The Behavior of the Traction Power Supply System of AC 25 kV 50 Hz During Operation

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Abstract This paper studies the AC 25 kV 50 Hz traction power supply system, which is used, in particular, in the Czech Republic. Nowadays, railway operation is very complicated and sophisticated, both from the viewpoint of railway infrastructure and means of transport. New technologies, devices and standards bring new problems for rail operation, including coupling to surrounding elements in the traction power supply system, transient effects during the recuperation mode of traction vehicles, the influence of neighboring track contact lines when disconnecting contact lines, etc. The main findings of these problems are detailed in this paper.

Keywords Traction power supply systems · Transient effects · Recuperation · Short-circuit

1 Introduction

Rail transport uses new technologies and devices in its infrastructure and transport systems for improving the safety, comfort, and speed of services. Therefore the AC 25 kV 50 Hz traction power supply system, which, in addition to other necessary
systems for signaling, safety and communication systems, is used in the Czech Republic, has some specific characteristics during operation. One of these is transient effects, which can lead to non-standard dangerous conditions in the traction systems. It is necessary to maintain the standard requirements for all devices and equipment during their operation. For this reason, related engineering departments at the University of Pardubice have focused their research activities on studying the behavior of the traction system during faults or selected operation states. Therefore, it was necessary to analyze the particular elements and parts of the AC 25 kV 50 Hz traction power supply system and their couplings. The above-mentioned Czech traction power supply system is a little different from other traction systems. This system uses a specific filter-compensation device (FCD) in traction substations.

2 Configuration of the Traction Power Supply System

The general configuration of the Czech AC 25 kV 50 Hz traction power supply system with rail vehicles for transport in Fig. 1 contains the following:

- contractor feeding line of 110 kV,
- traction substation with FCD,
- catenary (the whole structure—contact line) and
- electric vehicles.

The electromagnetic compatibility is discussed more and more. Therefore, usage of the FCD in traction substations is necessary. This FCD is utilized for power factor corrections and to reduce current harmonics caused by older electric

![Diagram of Traction Power Supply System](image)

**Fig. 1** The AC 25 kV 50 Hz traction power supply system in the Czech Republic
single-phase locomotives with diode converters, which are still very common on Czech railways, see Figs. 2 and 3. This FCT during transient effects also influences the behavior of the traction system, see Fig. 4.

**Fig. 2** Locomotive type 242 with a diode rectifier, \((\text{DPF} \cong 0.84)\) and locomotive type 263 with a thyristor-diode bridge, \((\text{DPF} \cong 0.77)\)

**Fig. 3** Locomotive type 210 with a thyristor controller and uncontrolled rectifier, \((\text{DPF} \cong 0.76)\) and locomotive type 362 with an input converter filter, \((\text{DPF} \cong 0.87)\)

**Fig. 4** Locomotive pantograph showing voltage at the 3rd harmonic
Electric locomotives with diode rectifiers generate all odd current harmonics (i.e. 3rd, 5th, 7th, and so on), see Fig. 5. The current harmonics pass through a catenary, traction substation transformer, a 110 kV contractor feeding line and then to the main power supply system.

The harmonics spectrum (i.e. harmonic numbers), which depends on types of rectifier connections, is formulated by equation

$$N = (p \cdot n) \pm 1,$$

where

- $p$ is pulse number of rectifier ($-$)
- $n$ integer number 1, 2, … $n$

The harmonics pass through the catenary section, independently of the impedance of the external main power supply system, and then they pass through a series of alternate impedance of the traction substation transformer, which are changed only by transmission ratio of this transformer. According to Ohm’s law, voltage harmonics originate from the input alternate impedance of the external main power supply system. The harmonic currents of various frequencies cause a voltage drop from the impedance of the main power supply system and voltage deformation [1, 2].

The direct results from the above lead to a

- rise in network losses,
- drop of active power supply, thereby resulting in an efficiency drop.

