The PLC Implementation of Fractional-Order Operator Using CFE Approximation

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Abstract. In the paper an implementation of an elementary fractional order, integro-differential operator at PLC platform is discussed. The considered element is approximated with the use of discrete CFE approximation. The operator we deal with is a crucial part of fractional order PID controller. Guidelines to PLC implementation with the use of object-oriented approach presented by IEC 61131.3 standard are given also. As an example the implementation at SIEMENS SIMATIC S7 1200 platform is presented. As a reference the analytical response of element was applied, the quality of model was estimated with use of typical MSE cost function. Results of experiments show, that the PLC implementation of the fractional order element is possible to make with the use of object-oriented approach and the accuracy of approximation is determined by its order.

Keywords: Fractional order systems · CFE approximation · PLC · IEC61131.3 standard

1 An Introduction

Main areas of application the fractional order calculus in automation are: fractional order control and modeling of processes with dynamics hard to describe with the use of another approaches. Fractional order control covers mainly particularly Fractional Order PID controllers (FO PID). FO PID controllers have been presented by many Authors and their usefulness has been proven (see for example: [4,7,20,22,24]). A PLC implementation of FO controller was presented for example in [23].

However, the practical implementation of FO controllers and models causes a number of problems, generated mainly by the fact, that the fractional order differentiation/integration operator is impossible to exact implementation and it requires to use approximations, possible to digital implementation. It can be done with the use of PSE (Power Series Expansion), CFE (Continuous Fraction Expansion) approximation or discrete version of ORA (Ostaloup Recursive Approximation) approximation.

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PLCs have been a workhorse of industrial automation for many years. Hardware and software of PLC systems are normalized (IEC standard 61131) and their programming platforms offer a powerful tool to implement each control algorithm. However, most implementations cover logic control, sequential control and PID control, although PLC platforms make possible to implement more complex tasks, for example model based control algorithms or model based fault detection systems.

This paper is intented to show possibilities of implementation a basic FO element s^α at PLC with respect to object-oriented approach, described in standard 61131. The considered element is an elementary "brick" to implement many fractional order controllers and models at PLC platform. To implement the CFE approximation was employed. This was caused by the fact that this approximant requires us to use significantly shorter memory length, than PSE (Power Series Expansion) method.

The paper is organized as follows: at the beginning any elementary ideas from non integer order calculus are remembered, next any general remarks about implementation of special control algorithms at PLC platforms with the use of object oriented approach are given. Next the experimental PLC platform using SIEMENS S71200 family is presented and finally experimental results and main conclusions are discussed.

2 Preliminaries

2.1 Elementary Ideas

The presentation of elementary ideas will be started with define a non integer order, integro-differential operator. It is expressed as follows (see for example [12]):

Definition 1. *The non integer order integro - differential operator*

$$ {}_0D_t^\alpha f(t) = \begin{cases} \frac{d^\alpha f(t)}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int\limits_a^t f(\tau)(d\tau)^{-\alpha} & \alpha < 0. \end{cases} \tag{1} $$

where a and t denote time limits to operator calculating, $\alpha \in \mathbb{R}$ denotes the non integer order of the operation.
Next an idea of Gamma Euler function (see for example [13]) can be given:

Definition 2. *The Gamma function*

$$ \Gamma(x) = \int\limits_0^\infty t^{x-1} e^{-t} dt. \tag{2} $$

The fractional-order, integro-differential operator (1) can be described by different definitions, given by Grünvald and Letnikov (GL definition), Riemann and Liouville (RL definition) and Caputo (C definition). The digital modeling of FO operator can be most naturally done with the use of GL definition and it will be presented here:

Definition 3. *The Grünvald-Letnikov definition of the FO operator* [4,19]

$$ {}_0^{GL}D_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{[\frac{t}{h}]} (-1)^j \binom{\alpha}{j} f(t - jh). \tag{3} $$

In (3) $\binom{\alpha}{j}$ is a generalization of Newton symbol into real numbers:

$$ \binom{\alpha}{j} = \begin{cases} 1, & j = 0 \\ \frac{\alpha(\alpha-1)\ldots(\alpha-j+1)}{j!}, & j > 0 \end{cases} \tag{4} $$

Definition 4. *The Riemann - Liouville definition of the FO operator*

$$ {}_0^{RL}D_t^{\alpha} f(t) = \frac{1}{\Gamma(N-\alpha)} \frac{d^N}{dt^N} \int_0^{\infty} (t-\tau)^{N-\alpha-1} f(\tau) d\tau. \tag{5} $$

where $N - 1 < \alpha < N$ *denotes the non integer order of operation and* $\Gamma(..)$ *is the complete Gamma function expressed by* (2).

