal curve of error possesses no special fitness for describing errors or deviations such as arise either in observing practice or in nature' (Pearson 1900).

For well over a century, evidence had been mounting that real-world behaviour in particular, behaviour of *systems*, whether natural, social, economic, or financial does not follow normal distribution characteristics. The first statistical tests for normality were first developed in the 1870s by German statistician Wilhelm Lexis who found that the only series which closely matched the Gaussian distribution was birth rates. The data sets used by Pierce (Wilson and Hilferty 1929) were reexamined in 1929 by E.B. Wilson and M.M. Hilferty using formal statistical techniques which ruled out normality. The following period saw increased evidence for non-normality in real-world settings and in its place an alternative distribution, namely the power-law distribution was shown to be exhibited by real-world systems in all fields of science (Haldane 2012).

2.4 Power Laws—History

Fractals conform to power laws. A power law is a relationship in which one quantity A is proportional to another B taken to some power n; that is, $A \sim B^n$ (Buchanan 2004). Power-law distributions are found in a vast variety of systems with apparently different characteristics and have a long history of being recorded in different fields of investigation (Montroll and Shlesinger 1982, 1983, 1984). One of the oldest scaling laws in geophysics is the Omori law (Omori 1894). This law describes the temporal distribution of the number of aftershocks, which occur after a larger earthquake (i.e. mainshock) by a scaling relationship. Richardson (1960) came close to the concept of fractals when he noted that the estimated length of an irregular coastline scales with the length of the measuring unit. Richardson (1926) measured the increasing span of plumes of smoke from chimneys subjected to fluctuating atmospheric wind fields. From his observations he speculated that the turbulent air speed, which was known to be non-differentiable, could be characterized by a Weierstrass function, i.e. a self-similar fractal. He observed that the span of the plume increased as t^{β} with $\beta \geq 3$, a value inconsistent with molecular diffusion for which $\beta = 1$. The eddy cascade model of turbulence invented by Kolmogorov (1941) was, in fact, a dynamic fractal so that turbulence has no characteristic space or time scale (West 2014). References to earliest known work on power-law relationships (Mitzenmacher 2003; Andriani and McKelvey 2007; Baek et al. 2011) are summarized as follows. Pareto (1896, 1897) first noticed power laws and fat tails in economics. Cities follow a power law when ranked by population (Auerbach 1913). Dynamics of earthquakes follow power law (Gutenberg and Richter 1944, 1956) and Zipf (1949) found that a power law applies to word frequencies [Estoup (1916) had earlier found a similar relationship]. Mandelbrot (1963) rediscovered them in the twentieth century, spurring a small wave of interest in finance (Fama 1965; Montroll and Shlesinger 1984). However, the rise of the 'standard' model (Gaussian) of efficient markets sent power-law models into obscurity. This lasted until the 1990s, when the occurrence of catastrophic events, such as the 1987 and 1998 financial crashes, that were difficult to explain with the 'standard' models (Bouchaud et al. 1998), rekindled the fractal model (Mandelbrot and Hudson 2004).

A power-law world is dominated by extreme events ignored in a Gaussian world. In fact, the fat tails of power-law distributions make large extreme events orders of magnitude more likely. Theories explaining power laws are also scale free. This is to say, the same explanation (theory) applies at all levels of analysis (Andriani and McKelvey 2007).

Cumulative probability distributions which follow power-law distributions are sometimes known as Pareto distribution or Zipf's law after the two early researchers Pareto (1897) and Zipf (1949) who first investigated these distributions. Power law for cumulative probability distribution implies power law for probability P also and therefore Zipf's law or the Pareto distribution apply to power-law distributions which give the probability of occurrence of particular value of some quantity as proportional to the inverse power of that value and is found in all branches of science and other fields of study. It may be of historical interest to note that the cumulative probability distribution $P(x)$ versus x was plotted with $P(x)$ on the vertical axis by Zipf while Pareto plotted $P(x)$ on the horizontal axis. The origin of power-law behaviour has been a field of intensive study in the scientific community for more than a century (Newman 2005).

2.5 Power-Law Distributions and Complex Systems

Mathematically, the power-law distribution is written as: $P(X > x) \sim x^{-\beta}$ or $f(x) = \mathrm{K}x^{-\beta}$.

In the above expression, the exponent β is a constant for the range of x values satisfying the power-law relation and K is the constant of proportionality. Power laws appear in many different contexts. The most common are that $f(x)$ describes a distribution of random variables or the autocorrelation function of a random process. The function $f(x)$ represents the probability P that random variable X exceeds some level x is proportional to $1/x^{\beta}$, i.e. the probability of large events decays polynomially with their size. For a Gaussian distribution, the probability of large events decays *exponentially* with their size so that the probability of large events decreases much more rapidly than in the case of power laws. A Gaussian distribution predicts near-zero probability for extreme events, while the power law predicts nonzero probability as happening in real-world extreme events such as earthquakes, catastrophic floods, etc. Normal distribution gives a complete description with only the mean and variance. Power laws exhibit fat tails and cannot be described completely with a single mean and variance. Power-law-distributed data have a well-defined mean only when β lies above unity and their variance only exists when β exceeds two. Therefore, Laplace's central limit theorem may not apply to power-law-distributed variables. The normal statistical probabilities of observed variables cannot be given with reference to standard deviation and departure from the mean. Physical laws generate *system*-wide nonlinear behaviour and generate the observed power laws or fat tail distributions. As compared to Gaussian distribution, power laws give a realistic higher probability for extreme events. Power laws are a signature of interdependence of component parts in real-world complex systems. Gaussian distribution is based on central limit theorem with assumption of independence of observations and therefore not applicable to real-world complex systems (Haldane 2012). Fractals, $1/f$ noise, and Zipf's laws ubiquitous to real-world systems represent three signatures of complex systems and are associated with scaling laws. Both $1/f$ spectra and Zipf's law can be converted into a self-similar hierarchy. The mathematical laws governing this hierarchical structure when identified can provide us with a unified view of looking at complexity and complex systems (Chen 2012). A complex system consists of a large number of nonidentical components with their own complex internal structure. Local and nonlinear interactions of the internal constituents result in new emergent dynamics most frequently exhibited as the heavy tailed power-law distribution for the variables characterizing the dynamics of the complex system. The probability of observing an extremely large value is more likely in a heavy tailed distribution than in an exponential distribution such as the statistical normal distribution (Markovic and Gros 2013).

2.6 Power Laws, Scale Invariance and Self-similarity

Kavvas et al. (2015) discuss the physics of scaling and self-similarity in hydrologic dynamics, hydrodynamics, and climate as follows. Since the work of Buckingham (1914) about a century ago on dimensional analysis, the role of scales in geophysical processes has been studied by many scientists and engineers. Geophysical processes exhibit self-similarity and scale invariance over a wide range of time and space scales. Identification of the physical mechanism underlying the observed ordered cooperative existence of fluctuations on all space-time scales will help formulate simple scale-free models for simulation and prediction of geophysical processes.