These direct results can create others problems with their system configurations

- Creation of system resonance, which usually produces increased current or voltage.
- Faulty function of protections, measuring equipment and registering equipment.
- Interference of telecommunication equipment and control circuits.
The requirements for the FCD are given according to [1–3]:

- Adjust to inductive power factor of fundamental harmonic of traction consumption of single-phase locomotives to the required value of contractor (i.e. with inductive character) in connecting the point of traction substation to guarantee sufficient compensating power.

Minimize transfer of current harmonics at the 3rd, 5th and, perhaps even, the 7th harmonic numbers to corresponding components in voltage of the connecting point of the traction substation were under the minimum values required by the contractor.

- Guarantee input impedance of the traction substation as complex (i.e. including catenary capacity and traction consumption of single-phase locomotives) for the operating frequency of the centralized ripple control of the contractor was higher than the required value (i.e. the level needed to prevent a drop of this operating frequency $f_{CRC} = 216.67$ Hz at the connecting point of the traction substation).

- These conditions have to be realized in all traction load ranges of the traction substations, according to the principle of a single-way power supply of the catenary section.

The FCD is designed under these conditions, see Fig. 6. The FCD contains two parallel series LC branches of the 3rd and the 5th harmonics with parallel connecting decompensation branches. The LC branches’ tuning does not consist of order number of harmonic exactly, but it consists of low-order values as $n_3 = 2.90–2.95$ and $n_5 = 4.98–5.00$. The required sufficient total input impedance $Z = 500–900$ Ω for $f_{CRC}$ are realized by the suitable option of $C_3$ and $C_5$ values in branches; this ensures that they are dependent on each other. A disconnecting switch connects the 5th harmonic LC branch, thereby the filtration requirement, which has to be started at the lowest number of harmonic, is carried out. The FCD structure provides to the 7th harmonic LC branch.

The decompensation branch comes with or without a reducing transformer, thyristor phase controller and decompensation chokes. Decompensation is handled by a decompensation choke, which is controlled. Thus control is realized with inductive power factor $DPF = 0.98$ of input power, which is measured at the

![Fig. 6 The structure of the FCD](image-url)
connection point of traction power supply substation. The creation of additional harmonics (i.e. primarily 3rd harmonics) into voltage of 27 kV bus is increased by partial controlling of the decompensation branch controller. The sum of both the 3rd harmonics controller and system are obtained amid congestion of the 3rd harmonic LC branch. Thus, LC branch tuning for FCD is under frequency 150 Hz.

3 Mathematical Model of Traction Power Supply System

Transient effects are analyzed at the linear systems that are defined equations. It was necessary to avoid building a physical model of a traction power supply system, because it would be costly or lead to the loss of process monitoring ability and the transient effects of circuit behavior. Therefore, we chose the program PSpice for mathematical simulation. PSpice utilizes input data to create a mathematical model of traction circuit elements, which represent the AC 25 kV 50 Hz traction power supply system. It was necessary to create quality models that represent real devices, because the simulation results could be distorted by unsuitable substitutions or simplifications of the traction circuit. The mathematical simulation results can be as exact as model elements and describe only effects that present using models. A creation of quality models that represent real devices well is the most important and most complicated problem of simulations of electronic circuits. Therefore mathematical models of the traction circuit were made for all parts of the AC 25 kV 50 Hz traction power supply system in the Czech Republic, as described below.

3.1 Model of Contractor Feeding Line of 110 kV

The 110 kV contractor feeding line and the catenary have the same character as that of a homogenous line with distributed electrical parameters. They can be considered to be a long electric line, which can be substituted in a two-port network as \( \pi \)-element or T-element with distributed electrical parameters or electrical long line with the following parameters: series specific resistance \( R_s \), series specific inductance \( L_s \), parallel specific capacity \( C_s \), parallel specific leakage \( G_s \) [4].