The Caputo definition is described as underneath:

Definition 5. *The Caputo definition of the FO operator*

$$ {}_0^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(N-\alpha)} \int_0^{\infty} \frac{f^{(N)}(\tau)}{(t-\tau)^{\alpha+1-N}} d\tau. \tag{6} $$

If the RL or C definition is considered, the Laplace transform can be also given (see for example [12]) as a generalization of Laplace transform for integer order case:

Definition 6. *Laplace transform of Riemann - Liouville operator*

$$ \begin{gathered} \mathcal{L}({}_0^{RL}D_t^{\alpha} f(t)) = s^{\alpha} F(s), \quad \alpha < 0 \\ \mathcal{L}({}_0^{RL}D_t^{\alpha} f(t)) = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^k {}_0D_t^{\alpha-k-1} f(0), \\ \alpha > 0, \quad n-1 < \alpha \leq n \in N. \end{gathered} \tag{7} $$

Definition 7. *Laplace transform of Caputo operator*

$$\mathcal{L}({}_0^C D_t^\alpha f(t)) = s^\alpha F(s), \quad \alpha < 0$$
$$\mathcal{L}({}_0^C D_t^\alpha f(t)) = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} {}_0 D_t^k f(0), \tag{8}$$
$$\alpha > 0, \quad n-1 < \alpha \leq n \in N.$$

Consequently an inverse Laplace transform can be given as underneath (see for example [13] p. 29):

$$\mathcal{L}^{-1}[s^\alpha F(s)] = {}_0 D_t^\alpha f(t) + \sum_{k=0}^{n-1} \frac{t^{k-1}}{\Gamma(k-\alpha+1)} f^{(k)}(0^+) \tag{9}$$
$$n-1 < \alpha < n, \quad n \in \mathbb{Z}.$$

For the element s^α, with the use of (9) the analytical form of step response is expressed as underneath:

$$y_{an}(t) = \frac{t^{-\alpha}}{\Gamma(1-\alpha)}. \tag{10}$$

The analytical formula of step response (10) will be interpreted as a standard to estimate an accuracy of tested PLC implementations.

2.2 The CFE Approximation

An implementation of operator (1) at each digital platform (PLC, microcontroller) requires us to apply an integer order, finite dimensional, discrete approximant. The most known are PSE (Power Series Expansion) and CFE (Continuous Fraction Expansion). They allow us to estimate a non integer order element with the use of digital FIR or IIR filter. The PSE approximant bases directly on discrete version of GL definition (3) and it has the form of FIR filter containing only zeros. However its digital implementation to keep a good quality requires us to apply long memory buffer (high order of the filter). The CFE approximant has the form of IIR filter containing both poles and zeros. It is faster covergent and easier to implement because its order is relatively low, typically not higher that 5.

The discretization of fractional order element s^α, $\alpha \in \mathbb{R}$ can be done with the use of the so called generating function $s \approx \omega(z^{-1})$. The new operator raised to power α has the following form (see for example [5,22], p. 119):

$$\begin{aligned}\left(\omega(z^{-1})\right)^\alpha &= \left(\tfrac{1+a}{h}\right)^\alpha CFE\{\left(\tfrac{1-z^{-1}}{1+az^{-1}}\right)^\alpha\}_{M,M} \\ &= \tfrac{P_{\alpha M}(z^{-1})}{Q_{\alpha M}(z^{-1})} = \left(\tfrac{1+a}{h}\right)^\alpha \tfrac{CFE_N(z^{-1},\alpha)}{CFE_D(z^{-1},\alpha)} = \frac{\sum_{m=0}^{M} w_m z^{-m}}{\sum_{m=0}^{M} v_m z^{-m}}.\end{aligned} \tag{11}$$

In (11) a is the coefficient depending on approximation type (for example: $a = 0$ for Euler approximation, $a = 1$ for Tustin approximation), h denotes the sample time, M is the order of approximation. Numerical values of coefficients w_m and v_m and different values of parameter a can be calculated for example with the use of MATLAB function given by Petras in [25]. This MATLAB function was applied in experiments described in the next section If the Tustin approximation is considered ($a = 1$) then $CFE_D(z^{-1}, \alpha) = CFE_N(z^{-1}, -\alpha)$ and the polynomial $CFE_D(z^{-1}, \alpha)$ can be given in the direct form (see [5]). Examples of polynomial $CFE_D(z^{-1}, \alpha)$ for $M = 1, 3, 5$ are given in Table 1.