Power laws are signatures of scale-free or scale-invariant phenomena, the functional form being independent of the magnitude of the scale (space or time) as explained in the following. In power laws, namely $f(x) = Kx^{-\beta}$ let the variable x undergo a scale transformation of the form $x \rightarrow cx$ where $c > 0$ is any constant. The function $f(x)$ is now transformed as $f(x) \rightarrow Kc^{-\beta}x^{-\beta} = c^{-\beta}f(x) = K_1 f(x)$, where K_1 is a constant equal to $c^{-\beta}$. The functional form of the power-law relation remains the same for scale change of the independent variable x, i.e. from x to cx. Thus power laws are a necessary and sufficient condition for scale-free behaviour (Amaral et al. 1997). Power laws imply that the physical mechanism required to generate system-wide scale-free behaviour is the same for all the scales from smallest to the largest. Power laws result from a linear relationship between logarithms such as $\log f(x) = -\beta \log x + \log K$ as seen from power law relation, namely $f(x) \sim x^{\beta}$ which is the basic definition of a power law (Farmer and Geanakoplos 2008). Scale-free behaviour signifies that the shape of the distribution $f(x)$ is unchanged, except for an overall multiplicative constant, i.e. functional form for statistical properties of the system is independent of scale size. The statistical properties of the system for any two scale sizes are related to each other by the ratio of the two scales and not on the scales themselves (Pruessner 2004).

2.7 Power Laws, Self-similarity, and Fractals

Mandelbrot (1977, 1983, 1997) from the late 1960s onwards was the first to notice and study the importance and ubiquity of scale-free behaviour in real-world systems, e.g. the coastline which exhibits a zigzag pattern on all scales of measurement. Mandelbrot gave the name 'fractals' to such self-similar structures, where the geometry of the larger structure and the enclosed smaller structures is the same. Fractals are non-differentiable geometric objects that satisfy the power-law scaling relation $f(x) = Kx^{-\beta}$ when β is not equal to an integer. Fractals are ubiquitous in nature, e.g. branches of plants, trees, river networks and tributaries, clouds, earthquakes, etc. Fractal objects cannot be described by traditional Euclidean geometry. It is found that a power-law relation exists between size and measurement resolution for a fractal object. The length of a coastline depends on the yardstick length, increasing when measured accurately with a smaller yardstick; the length varies as a power-law function of measurement resolution. Similarly, the volume of a cloud depends on the water concentration threshold and varies as a power law, inversely with this threshold. For earthquakes, floods, stock market indices, etc., the probability of large events greater than a given size, decreases with event size according to a power law. Mandelbrot brought to the world view for the first time that objects in nature are non-Euclidean and can be described by non-differentiable geometry only (Farmer and Geanakoplos 2008).

The apparently irregular space-time fluctuations exhibited by dynamical systems in nature such as rainfall, heartbeat intervals, stock market prices, etc., are associated with basic bifurcation or branching geometry of wrinkles or folds on all scales is associated with the symmetry of self-similarity under scales transformation or just self-similarity (Liu 1992).

2.8 Power Laws, 1/*f* Noise, and Long-Term Memory

For random processes that are correlated in space (or time) the autocorrelation function and the power spectrum (variance versus frequency *f*) decays as a power law signifying long-range dependence in space and time. Power spectrum exhibiting such inverse power-law form, namely $1/f^{\alpha}$, where *f* is the frequency and α, a constant, is also called 1/*f* noise. 1/*f* noise is exhibited by a wide range of systems in nature and signifies long-range correlations of fluctuations in space and time, identified as long-term memory or persistence. The physics of 1/*f* noise is not yet identified and is an area of intensive research in all disciplines in particular for predictability of future extreme events such as floods, earthquakes, business market crashes, etc. Power-law scaling applies strictly for the region, where the value of α remains a constant with a lower and upper level cutoff (Farmer and Geanakoplos 2008).

The association of fractals and scaling with long-range correlations or memory was brought to the attention of scientific community by the pioneering work of B. Mandelbrot in the early 1960s (Mandelbrot 1965a, b, c, 1983; Mandelbrot and Van Ness 1968; Mandelbrot and Wallis 1968, 1969a, b) to explain the Hurst effect (Hurst 1951) in hydrology related to water levels in the Nile river which exhibited anomalously fast growth of the rescaled range of the time series. Long memory signifies correlation between the present and all points in the past. A finite variance stationary process has long memory or persistence if its autocorrelation function (ACF) decays, as power law indicates fluctuations occur at all frequencies, in particular for low frequencies or large time periods. Because of this presence of low frequencies, one might observe long periods of 'high' values followed by long periods of 'low' values. This is in contrast to many standard (stationary) random processes, where the effect of each data point decays so fast that it rapidly becomes indistinguishable from noise. The long memory processes has important applications in many fields such as computer network traffic, econometrics, astrophysics, and geophysics (see e.g. Doukhan et al. 2003). The existence, or not, of long memory has special importance for predictability in weather systems particularly anthropogenic warming trend related climate change (Franzke 2012). According to the Weiner–Khintchine theorem, the spectral density function can be expressed as the Fourier transform of the autocorrelation function for a random process (Ghil and Robertson 2002). Power spectra of meteorological parameters for a wide range of space and time scales show 1/*f* noise or flicker noise, namely inverse power law form $f^{-\alpha}$, where *f* is the frequency and α the exponent for the variance spectrum indicating long-range correlations or memory for fluctuations in time (Lovejoy and Mandelbrot 1985; Lovejoy and Schertzer 1986a, b; Yano and Nishi 1989; Fraedrich and Larnder 1993; Fraedrich and Blender 2003; Blender and Fraedrich 2003). Traditional meteorological concepts assume characteristic scales for meteorological phenomena while observed power-law spectra of meteorological parameters show lack of characteristic scales in both space and time. Fractals, power laws and long-range correlations are signatures of nonlinear dynamics in atmospheric flows and are an intensive field of study since the 1980s (Yano et al. 2004; Bove et al. 2006; Samorodnitsky 2006, 2007; Graves et al. 2014).