The model of the 110 kV feeding line was based on a long homogenous electric line with distributed specific electrical parameters inductance \( L_L \), capacity \( C_L \), and resistance \( R_L \) without line leakage \( G_L \), Fig. 7. The specific electrical parameters of the 110 kV line of depend on construction and materials used. A standard three-phase overhead line represents this line.

This d prevents industrial interference of voltage or current and non-symmetrical lines.
3.2 Model of Traction Substation

The traction substation contains the 110/27 kV traction transformer with 10 MVA and the FCD. The 110/27 kV traction transformer can be presented only by one series for 50 Hz. The inductance $L_{TT}$ is given a short-circuit voltage for the traction transformer and series resistance $R_{TT}$, which represents active losses. The values of alternate series inductance depend on the used tap of the transformer, because the transformer ratio can be a little bit different for each transformer, see Fig. 8. These transformers have a wide regulation range of output voltage (i.e. $2 \times 8$ taps), which can be changed under power. Current harmonics pass through the traction transformer, and they are changed only by the used winding ratio.

The characteristic parameters of model transformer are

- short-circuit active losses 53 kW,
- series inductance $L_{TT} = 24$ mH,
- substitute resistance $R_{TT} = 0.39$ $\Omega$.

Two series LC branches of the 3rd and the 5th harmonic and the decompensation branch represent the model of the FCD. The tuning of the LC branches is not adjusted to the number of the harmonic exactly, but it has to set at a lower value. This adjustment of the LC branches is necessary, because harmonics from the 110 kV feeding line could overload these LC branches. The FCD final parameters
are set according to the location of the traction substation and local parameters, so that the FCD power range can be from 1 to 8 MVA. Refer to the example of the FCD in the traction substation [5]:

The 3rd harmonic LC branch and the 5th harmonic LC branch

- Total capacity $C_3 = 8.5 \ \mu F$ and $C_5 = 2.4 \ \mu F$
- Choke inductance $L_3 = 137 \ \text{mH}$ and $L_3 = 169 \ \text{mH}$
- Choke resistance $R_3 = 1.43 \ \Omega$ and $R_5 = 1.77 \ \Omega$
- Resonance frequency $f_3 = 147.5 \ \text{Hz}$ and $f_5 = 249.9 \ \text{Hz}$

Decompensation branch

- Reducing transformer 27/6 kV
- Air-core choke
- Decompensation branch at site 27 kV with total inductance $L_{\text{DEC}} = 0.596 \ \text{H}$ and resistance $R_{\text{DEC}} = 6.24 \ \Omega$
- Phase controller COMPACT, its control angle is calculated from values of instrument voltage transformer and instrument current transformer, so in order to values of power factor will be $\text{DPF} = 0.98$. This value is measured at the connecting point of the traction substation and the 110 kV contractor feeding line.

3.3 Model of Catenary

The catenary is also an electrical homogenous line with distributed electrical parameters, and it can be presented as a long electrical line [6, 7]. This presumption can be taken, because sections of the catenary are longer in comparison with sections of the station catenary. As mentioned previously, the model of the homogenous line has four parameters: series specific resistance $R_{\text{CL}}$, series specific inductance $L_{\text{CL}}$, parallel specific capacity $C_{\text{CL}}$, and parallel specific leakage $G_{\text{CL}}$. The $G_{\text{CL}}$ leakage of the catenary and $G_{\text{CL}}$ leakages of other lines, which are connected with the catenary, are left out of the calculations, because they have very high values. Line insulators provide excellent isolation of the contact line [8, 9].