Table 1. Coefficients of CFE polynomials $CFE_{N,D}(z^{-1}, \alpha)$ for Tustin approximation with respect to [5]

Order M	w_m	v_m
$M = 1$	$w_1 = -\alpha$	$v_1 = \alpha$
	$w_0 = 1$	$v_0 = 1$
$M = 3$	$w_3 = -\frac{\alpha}{3}$ $w_2 = \frac{\alpha^2}{3}$	$v_3 = \frac{\alpha}{3}$ $v_2 = \frac{\alpha^2}{3}$
	$w_1 = -\alpha$	$v_1 = \alpha$
	$w_0 = 1$	$v_0 = 1$
$M = 5$	$w_5 = -\frac{\alpha}{5}$	$v_5 = \frac{\alpha}{5}$
	$w_4 = \frac{\alpha^2}{5}$	$v_4 = \frac{\alpha^2}{5}$
	$w_3 = -\left(\frac{\alpha}{5} + \frac{2\alpha^3}{35}\right)$	$v_3 = -\left(\frac{-\alpha}{5} + \frac{-2\alpha^3}{35}\right)$
	$w_2 = \frac{2\alpha^2}{5}$ $w_1 = -\alpha$	$v_2 = \frac{2\alpha^2}{5}$ $v_1 = \alpha$
	$w_0 = 1$	$v_0 = 1$

The time response of the approximated FO element (11) in k-th time moment is expressed as underneath:

$$y^+_{CFE}(k) = \frac{1}{v_0}\left[-\sum_{m=1}^{M} v_m y^+(k-m) + \sum_{m=0}^{M} w_m u^+(k-m)\right]. \tag{12}$$

where $y^+_{CFE}(k-m)$ and $u^+(k-m)$ denote the output and input signals in k–m-th time moments respectively, v_m and w_m are coefficients of CFE approximation, given for example in Table 1. The Eq. (12) will be directly implemented as function block (FB) at PLC. The use of FB is caused by the fact, that the correct calculation of (12) requires us to know M previous steps of output and control signals and a FB is the smallest Program Organization Unit (POU) assuring the "memory function" for its variables.

The accuracy of approximation (12) will be estimated with the use of typical MSE (Medium Square Error) cost function:

$$MSE = \frac{1}{K_s}\sum_{k=1}^{K_s}\left(y(kh) - y^+_{CFE}(k)\right)^2. \tag{13}$$

where K_s is a number of all collected samples, y is the analytical time response calculated in discrete time steps kh, y^{+}_{CFE} is the time response of approximation, calculated along the same time grid and with respect to (12). If we assume that the input signal $u(t)$ is a Heviside function: $u(t) = 1(t)$, then $y(t) = y_{an}(t)$, where y_{an} is expressed by (10).

3 The Experimental System

3.1 General Remarks About Implementation of Special Control Algorithms at PLC Platform

A general frame of PLC software is given by IEC 61131-3 standard (the last update was published in November 2013). This standard was described by fundamental book [14], the previous version of standard was discussed also in book [11]. The use of formal methods in PLC programming was considered for example in [10], an example of implementation for special, model based control algorithm at SIEMENS soft PLC platform was presented for example in [16].

However the IEC 61131 standard describes all aspects of PLC programming, each real PLC system contains also a number of "specific" details non compatible to it. This must be taken into consideration during each real implementation. For example the system SIEMENS 300/400/1200/1500 provides Organisation Blocks (OB), Data Blocks (DB) and PLC Data types.

The implementation of special control algorithm at PLC should be done with respect to general guidelines formulated in the standard. However in particular situation they are not sufficient and any additional assumptions should also be met:

- Calculations should be divided into smaller, simpler parts, implemented as separated POUs (FC, FB),
- All calculations not necessary while real time control running: values of coefficients, self-tuning, etc., should not be calculated in each programming cycle. The following ideas can be given here:

  The first one is to use SCADA scripts or MATLAB and next send results to a PLC.

  An alternative idea is to calculate these coefficients at PLC once a time (for example during tuning of controller or after parameters change) and save they to a retentive memory.
- Scan time and response time of the PLC should be estimated or tested to avoid time errors.
- Complex mathematical expressions should be written with the use of textual languages (IL, ST), the assembling of the whole program should be done with the use of graphical languages (LD, FBD) or SFC,
- the algorithm must be evaluated with constant sample time T_s, assured by real-time clock of CPU. Detailed methods to assure it depend on particular system. For example at SIEMENS platform this functionality is delivered by organisation blocks OB30-OB38.

