# Preface

## Purpose

Our objective is to provide a post-calculus introduction to the subject of probability that

- Has mathematical integrity and contains some underlying theory
- Shows students a broad range of applications involving real problem scenarios
- Is current in its selection of topics
- Is accessible to a wide audience, including mathematics and statistics majors (yes, there are a few of the latter, and their numbers are growing), prospective engineers and scientists, and business and social science majors interested in the quantitative aspects of their disciplines
- Illustrates the importance of software for carrying out simulations when answers to questions cannot be obtained analytically

A number of currently available probability texts are heavily oriented toward a rigorous mathematical development of probability, with much emphasis on theorems, proofs, and derivations. Even when applied material is included, the scenarios are often contrived (many examples and exercises involving dice, coins, cards, and widgets). So in our exposition we have tried to achieve a balance between mathematical foundations and the application of probability to real-world problems. It is our belief that the theory of probability by itself is often not enough of a "hook" to get students interested in further work in the subject. We think that the best way to persuade students to continue their probabilistic education beyond a first course is to show them how the methodology is used in practice. Let's first seduce them (figuratively speaking, of course) with intriguing problem scenarios and applications. Opportunities for exposure to mathematical rigor will follow in due course.

## Content

The book begins with an Introduction, which contains our attempt to address the following question: "Why study probability?" Here we are trying to tantalize students with a number of intriguing problem scenarios—coupon collection, birth and death processes, reliability engineering, finance, queuing models, and various conundrums involving the misinterpretation of probabilistic information (e.g., Benford's Law and the detection of fraudulent data, birthday problems, and the likelihood of having a rare disease when a diagnostic test result is positive). Most of the exposition contains references to recently published results. It is not necessary or even desirable to cover very much of this motivational material in the classroom. Instead, we suggest that instructors ask their students to read selectively outside class (a bit of pleasure reading at the very beginning of the term should not be an undue burden!). Subsequent chapters make little reference to the examples herein, and separating out our "pep talk" should make it easier to cover as little or much as an instructor deems appropriate.

Chapter 1 covers sample spaces and events, the axioms of probability and derived properties, counting, conditional probability, and independence. Discrete random variables and distributions are the subject of Chap. 2, and Chap. 3 introduces continuous random variables and their distributions. Joint probability distributions are the focus of Chap. 4, including marginal and conditional distributions, expectation of a function of several variables, correlation, modes of convergence, the Central Limit Theorem, reliability of systems of components, the distribution of a linear combination, and some results on order statistics. These four chapters constitute the core of the book.

The remaining chapters build on the core in various ways. Chapter 5 introduces methods of statistical inference—point estimation, the use of statistical intervals, and hypothesis testing. In Chap. 6 we cover basic properties of discrete-time Markov chains. Various other random processes and their properties, including stationarity and its consequences, Poisson processes, Brownian motion, and continuous-time Markov chains, are discussed in Chap. 7. The final chapter presents some elementary concepts and methods in the area of signal processing.

One feature of our book that distinguishes it from the competition is a section at the end of almost every chapter that considers simulation methods for getting approximate answers when exact results are difficult or impossible to obtain. Both the R software and Matlab are employed for this purpose.

Another noteworthy aspect of the book is the inclusion of roughly 1100 exercises; the first four core chapters together have about 700 exercises. There are numerous exercises at the end of each section and also supplementary exercises at the end of every chapter. Probability at its heart is concerned with problem solving. A student cannot hope to really learn the material simply by sitting passively in the classroom and listening to the instructor. He/she must get actively involved in working problems. To this end, we have provided a wide spectrum of exercises, ranging from straightforward to reasonably challenging. It should be easy for an instructor to find enough problems at various levels of difficulty to keep students gainfully occupied.