Specific resistance $R_{\text{CL}}$ and specific inductance $L_{\text{CL}}$, which are dependent on frequency, enter into the calculation. The current, which passes through the conductor, is pushed out on conductor surface (i.e. skin-effect) by the increasing frequency. Then the useful section of conduction (i.e. effective section of conduction) is decreased and specific resistance $R_{\text{CL}}$ is increased. The current is decreased by the skin-effect, so the loop area decreases, too. The specific inductance $L_{\text{CL}}$ decreases until the definite frequency, where remains constant, see Fig. 9. The specific capacity $C_{\text{CL}}$, which consists of the capacity of all conductors that have traction voltage, is measured by the returned line, which represents the ground. Its numerical values will depend on the number of conductors, their height, their external diameter and the configuration of the neighboring of electrified railway track (tunnel, railway cutting, railway embankment, station and so on).
The model of the catenary with a standard structure of 100 Cu + 50 Bz, see Fig. 10, with an intensive line has the same characteristics as a homogenous long electric line with distributed electrical parameters, but there are main parameters (i.e. resistance, inductance, capacity and leakage), see Fig. 11. This presumption can be taken, because sections of the catenary are longer in comparison with the sections of the station catenary. The parameter values used for this model are (for the whole length of catenary)

- series specific resistance $R_{CL} = 0.4 \ \Omega \ \text{km}^{-1}$,
- series specific inductance $L_{CL} = 1.0 \ \text{mH} \ \text{km}^{-1}$,
- parallel specific capacity $C_{CL} = 20 \ \text{nF} \ \text{km}^{-1}$ (with intensive line),
- parallel specific leakage $G_{CL} = 0 \ \text{S} \ \text{km}^{-1}$.

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Fig. 9 The dependence of $L$ and $R$ parameters on frequency

Fig. 10 The main structure of the catenary

Fig. 11 The model of the catenary
3.4 Model of Electric Locomotive

The electric locomotive is one of the most complicated parts, due to the variable parameters, because the model parameters are changed during locomotive operation [10–13]. The model of the traction electric vehicle is represented by the power source with waveforms corresponding to recuperation vehicle with semiconductor converter and recuperation power 0.5 MW.

4 Analysis of Transient Effects

4.1 Short-Circuit at Traction Substation

See Fig. 12.

The current $I_{CL\_beginning}$ in the catenary comes out from the initial value 9.8 A, which represents a value of capacitive current passing through the catenary. The peak value of this current in the catenary is 146 A, see Fig. 13. The series inductances of the catenary cannot be used at the shorted current, because they have low values. The current is divided among parallel capacities. The initial value of the capacitive current has simulated value of 9.8 A. It has a higher calculated value, 9.75 A. This difference is caused by the accuracy of simulation models. The arrival of the wave at the end of the catenary, which is not mismatched, is reflected. The time of the passing wave (i.e. delay time) to one direction for selected section of catenary is given by the equation

$$TD = l_{CL} \cdot \sqrt{L_{CL} \cdot C_{CL}} = 206 \mu s,$$

(2)
where

\( l_{CL} \)  

is selected section length of catenary (km), \( l_{CL} = 53.2 \text{ km} \)

\( L_{CL} \)  

series specific inductance of catenary (mH km\(^{-1}\)), \( L_{CL} = 1.0 \text{ mH km}^{-1} \)

\( C_{CL} \)  

specific capacity of catenary (nF km\(^{-1}\)), \( C_{CL} = 15 \text{ nF km}^{-1} \).

The time of the passing wave in the catenary takes 412 \( \mu \text{s} \) both directions (i.e. from the beginning of the catenary to the end of the catenary and from the end of the catenary to the beginning of the catenary). Voltage \( V_{CL\_end} \) at the end of catenary comes out from initial value 38.9 kV and this value does not get again, Fig. 13. The whole effect subsides after 1.6 s. The character of short-circuit and passage time of the wave are the same for various types of FCD connections.

### 4.2 Short-Circuit at the End of Catenary

See Fig. 14.