2.9 Power Laws, Phase Transitions, and Critical Phenomena

Power-law distributions associated with phase transitions in critical phenomena are a well-established field in equilibrium statistical physics. The first microscopically based understanding of phase transitions is due to van der Waals, who in 1873 presented a primitive theory of the gas–liquid transition. Matter exists in three states, namely solid, liquid, and gas. The phase of a macroscopic substance is determined by a few macroscopic parameters such as the temperature and the pressure. A change in parameters such as the temperature across a phase boundary causes a sudden change in the phase of the substance. For example a solid phase changes into a liquid phase at the melting temperature. This is a phase transition. The density of the fluid has a jump at the liquid–gas phase transition (evaporation). This is the best example of a first-order phase transition, where all the physical quantities characterizing the material undergo a sudden change. Generally, the two phases are quite different at first-order transitions, and thus it takes a finite amount of energy to convert the substance from one phase to the other. This is the latent heat. It is interesting to note that the density jump at the liquid–gas transition decreases at higher pressures and temperatures. The first-order line ends at the liquid–gas critical point. The approach to this point is a second-order phase transition. In its vicinity, the fluid cannot seem to 'decide' what to become: a liquid or a gas. Large density fluctuations emerge leading to a 'milky' appearance of the fluid: the critical opalescence. Critical phenomena near continuous phase transitions at the critical points are typically observed on the scale of wavelengths of visible light (4×10^{-7} to 7×10^{-7} m) which is about three orders of magnitude larger than the random molecular scale fluctuations (H_2O molecular diameter about 2.75×10^{-10} m) of the liquid. Near the critical point, the invisible molecular scale random fluctuations form organized larger scale coherent structures visible as milky white critical opalescence.

Critical points at continuous phase transitions are one of the most interesting and important topics in equilibrium statistical physics. A phase boundary sometimes disappears at a critical point, where the two phases become indistinguishable and the substance shows anomalous behaviour, namely, the atomic/molecular scale random fluctuations organize to form macroscale coherent structures exhibiting fluctuations on all length scales. These scale-free or scale invariant fluctuations can be quantified by a power-law function with well-defined exponent called the critical exponent. All the thermodynamic properties of the system at critical point are due to the so-called scale invariance of critical phases. It is noteworthy that for a large class of very different experimental systems, the values of critical exponents are the same, independent of specific details of interactions depending only on very general properties, such as dimensionality. Near the critical point, the collective organization of individual random atomic/molecular scale fluctuations organizes to form coherent structures of all size scales observed in the scale-free fluctuations of thermodynamic parameters. Systems sharing the same values of critical exponents are said to belong to the same universality class. Universality is a well-established central concept of equilibrium physics. Systems far from equilibrium also exhibit such universality classes. However the physics of such nonequilibrium universality is not yet identified. Popkov et al. (2015) show that the two best-known examples of nonequilibrium universality classes, the diffusive and Kardar–Parisi–Zhang classes, are only part of an infinite discrete family. The members of this family are identified by their dynamical exponent, which can be expressed by a Kepler ratio of Fibonacci numbers which progressively approach the golden ratio equal to $(1 + \sqrt{5})/2$ (≈ 1.618).

It is to be noted that in equilibrium systems, the thermodynamic parameters have to be tuned precisely to observe the power-law distribution of scale-free fluctuations typically observed on the scale of wavelengths of visible light (4×10^{-7} to 7×10^{-7} m). However, spontaneous occurrence of power-law distributions has been reported in nonequilibrium systems such as turbulent atmospheric flows where coherent large-scale organization of scale-free fluctuations is observed in the size scale of up to thousands of kilometres. Peters and Neelin (2006) report similar scale-free phenomena for atmospheric precipitation on scales of tens of kilometres.

There are a number of problems in science which have as a common characteristic that complex microscopic behaviour underlies macroscopic effects. In simple cases, the microscopic fluctuations average out when larger scales are considered and the averaged quantities satisfy classical continuum equations. Hydrodynamics is a standard example of this, where atomic fluctuations average out and the classical hydrodynamic equations emerge. Unfortunately, there is a much more difficult class of problems, where fluctuations persist out to macroscopic wavelengths, and fluctuations on all intermediate length scales are important too. In fully developed turbulence in the atmosphere, global air circulation becomes unstable, leading to eddies on a scale of thousands of miles. Chaotic motions on all length scales down to millimetres are excited. Theorists have difficulties with these problems because they involve very many coupled degrees of freedom (Wilson 1982).

Mandelbrot had shown that fractal fluctuations underlie scale-free power-law distributions of thermodynamic parameters. Fractals are characterized by power-law frequency-size distributions indicating scale invariance, i.e. do not possess a characteristic length scale (Turcotte 1999). Fractals in nature are self-similar, i.e. scale invariant and do not possess a characteristic length scale, e.g. branching structure of trees and plants, river tributaries, clouds on all size scales, etc. The power-law distribution is the only function satisfying the scale-free criterion, i.e. power law is the only distribution that remains the same for any scale size.

Popkov et al. (2015) show that in nonequilibrium phenomena governed by NLFH (nonlinear fluctuating hydrodynamics) with n conservation laws, mode coupling theory predicts a family of dynamical universality classes with dynamical exponents given by the sequence of consecutive Kepler ratios of Fibonacci numbers which progressively approach the golden mean $(1 + \sqrt{5})/2$ ($\approx$1.618).

2.10 Power Laws and Self-organized Criticality

About 25 years ago, the concept of self-organized criticality (SOC) emerged (Bak et al. 1987), initially envisioned to explain the ubiquitous $1/f$-power spectra, which can be characterized by a power-law function $P(f) \propto f^{-1}$. The term $1/f$ *power spectra* or *flicker noise* should actually be understood in broader terms, including power spectra with pink noise ($P(f) \propto f^{-1}$), red noise ($P(f) \propto f^{-2}$), and black noise ($P(f) \propto f^{-3}$), essentially everything except white noise ($P(f) \propto f^{0}$). While white noise represents traditional random processes with uncorrelated fluctuations, $1/f$ power spectra are a synonym for time series with nonrandom structures that exhibit long-range correlations (Aschwanden et al. 2016).

The signature of fractals, namely, inverse power-law form for power spectra of fluctuations was identified for isotropic homogeneous turbulence by Kolmogorov in the 1940s. The concept of fractals and its quantitative measure for space-time fluctuations of all scales was introduced by Mandelbrot in the late 1960s. The robust pattern of self-similar space-time fluctuations was identified by Bak, Tang and Wiesenfeld in the late 1980s as self-organized criticality (SOC), whereby the cooperative existence of fluctuations of all space-time scales maintains the dynamical equilibrium in dynamical systems.

Scaling behaviour is the hallmark of criticality (Fisher 1967), as it implies scale invariance: The statistical properties of a system on one scale are, apart from some factors, identical to the statistical properties of the system on another scale. The factors depend only on the ratio of the two scales and not on the scales themselves.

Power-law behaviour is observed in many different systems in nature signifying a common unifying principle underlying such scale-free organization of fluctuations. Bak et al. (1987, 1988) in their seminal work provided the central hypothesis that complex systems with many interacting components will spontaneously organize to give scale-free fluctuations such as that observed in a equilibrium thermodynamic system near a second-order phase transition known as critical phenomena, an established field of study for more than 150 years. Critical phenomena were discovered by Cagniard de la Tour in 1822 (Berchea et al. 2009). Such complex behaviour in nature occurs without the need for any fine tuning of parameters and therefore named self-organized criticality (SOC). The physics of the collective organization of short range microscopic fluctuations to result in large-scale fluctuations of all length scales with long-range correlations is not yet identified.