## Mathematical Level

The challenge for students at this level should be to master the concepts and methods to a sufficient degree that problems encountered in the real world can be solved. Most of our exercises are of this type, and relatively few ask for proofs or derivations. Consequently, the mathematical prerequisites and demands are reasonably modest. Mathematical sophistication and quantitative reasoning ability are, of course, crucial to the enterprise. Univariate calculus is employed in the continuous distribution calculations of Chap. 3 as well as in obtaining maximum likelihood estimators in the inference chapter. But even here the functions we ask students to work with are straightforward—generally polynomials, exponentials, and logs. A stronger background is required for the signal processing material at the end of the book (we have included a brief mathematical appendix as a refresher for relevant properties). Multivariate calculus is used in the section on joint distributions in Chap. 4 and thereafter appears rather rarely. Exposure to matrix algebra is needed for the Markov chain material.

## Recommended Coverage

Our book contains enough material for a year-long course, though we expect that many instructors will use it for a single term (one semester or one quarter). To give a sense of what might be reasonable, we now briefly describe three courses at our home institution, Cal Poly State University (in San Luis Obispo, CA), for which this book is appropriate. Syllabi with expanded course outlines are available for download on the book's website at Springer.com.

| Title: | *Introduction to Probability and Simulation* | *Introduction to Probability Models* | *Probability and Random Processes for Engineers* |
|---|---|---|---|
| Main audience: | Statistics and math majors | Statistics and math majors | Electrical and computer engineering majors |
| Prerequisites: | Univariate calculus, computer programming | Univariate calculus, computer programming, matrix algebra | Multivariate calculus, continuous-time signals incl. Fourier analysis |
| Sections covered: | 1.1–1.6 | 1.1–1.6 | 1.1–1.5 |
| | 2.1–2.6, 2.8 | 2.1–2.5, 2.8 | 2.1–2.5 |
| | 3.1–3.4, 3.8 | 3.1–3.4, 3.8 | 3.1–3.5 |
| | 4.1–4.3, 4.5 | 4.1–4.3, 4.5, 4.8 | 4.1–4.3, 4.5, 4.7 |
| | | 6.1–6.5 | 7.1–7.3, 7.5–7.6 |
| | | 7.5 | 8.1–8.2 |

Both of the first two courses place heavy emphasis on computer simulation of random phenomena; instructors typically have students work in R. As is evident from the lists of sections covered, *Introduction to Probability Models* takes the earlier material at a faster pace in order to leave a few weeks at the end for Markov chains and some other applications (typically reliability theory and a bit about Poisson processes). In our experience, the computer programming prerequisite is essential for students' success in those two courses.

The third course listed, *Probability and Random Processes for Engineers*, is our university's version of the traditional "random signals and noise" course offered by many electrical engineering departments. Again, the first four chapters are covered at a somewhat accelerated pace, with about 30–40% of the course dedicated to time and frequency representations of random processes (Chaps. 7 and 8). Simulation of random phenomena is not emphasized in our course, though we make liberal use of Matlab for demonstrations.

We are able to cover as much material as indicated on the foregoing syllabi with the aid of a not-so-secret weapon: we prepare and require that students bring to class a course booklet. The booklet contains most of the examples we present as well as some surrounding material. A typical example begins with a problem statement and then poses several questions (as in the exercises in this book). After each posed question there is some blank space so the student can either take notes as the solution is developed in class or else work the problem on his/her own if asked to do so. Because students have a booklet, the instructor does not have to write as much on the board as would otherwise be necessary and the student does not have to do as much writing to take notes. Both the instructor and the students benefit.

We also like to think that students can be asked to read an occasional subsection or even section on their own and then work exercises to demonstrate understanding, so that not everything needs to be presented in class. For example, we have found that assigning a take-home exam problem that requires reading about the Weibull and/or lognormal distributions is a good way to acquaint students with them. But instructors should always keep in mind that there is never enough time in a course of any duration to teach students all that we'd like them to know. Hopefully students will like the book enough to keep it after the course is over and use it as a basis for extending their knowledge of probability!

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## A Final Thought

It is our hope that students completing a course taught from this book will feel as passionately about the subject of probability as we still do after so many years of living with it. Only teachers can really appreciate how gratifying it is to hear from a student after he/she has completed a course that the experience had a positive impact and maybe even affected a career choice.

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