The current \( I_{CL\_beginning} \) in catenary comes out from the initial value 9.8 A and its peak value gets 1.19 kA. The voltage \( V_{CL\_beginning} \) at the beginning of the catenary comes out from the initial value 38.9 kV. Its value gets 90.3 kV in time \( c. 620 \text{ ms} \) which depends on section of catenary (in this case) due to reflection of wave at the end of the open catenary then voltage falls consecutively, Fig. 15. Theoretically, peak voltage values at output of the traction substation can be achieved threefold the peak value for traction voltage (i.e. 116.7 kV). This value is obtained in triple the time of the wave passage. The catenary as a long line is ended inductance, which is represented substitutional inductance of traction transformer.
$L_{TT} = 24$ mH. Internal impedance of source (i.e. 38.9 kV) can be considered as zero impedance. This inductance seems to be an infinite impedance during a few milliseconds [14].

Wave reflection impedance of the traction substation does not depend on the number of LC branches of the FCD, because the traction substation in terms of the catenary, consists of the following parallel inductances: inductance of the traction transformer $L_{TT} = 24$ mH, inductance of the 3rd harmonic LC branch $L_3 = 137$ mH and inductance of the 5th harmonic LC branch $L_5 = 169$ mH.

The wave comes to the open end of the homogenous line it reflects with the same polarity as the original wave. Unlike the original wave, the wave coming to the
shorted end of the homogenous line is reflected with a reversed polarity. The reflection wave has the same polarity and the same amplitude. After short-circuiting, it is possible to suppose a constant value of voltage during a few milliseconds at the traction substation. Wave passage time is the sum of the following:

- first passage time of wave from the shorted end of the catenary to the traction substation,
- second passage time of wave from the traction substation to the shorted end of the catenary,
- third passage time of wave back to the traction substation.

### 4.3 Short-Circuit at the End of Catenary as One Section

See Figs. 16, 17 and 18.

**Fig. 16** The circuit diagram at short-circuit of the end of the catenary as one section

**Fig. 17** The voltage for the catenary as one section
4.4 **Short-Circuit at the End of the Catenary as Two Symmetrical Sections**

See Figs. 19 and 20.

4.5 **Short-Circuit at the End of Catenary as Two Non-symmetrical Sections**

See Figs. 21, 22 and 23.

4.6 **Short-Circuit at the End of the Catenary as Sections of a Railway Station**

See Figs. 24, 25 and 26.
Fig. 19 The circuit diagram at short-circuit of the end of the catenary as two symmetrical sections

Fig. 20 The voltage at the traction substation and the voltage at the end of catenary A after a short-circuit at the end of catenary B
Fig. 21 The traction circuit diagram by short-circuit at the end of the catenary as two non-symmetrical sections

Fig. 22 The voltage at traction substation and the voltage at the end of the catenary A after short-circuit at the end of the catenary B
4.7 Findings of Transient Effects During Short-Circuits

- The electrical values of output voltage of the traction substation depend on the original conditions of short-circuit at the catenary. The short-circuit origin is always maximized by traction voltage. The presented cases of short-circuits are made without any traction consumption.
- Regarding overvoltage, the worst case is when the catenary is represented by one section. In this case, the voltage surge is reflected at the traction substation. Theoretically, traction substation voltage can rise up to threefold higher than traction voltage because of wave interference. The catenary as a long lossy line is ended by inductance represented by inductance of the traction transformer $L_{TT} = 24 \text{ mH}$. Internal impedance of the 38.9 kV source can be considered as zero impedance. This inductance seems to be infinite impedance during a few milliseconds. The reflection of the voltage surge on impedance of the traction substation does not depend on the number of LC branches of the FCD, because the traction substation from the viewpoint of catenary consists of parallel inductances which are: traction transformer inductance with $L_{TT} = 24 \text{ mH}$, inductance of the 3rd harmonic LC branch with $L_3 = 137 \text{ mH}$ and inductance of the 5th harmonic LC branch with $L_5 = 169 \text{ mH}$. The real peak of voltage at output of traction substation is about 80 kV due to line losses that are higher than in the simulation model.