3.2 The Hardware

Experiments were done with the use of PLC system shown in Fig. 1. It contains the following elements: PLC SIEMENS 1200 with CPU 1212C, HMI panel SIEMENS KTP400 and industrial switch CSM1277. The system is connected to PC with software SIEMENS TIA PORTAL V13 via PROFINET. All parameters to experiments were introduced with the use of HMI, it was employed also to write results at pendrive in text format.

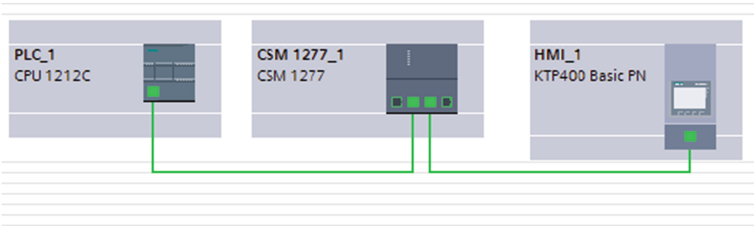


Fig. 1. The experimental system

3.3 The Software

The software implementing the tested FO element was prepared with the use of standard elements available at platform TIA PORTAL v13 with respect to general remarks given above. Main components of the software are described in Table 2, PLC and HMI tags are described in Tables 3 and 4 respectively. All elements of program were connected via PLC tags, which are equivalent to Directly Represented Variables described by IEC61131.3 standard.

4 Experimental Results

Experiments were done with the use of experimental system shown in the previous section. The performance of CFE approximation was estimated with the use of MSE cost function (13). Tests were done for different values of fractional order α and different values of CFE approximation order $M = 1...5$. The Tustin approximation was applied ($a = 1$) with the sample time h equal 1[s], the number of collected samples K_s was equal 50. The positive values of α (differentiator) and negative values (integrator) were tested separately. Values of cost function (13) for all tests are given in Tables 5 and 6, diagrams of step responses are given in Table 7.

Table 2. Software components

POU	Formal name	Symbolic name	Description
Function (FC)	FC1	CFEcalc	Function calculating the coefficients of CFE approximant with respect to [25]
Function Block (FB)	FB1	CFE	Function Block calculating the response of FO element with respect to (12)
Organisation Block (OB)	OB1	Main	Main program organisation block. It contains the conditional call of function CFEcalc. The call is run via button F1 on HMI, results are written in retentive data block DB1
Organisation Block (OB)	OB30	Cyclic interrupt	Organisation block fired by clock interrupt with constant period, equal sample time. It calls the instance of FB1, calculating the step response of tested FO element
Data Block (DB)	DB1	CFEcoeff	Data block saving the calculated coefficients of approximation. It is located in the retentive memory
Data Block (DB)	DB2	CFEDB	Data block associated to Function Block CFE

Table 3. PLC tags (directly represented variables)

Name	Data type	Logical address	Comment
calculate_CFE	Bool	%M0.0	Conditional CFE coefficients calculating
Yout	Real	%MD4	Output signal
Uin	Real	%MD8	Input signal
M	USint	%MB1	Order of CFE approximation
a	Real	%MD12	a coefficient
alfa	Real	%MD24	Fractional order
control_ON	Bool	%M0.1	Input signal ON
Samples_count	Int	%MW128	Number of samples

Table 4. HMI tags

Name	Connection	PLC tag	DataType	Length	Access method	Acquisition mode	Acquisition cycle
a	HMI_Connection_1	a	Real	4	Symbolic access	Cyclic in operation	1 s
M	HMI_Connection_1	M	USInt	1	Symbolic access	Cyclic in operation	1 s
alfa	HMI_Connection_1	alfa	Real	4	Symbolic access	Cyclic in operation	1 s
calculate_CFE	HMI_Connection_1	calculate_CFE	Bool	1	Symbolic access	Cyclic in operation	1 s
control_ON	HMI_Connection_1	control_ON	Bool	1	Symbolic access	Cyclic in operation	1 s
Yout	HMI_Connection_1	Yout	Real	4	Symbolic access	Cyclic in operation	1 s
Uin	HMI_Connection_1	Uin	Real	4	Symbolic access	Cyclic in operation	1 s
pen_removed	<No Value>	<No Value>	Bool	1	<No Value>	Cyclic in operation	1 s
sample_count	HMI_Connection_1	Samples_count	Int	2	Symbolic access	Cyclic in operation	1 s