In equilibrium thermodynamics critical phenomena are associated with the notion of universality. Systems belonging to many different phase transitions are grouped into a small number of universality classes, each class having the same values of critical exponents and scaling functions. The collective behaviour of the system becomes independent of its microscopic details and simple models can describe the observed scale-free fluctuations. Physical systems exhibiting SOC are nonequilibrium systems, since there is a constant flux of matter and energy exchange with the environment. Universality classes, if any, such as in equilibrium phase transitions, have not yet been identified and also the dynamical processes governing the nonequilibrium phase transitions are not known (Markovic and Gros 2013).

The concept of self-organized criticality (SOC) by Bak et al. (1987, 1988) helped to bring together as a multidisciplinary field the results of earlier studies relating to power laws, 1/*f* noise, space-time fractals in disparate fields of research (e.g. van der Ziel 1950; Schick and Verveen 1974; Weissman 1988). Scale-free power-law distributions for dynamical processes indicate simple physical interactions on the smallest scale are carried over to the largest scales. SOC may therefore provide the theoretical framework for investigating in a scale-free multi-scale phenomena, the nature of the physical processes at the smallest scale (unscaled primary) operating globally at all larger scales in the continuum as visualized by Anderson (1972) and Wilson (1979) (Watkins et al. 2016).

2.11 Current Status of Power-Law Distributions

Power-law distributions, also known as heavy tail distributions, Pareto-like laws, or Zipf-like laws have been reported in real-world phenomena in different contexts. However, they are discussed only as statistical phenomena and how well they fit Pareto or Zipf laws without positive consensus regarding the fit (Clauset et al. 2009; Pinto et al. 2012; Stumpf and Porter 2012). Sornette and Ouillon (2012) state that many phenomena in nature exhibit power-law characteristics which can be related to many different physical mechanisms (Mitzenmacher 2003; Newman 2005; Sornette 2006). Scale-free distributions suggest the same dynamical mechanism for the growth of fluctuations from smaller to larger scales including extreme catastrophic events. A statistically sound power law even without a supporting rigorous theory still gives useful information regarding frequency of occurrence of extreme events (Stumpf and Porter 2012). Willinger et al. (2004) state that Mandelbrot (1997) has provided for the last 40 years mathematical, statistical, and data-analytic arguments that demonstrate that highly variable event sizes are in a sense just as ‘normal’—or even more ‘normal’—than Gaussian-type event sizes. Baek et al.

(2011) have given a summary of power-law distribution history and remarks that fat tails are a common feature of power-law distributions encountered in disparate contexts. The question remains whether 'fat tails' represent a global, ubiquitous nonsystem-specific feature.

2.12 Power-Law Relations (Bivariate) and Power-Law (Probability) Distributions

Reported power laws are of two different types: power-law probability distributions like the rainfall rates and bivariate power laws like allometric scaling (Stumpf and Porter 2012). Allometry designates the changes in relative dimensions of parts of the body that are correlated with changes in overall size. Julian Huxley and Georges Teissier coined this term in 1936 (Gayon 2000).

Power-law probability relations are associated with critical points and fractals. Fractals signify objects with complex geometrical properties resulting from the coexistence of many scales (Mandelbrot 1983). In statistical physics, power laws are associated with second-order phase transitions in the neighbourhood of critical points, where scale-free phenomena occur (Stumpf and Porter 2012). Power-law probability relations are associated with critical points and fractals. Bivariate power-law relation between two variables on the other hand is equivalent to a mathematical function characterizing the relationship (Sornette 2012).

Some historical bivariate power-law relations in science are (i) Coulomb's inverse square law of electrostatic force (first published in 1785). (ii) Kepler's laws of planetary motion (published between 1609–1619). (iii) Stefan–Boltzmann law of radiation (in 1879 on the basis of experimental measurements). (iv) allometric scaling (first outlined by Snell (1892), D'Arcy Thompson (1917) in *On Growth and Form*) in biology and botany.

These bivariate power-law relations are not statistical distributions, but are well established and accepted with strong data base support (Miramontes et al. 2012).

2.13 Allometric Scaling and Fractals

In biology, the allometric power-law relationship between body size x and metabolic performance y has been supported empirically over many orders of magnitude (from bacteria to whales) (West et al. 1997). West et al. (1997) proposed that one of the common mechanisms underlying allometric relationship is based on the unifying principle or assumption that a space-filling fractal-like branching structure (Mandelbrot 1977) is required for the network supplying the entire volume of the organism. Scaling laws arise from the interplay between physical and geometric constraints (West et al. 1997).

Some of the documented power laws appear to be universal to plants, animals and microbes, to terrestrial, marine and freshwater habitats; and to human-dominated as well as 'natural' ecosystems. The basic self-similar fractal architecture underlying the power laws indicates ordered organization of components of complex systems governed by a few basic physical, biological, and mathematical principles. Self-similarity over many orders of magnitude for geometrical structures and dynamical processes enable extrapolating between scales. Classic scaling relations have been described for river networks (Horton 1945; Gupta and Waymire 1989, 1998a, b; Peckham and Gupta 1999). Variations of flows, velocities, depths, widths, and slopes take the general form $Y \sim Q^b$, where Y is the hydraulic–geometric variable, Q is stream discharge and related to size of a basin, and b is a scaling exponent (Leopold and Miller 1956; Ibbitt et al. 1998; Brown et al. 2002).

Fractals are hierarchical branching networks. Nature has evolved fractal-like networks in living systems (animals and plants), where maximum surface area for energy exchange with minimum transport distance helps maximize both metabolic capacity and internal efficiency. Scaling laws typically reflect underlying general features and principles that are independent of detailed structure, dynamics or other specific characteristics of the system, or of the particular models used to describe it (West et al. 1997, 1999; West and Brown 2005).