Fig. 23 The comparison of voltage for symmetrical and non-symmetrical sections
Fig. 24 The circuit diagram at short-circuit of the end of the catenary with railway station
Fig. 25  The voltage for catenary sections with a railway station

Fig. 26  The current for catenary sections with a railway station
The restoration of traction voltage is achieved by transient effect after reflection of the voltage surge. This transient effect depends on the parameters of the catenary and the whole contact line, parameters of the 110 kV contractor supply line and the 110/27 kV traction transformer. The voltage surge, which comes at the short-circuit end of the catenary, reflects at this end of catenary with inversed polarity. This reflected wave comes to the open end of the catenary, where it reflects with the same polarity again. In this time, this wave can achieve triple peak value of traction voltage.

A similar situation occurs when the wave comes at catenary sections at the railway station. The voltage surge can triple the peak value of the traction voltage theoretically as well. The problem occurs at separate voltage surges at station sections. Overvoltage does not depend on the number of station tracks, but it depends on the station tracks length.

5 Analysis of Transient Effects During Recuperation

The efficiency of traction energy usage is increasingly discussed in rail transport. This is related to the usage of energy from recuperation braking, which can be consumed by an electric rail vehicle or transformed to the 110 kV contractor mean network. The recuperation energy can be represented by units of MW during units of minutes. These energy sources in the traction power supply system bring new requirements for protection settings for maintaining standards and operational regulations. For understanding of behavior of traction circuits during these transient effects the mathematical simulations are used and allow one to monitor the states and points of traction circuits, which cannot be monitored during operation conditions in real traction circuit (e.g. identifying the root causes of short-circuits) [13, 15–17]. Traction circuit of the AC 25 kV 50 Hz traction power supply system for this study effect contains the main elements, Fig. 27.

All elements of the traction circuit can affect the behavior of the whole traction systems and each section of electrified track has its own specific characteristics. The recuperation of the 110 kV network, which is not allowed for all traction substations in the CR network, is solved on the side of contractor [14–16, 18]. The utilized models of the traction circuit made for all parts of the AC 25 kV traction power supply system have similar problems with transient effects.

Model of feeding line 110 kV is based on a homogenous long electric line with distributed electrical parameters (i.e. inductance, capacity and resistance), without line leakage. This line is as standard three-phase overhead line. This means it is without industrial interference of voltage or current and has a non-symmetrical character of line.

The model of the traction substation contains the 110/27 kV traction transformer with 10 MVA and filter-compensation device (FCD). The model of the 110/27 kV traction transformer can be presented by a series inductance \( L_{TT} \) in energetic
harmonic area, which is given short-circuit voltage of traction transformer and series resistance $R_{TT}$ represents active losses. In the case of higher harmonics, it is necessary to add a passive filter with inductance and series resistance. The current harmonics pass through traction transformer and they are changed by using only a winding ratio. The characteristic parameters of the model transformer are short-circuit active losses of 53 kW, series inductance $L_{TT} = 28$ mH, resistance $R_{TT} = 0.42$ Ω.

The two series LC branches of the 3rd and the 5th harmonics and a decompensation branch represent the FCD model placed into the traction substation. The tuning of the LC branches is not adjusted to the number of the harmonic exactly, but it has to be adjusted for lower values. This adjustment of LC branches is necessary because harmonics from 110 kV feeding line could overload these LC branches. The following are the parameter values used for this model: capacity $C_3 = 8.8$ µF, inductance $L_3 = 132$ mH, $R_{L3} = 1.3$ Ω resonance frequency $f_3 = 147.7$ Hz and $C_5 = 2.5$ µF, inductance $L_5 = 163$ mH, $R_{L5} = 1.7$ Ω resonance frequency $f_5 = 249.5$ Hz. The decompensation branch has a reducing transformer 27 kV/6 kV, air-core choke and semiconductor controller which control power factor angle near to DPF = 0.98.