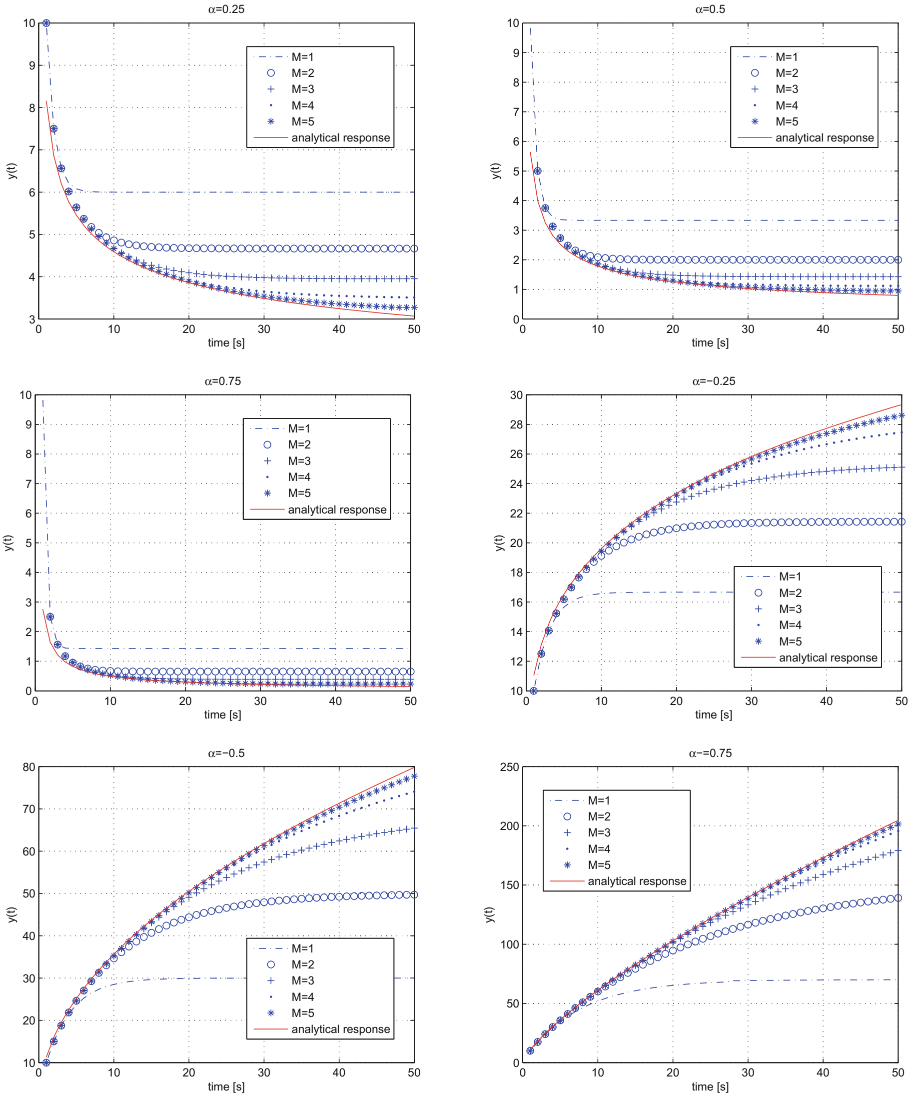
Table 5. MSE cost function (13) for positive values of α

	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
$M = 1$	5.0838	4.6758	2.2865
$M = 2$	1.1845	1.1265	1.2097
$M = 3$	0.3229	0.5522	1.0920
$M = 4$	0.1256	0.4357	1.0720
$M = 5$	0.0885	0.4136	1.0690

Table 6. MSE cost function (13) for negative values of α

	$\alpha = -0.25$	$\alpha = -0.50$	$\alpha = -0.75$
$M = 1$	69.0932	899.0891	5281.0921
$M = 2$	19.7334	235.8201	906.2536
$M = 3$	4.1168	40.2426	109.7010
$M = 4$	0.6651	5.3124	11.6897
$M = 5$	0.1263	0.7404	1.4017

Table 7. The step responses: of the plant (red line) and of model with different α and different orders M of CFE approximant



5 Conclusions

Final conclusions from the paper can be formulated as underneath:

– The elementary fractional order plant, expressed by s^{α} transfer function can be implemented at PLC platform with the use of normalized software tools,

– the accuracy of model is determined by the order of approximation: higher order gives the better accuracy,
– the use of CFE approximant allows us to obtain properly working fractional order element with sensible order. This can be pointed as advantage of this method in contrast to PSE approximation, where similar accuracy requires us to use much more higher (and complex) model.

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