2.14 Fractals and the Golden Section in Plant Growth

Phyllotaxis is a subdivision of plant morphology in Botany and refers to the study of repeated units such as leaves around a stem, scales on a pine cone or on a pineapple, florets in the head of a daisy, and seeds in a sunflower. These units self-organize to form repeated units of beautiful geometric patterns of spirals or helices closely resembling crystals. Adler et al. (1997) have given a comprehensive history of the study of phyllotaxis. In phyllotaxis, the angular position of leaves on the stem follows a precise mathematical spiral pattern such that the divergence angle of subsequent leaves divide the whole circle (360°) into regular fractions (Cummings and Strickland 1998). Longstanding observations show that these fractions belong to a series, where numerator and denominator are Fibonacci numbers, for example, 1/2, 1/3, 2/5, 3/8, 5/13, …. In the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, …, each term after the second is the sum of the two that precede it (Adler et al. 1997) and the ratio of each term to the preceding approaches the golden section (golden ratio or golden mean) τ equal to $(1 + \sqrt{5})/2$ ($\approx$1.618), an irrational number. The earliest paper on the golden ratio by Ackermann (1895) attracted the interest of scientists and engineers in various fields of science and engineering (Li and Zhao 2013a, b). Leonardo Fibonacci of Pisa (1202) developed the Fibonacci series as a solution to the problem of monthly growth of rabbit population. The series 1/2, 1/3, 2/5, 3/8, 5/13, converges to the irrational limit of $1/\tau^2$ equal to 0.382…, which corresponds to the 'golden angle' equal to 137.5°, the golden section of full circle (360°). Serious study of phyllotaxis as a subject has a history of more than 300 years with observational phyllotaxis by Bonnet (1754), pioneering work on mathematical phyllotaxis by Braun (1831), Schimper (1836) and Bravais and Bravais (1837). Jean (1994) gives a complete survey of the studies in phyllotaxis. The physical mechanism creating these beautiful mathematical patterns of phyllotaxis is still not identified (King et al. 2004). In plants, the golden section is an essential feature found to occur with mathematical precision in the fractal-like branching patterns of trees, shrubs, etc., in the arrangement of petals/seeds and often the spiral arrangement of plant organs (Zeng and Wang 2009).

2.15 Turbulent Fluid Flow Structure, Fractals, and the Golden Ratio

von Karman (1930) showed that turbulent fluid flow dynamics near a wall follow the 'law of the wall', namely the average velocity of a turbulent flow at a certain point is proportional to the logarithm of the distance from that point to the 'wall', or the boundary of the fluid region. Recently, Li and Zhao (2013a, b) have shown that the power spectra for turbulent fluid flows derived by Kolmogorov (1941) and von Karman (1948) exhibit self-similar, i.e. fractal character of turbulent eddy fluctuations and can be expressed in terms of the golden ratio. Therefore, the trajectory of turbulent fluid flows near a boundary or wall follows a logarithmic spiral with the golden mean as the winding number similar to the spiralling arrangement of successive leaves in plant growth (Jean 1994). Selvam (1990) has shown that the atmospheric flow structure follows a logarithmic spiral trajectory with crossing angle equal to the golden mean.

2.16 Fractal Space-Time and the Golden Section

El Naschie (1994) refers to earlier work of Mauldin and Williams (1986) to show that a random Cantor set (fractal) with a Golden Mean Hausdorff dimension is a model for micro space-time.

Self-similar fractal space-time fluctuations result from the superimposition of eddy continuum fluctuations. A general systems theory model for fractal space-time fluctuations in turbulent fluid flows such as atmospheric flows (Selvam 1990, 2007, 2015) predicts that the golden mean is incorporated in the growth processes of eddy continuum fluctuations observed in atmospheric flows.

2.17 Power-Law (Probability) Distributions in Meteorological Parameters

Fraedrich and Larnder (1993) discuss earlier studies of power spectral analyses and identification of power-law distribution characteristics of meteorological parameters as follows. Power spectral analysis, namely the spectral variance density distribution of an observed time series, is used to locate dominant timescales which exhibit peaks in their contribution to the total variance. A famous example is the spectrum of high resolution with measurements at experimental sites (Van der Hoven 1957; Fiedler and Panofsky 1970; Vinnichenko 1970). Other examples, to mention a few, are the frequency spectra deduced from daily rawin observations at weather ships and continental stations (Hartmann 1974), wavenumber–frequency spectra of travelling and quasi-stationary disturbances (Fraedrich and Bottger 1978).

It is now recognized that self-similar fractal fluctuations are ubiquitous to dynamical systems such as atmospheric flows in nature. The atmospheric flow dynamics is characterized by a scale invariant broadband spectrum and cannot be completely defined by one or several prominent spectral peaks or modes. Scale invariance or scaling behaviour within a waveband implies long-range correlations between the fluctuations of the larger and smaller scale components. Thus, the scaling behaviour within a frequency band (and not the spectral peak) may be more useful and appropriate to characterize the timescales of atmospheric dynamics.

Earlier scale invariance in atmospheric flow structure was identified by (i) Richardson (1926) who applied the Lagrangian two-particle dispersion method to two-dimensional diffusion leading to his celebrated scaling law (ii) the Kolmogoroff 5/3-spectrum for isotropic three dimensional turbulence (iii) Charney's Ansatz (Charney 1971) for geostrophic turbulence, etc. More recently, in their analysis of temperatures, Lovejoy and Schertzer (1986b) characterize climatological regimes by scale invariance; this was extended to rainfall (Ladoy et al. 1991) with the emphasis to describe extreme variability by scaling and intermittency.

Fraedrich and Larnder (1993) analysed the scaling behaviour of rainfall both for a range of scales in time and for a given scale in intensity using the statistics of the Fourier transform and the cumulative probability distribution. The data sets used are (i) sets of long time series of daily rainfall at 13 European stations and (ii) sets of 5-min totals thus covering a wide scaling range.

Some early studies on application of self-organized criticality to atmospheric flows are given in the following. Turcotte (1999) has cited the following references for applications of the concept of self-organized criticality in atmospheric sciences: (i) Grieger (1992) has discussed applications to climate fluctuations (ii) Nagel and Raschke (1992) consider cloud formation processes (iii) Andrade et al. (1998) have analysed rainfall records for possible relation to self-organized criticality.

Yano et al. (2004) state that numerous studies show that meteorological phenomena exhibit scale invariant behaviour manifested as power laws in spectra (Lovejoy and Mandelbrot 1985; Lovejoy and Schertzer 1986a, b; Yano and Nishi

1989; Fraedrich and Larnder 1993; Fraedrich and Blender 2003; Blender and Fraedrich 2003). The observed scale invariant behaviour implies lack of characteristic scales in both time and space in direct contrast with traditional meteorological view of characteristic scales for individual phenomena.

Earlier Yano et al. (2001) found 1/*f* noise, a signature of scale invariance in tropical convective variability. The 1/*f* noise is unique among the power-law spectra in the sense that it contains equal variability for all timescales (Keshner 1982).

Blender et al. (2011) summarized observations and modelling of 1/*f*-noise in weather and climate as follows. Meteorological and hydrological parameters exhibit fluctuations on all scales (Jiang et al. 2005; Yano et al. 2004; Fraedrich and Blender 2003) and in the absence of external influences may be attributed to the internal variability of weather and climate. The low frequency part of the power spectra of these time series exhibit approximate inverse power-law form $f^{-\alpha}$ with $\alpha > 0$ and signifies long-range space-time correlations or long-term memory.