The model of catenary with standard structure of 100 Cu + 50 Bz with intensive line has the same character as homogenous long electric line with distributed electrical parameters, but there are considered all main parameters (i.e. resistance, inductance, capacity and leakage). This presumption can be taken because sections of catenary are longer in comparison with sections of station catenary. The following parameter values are used for this model: series specific resistance $R_C = 0.3$ Ω km$^{-1}$ series specific inductance $L_C = 0.8$ mH km$^{-1}$ and parallel specific capacity $C_C = 20$ nF km$^{-1}$.
The model of the traction electric vehicle is represented by power source with waveforms corresponding to recuperation vehicle with semiconductor converter and recuperation power 0.5 MW.

5.1 Simulations of Transient Effects During Recuperation

The simulations are carried out for transient effects during the recuperation mode of traction vehicles at short-circuit in the most serious points of traction circuit. For these effects in the traction circuits, the protection setting has to able to cover the whole spectrum of effect characteristics. This problem can be studied only by mathematical simulation in the given range, with partial verification of results by real measurements. The output data of the simulation program represent voltage and current waveforms. The critical states are deduced from knowledge of individual waveforms. The input parameters for protection settings of traction circuit can be gained by the analysis of these states [19]. When there is a short-circuit between a traction vehicle in recuperation mode and a traction substation, rapid exchanges of energy occur between the traction vehicle in recuperation mode and the short-circuit starting point as well as between the traction substation with FCD and the short-circuit starting point. At the beginning, the short-circuit is fed from two points and the energy from the recuperation vehicle is absorbed in units of ms, see Fig. 28.

In the case of the energy from the traction substation, the current wave is absorbed in tenths of ms (without protection off). During this time, a large amount of energy is transferred through the catenary; for example, the FCD supplies almost 20 kJ during the first units of ms to this short-circuit.

Fig. 28 The voltage at traction substation and voltage at pantograph of locomotive with recuperation
The energy transferred from the network to the traction power supply system is very high for this case. At the short-circuit, the characteristics of the 110/27 kV traction transformer and the impedance of lines are the only limiting circuit elements. The resistive impedances are decreased when the traction system achieves high efficiency, but these impedances are suitable for the short-circuit, when one considers the lowering of the short-circuit current and harmonics by modern electric vehicles propagating in the network (Fig. 29).

5.2 Findings for Transient Effects During Recuperation

The critical states are deduced from knowledge of individual waveforms. The input parameters for adjustment protections of traction circuit were obtained by the analysis of these states. The simulation diagrams, which are represented by voltage and current waveforms, can be also used as a main tool for particular project of traction substation of protection settings process. The design of the protections utilizes the traction substation design with FCD or without FCD.

6 Analysis of Voltage and Current at Contact Line at Track Closure

For disconnecting the contact line (i.e. track closure), of the AC 25 kV 50 Hz traction power supply system, it is necessary to connect the line with rail at both the front and at the end of the protected work area by two grounded rods. For this problem, it is necessary to analyze the influence (i.e. coupling ways) of the neighboring track contact line under the conditions that occur during the common
operation or during fault current. For this case, the contact lines of both tracks can be considered as a special case of an air transformer with free coupling between primary and secondary winding \([20, 21]\). The contact line of the operating track and the secondary winding of the track closure contact line represent the primary winding.

Voltage between track and contact line at section closure can occur for two reasons:

- electrostatic field effect caused by the contact line of the operating track,
- magnetic field effect caused by current passing through the operating track.

Both of these effects occur during the accidental connection of the contact line, but their qualitative influence is very different and depends on circuit diagram of the contact line section closure.

For an electrical connection of the contact line of work section closure, it is necessary to take into consideration three circuit diagrams that arise at section closure.

- The contact line closure is disconnected from the feeder, but it is not connected to the rails; it hangs as an insulated line above the rails, see Fig. 30.
- This contact line is connected at one end to the rail, see Fig. 31.
- This contact line is connected at both ends to the rail, see Fig. 32.

