

Contents

1	Introduction	1
1.1	Main Application of the Theory of GLT Sequences.	1
1.2	Overview of the Theory of GLT Sequences.	4
2	Mathematical Background	7
2.1	Notation and Terminology	7
2.2	Preliminaries on Measure and Integration Theory	10
2.2.1	Essential Range.	10
2.2.2	L^p Spaces	12
2.2.3	Convergence in Measure, a.e., in L^p	14
2.2.4	Riemann-Integrable Functions.	19
2.3	Preliminaries on General Topology	20
2.3.1	Pseudometric Spaces.	20
2.3.2	The Topology τ_{measure} of Convergence in Measure	22
2.4	Preliminaries on Matrix Analysis	27
2.4.1	p -norms.	27
2.4.2	Singular Value Decomposition	29
2.4.3	Schatten p -norms	31
2.4.4	Singular Value and Eigenvalue Inequalities	34
2.4.5	Tensor Products and Direct Sums.	40
2.4.6	Matrix Functions.	41
3	Singular Value and Eigenvalue Distribution of a Matrix-Sequence	45
3.1	The Notion of Singular Value and Eigenvalue Distribution	45
3.2	Rearrangement	47
3.3	Clustering and Attraction	49
3.4	Zero-Distributed Sequences	52

4	Spectral Distribution of Sequences of Perturbed Hermitian Matrices	57
4.1	Preliminary Results	57
4.2	Main Results	62
5	Approximating Classes of Sequences	65
5.1	The a.c.s. Notion	65
5.2	The a.c.s. Topology $\tau_{a.c.s.}$	66
5.2.1	Construction of $\tau_{a.c.s.}$	67
5.2.2	Expression of $d_{a.c.s.}$ in Terms of Singular Values	71
5.2.3	Connection Between $\tau_{a.c.s.}$ and τ_{measure}	72
5.3	The a.c.s. Tools for Computing Singular Value and Eigenvalue Distributions	74
5.4	The a.c.s. Algebra	83
5.5	Some Criteria to Identify a.c.s.	88
5.6	An Extension of the Concept of a.c.s.	92
6	Toeplitz Sequences	95
6.1	Toeplitz Matrices and Toeplitz Sequences	95
6.2	Basic Properties of Toeplitz Matrices	97
6.3	Schatten p -norms of Toeplitz Matrices	100
6.4	Circulant Matrices	106
6.5	Singular Value and Spectral Distribution of Toeplitz Sequences: An a.c.s.-Based Proof	108
6.6	Extreme Eigenvalues of Hermitian Toeplitz Matrices	111
7	Locally Toeplitz Sequences	115
7.1	The Notion of LT Sequences	115
7.2	Properties of the LT Operator	121
7.3	Fundamental Examples of LT Sequences	125
7.3.1	Zero-Distributed Sequences	125
7.3.2	Sequences of Diagonal Sampling Matrices	126
7.3.3	Toeplitz Sequences	130
7.4	Singular Value and Spectral Distribution of a Finite Sum of LT Sequences	133
7.5	Algebraic Properties of LT Sequences	135
7.6	Characterizations of LT Sequences	136
8	Generalized Locally Toeplitz Sequences	143
8.1	Equivalent Definitions of GLT Sequences	143
8.2	Singular Value and Spectral Distribution of GLT Sequences	144
8.3	Approximation Results for GLT Sequences	146
8.3.1	Characterizations of GLT Sequences	151
8.3.2	Sequences of Diagonal Sampling Matrices	152

- 8.4 The GLT Algebra. 154
- 8.5 Algebraic-Topological Definitions of GLT Sequences 163
- 9 Summary of the Theory** 165
- 10 Applications.** 173
 - 10.1 The Algebra Generated by Toeplitz Sequences 173
 - 10.2 Variable-Coefficient Toeplitz Sequences. 175
 - 10.3 Geometric Means of Matrices 183
 - 10.4 Discretization of Integral Equations 185
 - 10.5 Finite Difference Discretization of Differential Equations. 189
 - 10.5.1 FD Discretization of Diffusion Equations 191
 - 10.5.2 FD Discretization of Convection-Diffusion-Reaction Equations 198
 - 10.5.3 FD Discretization of Higher-Order Equations 210
 - 10.5.4 Non-uniform FD Discretizations 212
 - 10.6 Finite Element Discretization of Differential Equations 218
 - 10.6.1 FE Discretization of Convection-Diffusion-Reaction Equations 218
 - 10.6.2 FE Discretization of a System of Equations 225
 - 10.7 Isogeometric Analysis Discretization of Differential Equations 229
 - 10.7.1 B-Spline IgA Collocation Discretization of Convection-Diffusion-Reaction Equations. 229
 - 10.7.2 Galerkin B-Spline IgA Discretization of Convection-Diffusion-Reaction Equations. 244
 - 10.7.3 Galerkin B-Spline IgA Discretization of Second-Order Eigenvalue Problems 254
- 11 Future Developments** 261
- 12 Solutions to the Exercises.** 265
- References.** 299
- Index** 305



<http://www.springer.com/978-3-319-53678-1>

Generalized Locally Toeplitz Sequences: Theory and
Applications

Volume I

Garoni, C.; Serra-Capizzano, S.

2017, XI, 312 p. 16 illus. in color., Hardcover

ISBN: 978-3-319-53678-1