Data with power spectra close to $S(f) \sim 1/f$ is denoted as 1/*f* or flicker noise. Examples for 1/*f* spectra are found in wide ranges from minutes to millennia: (i) tropical boundary layer observations (TOGA/COARE, Yano et al. 2004) (ii) the discharge of the Yangtze river in the intra-annual frequency range (Wang et al. 2008) (iii) the sea surface temperature in a region of the North Atlantic and in the southern ocean [reproduced by models (Fraedrich and Blender 2003)] (iv) reconstructed near surface temperatures up to the Milankovitch cycle (Huybers and Curry 2006).

Andrade et al. (1998) state that long-term data sets from weather stations around the world exhibit self-organized critical dynamics, namely, the distribution of droughts in semiarid regions obeys a clear-cut power law. The statistics for rain intensity, on the other hand, exhibit two distinct scaling regimes. These authors cite the following studies which show scale-free power-law distributions, a signature of SOC in meteorological parameters.

1. Analysis of historical records of the Southern Pacific sea surface temperature by Andrade et al. (1995) indicate that the El Nino may be an example of SOC in climate phenomena.
2. It is now recognized that several properties of the statistical distributions of pertinent meteorological fields (temperature, air humidity, etc.) are independent of the particular (time or length) scale at which they are observed (Lovejoy and Schertzer 1991, 2010; Vattay and Harnos 1994; Fraedrich and Larnder 1993; Ladoy et al. 1991), i.e. they obey scale-free distributions.
3. Former identifications of power-law behaviour in single precipitation events (Ladoy et al. 1991; Lovejoy and Mandelbrot 1985), and work of Fraedrich and Larnder 1993) which detects two different scaling domains in the rainfall spectrum.

2.17.1 Power-Law (Probability) Distributions in Precipitation

Arakawa (2006) remarks that despite the complexity of the processes involved, the statistics of the tropical rain rate are remarkably similar to those of critical phenomena near continuous phase transitions in other—much smaller—physical systems.

Deluca et al. (2015) state that coupling between several nonlinear mechanisms with different spatial and temporal characteristic scales gives rise to structures and correlations across long ranges in space and time in the observed characteristics of precipitation (Lovejoy 1982; Vattay and Harnos 1994; Yano et al. 2004; Bodenschatz et al. 2010). Strong statistical regularities are presented by individual rain events from diverse backgrounds (Peters et al. 2002, 2010; Peters and Neelin 2006; Neelin et al. 2008; Deluca and Corral 2014; Deluca et al. 2016) giving support to the hypothesis that atmospheric convection and precipitation may be a real-world example of self-organized criticality (SOC).

Jordan (2008) gives a summary of observational studies of region wise evidence for the operation of SOC in atmospheric convective processes as follows.

(i) Vattay and Harnos (1994) were the first to make the suggestion that SOC is present in the atmosphere. They showed that the daily average air humidity fluctuations from central Europe over the interval of a year exhibited approximate $1/f$ behaviour.
(ii) Later Peters and Christensen (2002) using rainfall statistics at a site on the German Baltic coast over an 8 month period from January to July 1999 showed that no typical size or time scale exists for rain event sizes and durations confirming signature of SOC in atmospheric processes.
(iii) A detailed analysis of tropical convective variability using Tropical Ocean Global Atmosphere Coupled Ocean Atmosphere Response Experiment (TOGA-COARE) data was carried out by Yano et al. (2003). Using time series data from 13 sites in the western Pacific, approximate $1/f$ behaviour was found to occur in atmospheric surface variables (air temperature, moisture mixing ratio and wind speed) over an interval from 1 h to 10 days.
(iv) Critical phenomena in tropical rain was first studied by Peters and Neelin (2006) using tropical rainfall data. The data sets that the study analysed were comprehensive; satellite microwave data from each major global ocean basin over a 5 year period from 2000 to 2005 at a 20 km grid resolution. Analysing the relationship between vertically integrated water vapour and precipitation, they demonstrated that above a critical value of vertically integrated water vapour precipitation is intense, short-lived, and follows a power-law relationship. Below that critical value, it is weak but more persistent.

Andrade et al. (1998), Peters et al. (2002) and Peters and Christensen (2002, 2006) were the first to study rain events as avalanches in SOC of precipitation processes. These authors defined, independently, the event size as equal to the total amount of rain collected during the duration of the event. There was evidence for power-law distributions for event sizes and for dry-spells durations over several orders of magnitude. A later study by Peters et al. (2010) covering 10 sites across different climates using rain data from optical gauges showed unambiguous power-law distributions of event sizes and dry-spell durations. Peters et al. (2010) compared rain event size distributions derived from measurements in climatically different regions and found them to be well approximated by power laws of similar exponents over broad ranges. Differences were seen in the large-scale cutoffs of the distributions. Event duration distributions suggested that the scale-free aspects are related to the absence of characteristic scales in the meteorological mesoscale.

Deluca and Corral (2014) analysed distributions of rain event sizes, rain event durations, and dry-spell durations for data obtained from a network of 20 rain gauges scattered in a region of the NW Mediterranean coast. The distributions exhibit scale invariance, a signature of SOC in precipitation observed for medium resolution rain data.

Rain fields exhibit large spatial and temporal intermittency and extreme variability such that their intensity cannot be characterized by its mean value (Bodenschatz et al. 2010). The complex phenomena of cloud and rain formation processes exhibit surprising statistical regularities such as (i) numerous geometric and radiative properties of clouds present clear scaling or multiscaling behaviour (Lovejoy 1982; Cahalan and Joseph 1989; Peters et al. 2009; Wood and Field 2011) (ii) raindrop arrival times and raindrop sizes, are well characterized by power-law distributions over several of orders of magnitude (Olsson et al. 1993; Lavergnat and Gole 2006).

Peters and Christensen (2002) show that the number density of rain events per year is inversely proportional to the released water column raised to the power 1.4. This is the rain equivalent of the Gutenberg–Richter law (Gutenberg and Richter 1944, 1956) for earthquakes. The event durations and the waiting times between events are also characterized by scaling regions, where no typical time scale exists. Scale-free power-law behaviour is found to govern the statistics of rain over a wide range of time and event size scales and rain is an excellent example of a self-organized critical process.

Processes relevant for precipitation are associated with many different characteristic time and spatial scales, see, e.g. (Bodenschatz et al. 2010). The list of these scales has a gap, however, from a few kilometres (a few minutes) to 1000 km (a few days), spanning the so-called mesoscale.

Devineni et al. (2015) present the first ever results on a global analysis of the scaling characteristics of extreme rainfall areas for durations ranging from 1 to 30 days. Their findings lead to the question as to how the climate system organizes over these scales, overcoming the substantial apparent heterogeneity in process dynamics. They suggest that power-law scaling may also apply to planetary scale phenomenon, such as frontal and monsoonal systems, and their interaction with local moisture recycling. Such features may have persistence over large areas corresponding to extreme rain and regional flood events.

Peters et al. (2002) showed that power law is followed by precipitation observations, with the coefficient α being 1.36 and 1.42 for precipitation intensity and drought length, respectively. More recently, Bove et al. (2006) found power-law distributions of precipitation intensity and drought length in 15-min resolution rain gauge data, with coefficients of 2.35 and 2.1 for intensity and drought, respectively. Their analysis of rain data collected from automatic stations (Data Collection Platforms) spread on the Italian territory shows that the rain is a self-organized critical phenomenon. They show that power laws describe the number of rain events versus size and number of droughts versus duration. Anomalous Hurst coefficients and one-over-*f* (1/*f*) noise found are consistent with the concepts of criticality and self-similarity.

Peters and Neelin (2006) analysed satellite microwave estimates of rainfall rate, *P*, water vapour, *w*, cloud liquid water and sea surface temperature (SST) from the Tropical Rainfall Measuring Mission from 2000 to 2005. Observations from the western Pacific provided initial support for their conjecture, namely, a power-law pickup of precipitation (the order parameter) above a critical value, w_c, of water vapour (the tuning parameter).

Atmospheric convection and precipitation have been hypothesized to be a real-world realization of self-organized criticality (SOC). This idea is supported by observations of avalanche-like rainfall events (Andrade et al. 1998; Peters et al. 2002) and by the nature of the transition to convection in the atmosphere (Peters and Neelin 2006; Neelin et al. 2009).

Wang and Huang (2012) analysed the long-term rain records of five meteorological stations in Henan, a central province of China for rain duration, drought duration, accumulated rain amount characterizing these rain events processes. They found that the long-term rain processes in central China exhibit the feature of self-organized criticality.

Andrade analysed long-term daily rain records of weather stations around the world with a special emphasis on the semiarid regions and found that there existed some evidences of SOC with these data (Andrade et al. 1998). Peters et al. investigated the European rain and found that it exhibits the feature of SOC (Peters and Christensen 2002; Peters et al. 2002; Pruessner and Peters 2006).

Pelino et al. (2006) show that daily precipitation is characterized by a behaviour regulated by power laws for the frequency distribution of both event intensity and drought duration. In this respect, precipitation appears to follow self-organized criticality laws, much as other geophysical phenomena such as avalanches and earthquakes.

Peters et al. (2002) analysed data at 1-min resolution showed that this power law is followed by precipitation observations for precipitation intensity and drought length.

Bove et al. (2006) found power-law distributions of precipitation intensity and drought length in 15-min resolution rain gauge data. Scale-free power-law behaviour is found to govern the statistics of rain over a wide range of time and event size scales (Peters and Christensen 2006).

Therefore observed precipitation exhibits self-organized criticality and complexity-like behaviour, at least at fine temporal scales.

Sarkar and Barat (2006) analysed monthly rainfall records of All India (total) and different regions of India for the time period 1871–2002 and found that the distributions of the rainfall intensity exhibit perfect power-law behaviour. The scaling analysis revealed two distinct scaling regions in the rainfall time series.

2.17.2 Power-Law (Probability) Distributions in Temperature

Franzke's (2012) study investigates the significance of trends of four temperature time series—Central England Temperature (CET), Stockholm, Faraday–Vernadsky, and Alert. An analysis of the four temperature time series reveals evidence of long-range dependence (LRD) and nonlinear warming trends. There is increasing evidence that surface temperatures are long-range dependent (Koscielny-Bunde et al. 1998; Eichner et al. 2003; Gil-Alana 2005; Huybers and Curry 2006; Rybski et al. 2006; Fatichi et al. 2009; Franzke 2010).

Fraedrich et al. (2009) cite the following studies for long-term memory and $1/f$ noise in climate variability. The variability of the near surface temperature shows a continuum of long-term memory. In the atmosphere, the high frequency range is assessed by observations in the tropical western Pacific. Here, convective available potential energy (CAPE) shows a $1/f$ spectrum within 1–30 days (Yano et al. 2001), while temperature, wind speed, and moisture show this spectrum within 1 h to 10 days. The observed sea surface temperature in the North Atlantic shows a $1/f$ spectrum on intra-annual timescales (Fraedrich and Blender 2003). Monetti et al. (2003) studied the persistence in sea surface temperature (SST) records at many sites in the Atlantic and Pacific oceans. They find that for all timescales, the SST fluctuations exhibit stronger correlations than atmospheric land temperature fluctuations (Monetti et al. 2003). Greenland ice cores present a proxy for temperature variability revealing scaling long-term memory on millennial timescales (Blender et al. 2006).

Fraedrich et al. (2004) state that spectra of observed ocean surface temperatures follow $1/f$ (or flicker noise) beyond one year in northern and southern midlatitudes (Fraedrich and Blender 2003; Monetti et al. 2003).

Liu et al. (2014) investigated the frequency-size distribution of three climatic factors (average daily temperature, vapour pressure, and relative humidity) for the period 1961–2011 in Yanqi County, northwest China and found that they were well approximated by power-law distribution, which suggested that climatic factor might be a manifestation of self-organized criticality.

2.17.3 Power-Law (Probability) Distributions in Quaternary Ice Volume Fluctuations

Grieger (1992) shows that the concept of self-organized criticality due to Bak et al. (1987, 1988) offers a simple and appealing possibility to explain the power-law background spectrum of the quaternary ice volume fluctuations.

2.17.4 Power-Law (Probability) Distributions in Atmospheric Pollution

Kai et al. (2013) found long-range correlation at one-year temporal scale in their analysis of long-term time series of daily average PM10 (pollution index) concentrations in Chengdu city, China. Further spectral analysis of the time series indicated 1/*f* noise behaviour. The probability distribution functions of PM10 concentrations fluctuation have a scale-invariant structure. The observed scale-invariant structure of PM10 fluctuations indicates the operation of SOC in atmospheric dynamical processes.

Liu et al. (2015) analysed the temporal fluctuations of the three pollution indices (SO_2, NO_2 and PM10) and the daily air pollution indices (APIs) of Shanghai in China. The results show that the temporal scaling behaviours in all the four series exhibit two different power laws. Their findings suggest that SO_2, NO_2, and PM10 pollution is an example of a self-organized criticality (SOC) process in the atmosphere.

2.18 General Systems Theory Model for Self-organized Criticality in Atmospheric Flows

Selvam (1990, 2007, 2014, 2015) has presented a general systems theory model for turbulent fluid flows such as atmospheric flows. The three important model predictions are given as follows:

(a) The observed fractal fluctuations of meteorological parameters result from the superimposition of an eddy continuum fluctuations.
(b) The probability distribution P of amplitudes of component eddies represents the variance distribution also and is given in Eq. (2.1) as

$$P = \tau^{-4t}, \tag{2.1}$$

where τ (≈1.618) is the golden mean and t the normalized deviation (deviation/standard deviation) of the data used for the study. Popkov et al. (2015) show that in nonequilibrium phenomena governed by NLFH (nonlinear fluctuating hydrodynamics) with n conservation laws, mode coupling theory predicts a family of dynamical universality classes with dynamical exponents given by the sequence of consecutive Kepler ratios of Fibonacci numbers which progressively approach the golden mean $(1 + \sqrt{5})/2$ (≈1.618).

(c) A universal (scale independent) spectrum for suspended atmospheric particulate size distribution expressed as a function of the golden mean τ (≈1.618), the total number concentration and the mean volume radius (or diameter) of the particulate size spectrum. A knowledge of the mean volume radius and total number concentration is sufficient to compute the total particulate size spectrum at any location.

The model prediction at (b) implies that the additive amplitudes of eddies when squared (variance) represent the probability distribution of eddies and such a result is observed in the subatomic dynamics of quantum systems such as the electron or photon. Therefore atmospheric flows and in general turbulent fluid flows follow quantum-like dynamics.

The model predicted probability distribution P (Eq. 2.1) of fluctuation amplitudes is very close to the statistical normal distribution for values of normalized deviation t less than 2 and gives progressively larger values than the normal distribution for normalized deviation t values greater than 2. The statistical normal distribution gives near-zero probability of occurrence of extreme events ($t \gg 2$) while the model predicts appreciable probabilities as observed in practice.

The model predicted probability distribution for fractal fluctuations is shown to be the same as the Boltzmann's distribution for molecular scale energy distribution.

The general system theory predicted inverse power-law distribution for fluctuation amplitudes and variance distribution at Eq. (2.1) is a signature of self-organized criticality in atmospheric flows.

The following list of ten continuous periodogram power spectral analyses of data sets for different meteorological parameters for different time periods show that the variance spectra follow model predicted inverse power-law distribution at Eq. (2.1), a signature of self-organized criticality in atmospheric flows.

(i) Three sets each of 30 years (1871–1900, 1901–1930, 1956–1985) and one set of 25 years (1931–1955) summer monsoon rainfall time series for 29 meteorological subdivisions in the Indian region (Selvam et al. 1992).

(ii) 115 years (1871–1985) summer monsoon rainfall over the Indian region (Selvam 1993).

(iii) Sets of 50–364 daily mean atmospheric columnar total ozone content at 19 globally representative stations (Selvam and Radhamani 1994).

(iv) 28 years (1961–1988) of seasonal (September–November) mean COADS global surface (air and sea) temperature time series (Selvam and Joshi 1995).

(v) Daily values of atmospheric total ozone for five different stations were obtained from Ozone Data for the World (Dept. of Environment) 1988–91. Data sets used for the study are the following six sets of 20 to 25 daily or up to 6 days averages of atmospheric total ozone content. (1) Hobart (Australia): daily values for the period 1–25 February 1991, (2) Melbourne (Australia): 2-day mean values for the period 1 October–19 November 1990, (3) Melbourne (Australia): daily values for the period 1 February–20 February 1991, (4) Tateno (Japan): 6-day mean values for the period 1 March–30 April 1990, (5) Reykjavik (Iceland): 6-day mean values for the period 3 March–30 June 1990, (6) Varanasi (India) 4-day mean values for the period 1 March–19 May 1990 (Selvam and Radhamani 1995).

(vi) Two 50-years (1871–1920 and 1936–1985) of summer monsoon rainfall over the Indian region and one 84-years set (1893–1976) of winter half-year rainfall over England and Wales (Selvam et al. 1995).

(vii) Annual and seasonal mean global surface pressure time series for the 25 years 1964–1988 obtained from the Comprehensive Ocean Atmosphere Data Set (COADS) (Selvam et al. 1996).

(viii) The following data sets of climatological parameters from disparate climatic regimes: (1) Annual and Seasonal means of rainfall: Subdivision wise and total Indian region, 1871–1994 for 124 years, (2) Annual rainfall, England and Wales, 1766–1980 for 215 years, (3) Southern Oscillation Index (SOI): Seasonal, Tahiti−Darwin, 1852–1984 for 133 years, (4) Surface Temperature: Annual and Seasonal, Arctic, 1957–1981 for 25 years (5) Surface Temperature: Annual and Seasonal, Antarctic, 1957–1983 for 27 years (Selvam and Fadnavis 1998).

(ix) Two-day mean 00 GMT upper air (850, 500, and 200 mb) temperature data of TOGA (Tropical Ocean Global Atmosphere) temperature time series for latitude belts from 50°N to 50°S at 5° latitude and longitude intervals for the four seasons MAM, JJA, SON, DFJ respectively for the 5-year period 1986–1990 (Joshi and Selvam 1999).

(x) Global-gridded time series data sets of monthly mean temperatures for the period 1880–2007/2008 (Selvam 2011a).

General systems theory model prediction at (b) above that amplitudes of fractals fluctuations also follows the same inverse power-law form τ^{-4t} (Eq. 2.1) as the variance spectrum, where τ ($\approx$1.618) is the golden mean and t the normalized deviation of the data used for the study is seen in the following study. Historic month-wise temperature (maximum and minimum) and total rainfall for the four stations Oxford, Armagh, Durham and Stornoway in the UK region, for data periods ranging from 92 years to 160 years show that the fluctuations of temperature and rainfall follow the inverse power-law form τ^{-4t}, where τ is the golden mean ($\approx$1.618) and t is the normalized standard deviation equal to mean divided by the standard deviation of the data set (Selvam 2014).

The general systems theory model prediction at (c) above for fractal space-time fluctuations predicts a universal (scale independent) spectrum for suspended atmospheric particulate size distribution expressed as a function of the golden mean τ ($\approx$1.618), the total number concentration and the mean volume radius (or diameter) of the particulate size spectrum (Selvam 2015). A knowledge of the mean volume radius and total number concentration is sufficient to compute the total particulate size spectrum at any location. Model predicted spectrum is in agreement with the following four experimentally determined data sets:

(I) CIRPAS mission TARFOX_WALLOPS_SMPS aerosol size distributions (2) CIRPAS mission ARM-IOP (Ponca City, OK) aerosol size distributions (3) SAFARI 2000 CV-580 (CARG Aerosol and Cloud Data) cloud drop size distributions and (4) TWP-ICE (Darwin, Australia) rain drop size distributions (Selvam 2012a).

(II) Observations for atmospheric aerosol size distribution using VOCALS 2008 PCASP data (Selvam 2012b).

(III) The total averaged radius size spectra for the AERONET (aerosol inversions) stations Davos and Mauna Loa for the year 2010 and Izana for the year 2009 (Selvam 2013).

(IV) The following two experimentally determined atmospheric aerosol data sets, (1) SAFARI 2000 CV-580 Aerosol Data, Dry Season 2000 (CARG) (2) World Data Centre Aerosols data sets for the three stations Ny Ålesund, Pallas and Hohenpeissenberg (Selvam 2011b).

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