It is possible to consider the other circuit diagram represented by a partial parallel connection of line at the contact line closure at the double track, but this is the special case of full paralleling \([22]\). The circuit diagram of the contact line closure is changing progressively during the protection of the work section in the above-mentioned three circuit diagrams, according to ground rods location by workers. The Eqs. (3) and (4) come from a substitute circuit diagram of the above-mentioned situation, which is valid for

![Fig. 30](image)

*Fig. 30* The isolated contact line
\[ - \frac{dU_{2x}}{dx} = Z_{12} \cdot I_1 + Z_2 \cdot I_{2x} \quad (3) \]

\[ - \frac{dI_{2x}}{dx} = -Y_{12} \cdot U_1 + Y_2 \cdot U_{2x}, \quad (4) \]

where

- \( U_{2x} \) voltage in point from the second track (track closure), (V)
- \( I_{2x} \) current in the same point of the second track (track closure), (V)
\( I_1 \) current in the first track (operating track), (A)
\( U_1 \) voltage in the first track (operating track), (V)

\( Z_{12} \) mutual coupling reactance of both contact lines \( Z_{12} = j \omega \cdot M_{12}, \) (\( \Omega \))

\( Z_2 \) impedance of the second track, \( Z_2 = R_2 + j \omega \cdot L_2 \) (\( \Omega \)).

The solution of these equations

\[
U_{2x} = A \cdot e^{j \gamma x} + B \cdot e^{-j \gamma x} + k_2 \cdot U_1 \quad (5)
\]

\[
I_{2x} = \frac{1}{Z_{2v}} \left( A \cdot e^{j \gamma x} - B \cdot e^{-j \gamma x} \right) - k_1 \cdot I_1, \quad (6)
\]

where

\( A, B \) integrating constants, values of which are given by boundary conditions of electrical circuits, (–)

\( Z_{2v} \) wave reactance of the second track \( Z_{2v} = \sqrt{\frac{Z_2}{Y_2}}, \) (\( \Omega \))

\( \gamma \) constant of propagate wave \( \gamma = \sqrt{Z_2 \cdot Y_2}, \) (–)

\( k_1 = \frac{Z_{12}}{Z_2} \) and \( k_2 = \frac{C_{12}}{C_{12} + C_2} \) auxiliary constant (–).

All values of electrical parameters of contact line are considered per unit of contact line length, the best way is 1 km of length. Furthermore, it is necessary the approximate equation for \( e^{\gamma x} \).

For a length of work section, we use \( e^{\alpha x} \doteq 1 + a \cdot x \), according to the Taylor progression with a substation of original exponential function with accuracy 0.1% in range of exponent value from \(-0.045\) to \(0.045\) [23].

\( Y_{12} \) capacitive conductance between the two contact lines on condition that the ohmic component is much lower than the capacitive component, \( Y_{12} = j \omega \cdot C_{12}, \) \( C_{12} \) is the capacity between the contact lines [S], \( Y_2 \) is capacitive conductance of the second track, (i.e. between line and ground), on condition that the ohmic component is much lower than the capacitive component (S) \( Y_2 = j \omega \cdot (C_{12} + C_2) \), \( C_2 \) is the capacity between the track closure and the ground.

### 6.1 Isolated Contact Line

Boundary conditions for this circuit diagram come from the fact that the current cannot pass through at the ends of isolated section and then \( I_{20} = I_{2\ell} = 0 \). Under this presumption and equation usage (5) with using approximate equation for \( e^{\gamma x} \), we obtain
The findings for isolated contact line

- Voltage between isolated contact line and rail is achieved by only capacitive distribution independently on the position at contact line.
- Resistance of this voltage source is very high (i.e. low values of capacity), so this voltage is “soft” and under load is decreasing.
- No current passes through isolated contact line.

In operation, this mentioned voltage occurs between point of ground rod, which is connected to the rail, and contact wire during their approach.

6.2 Contact Line One-Sidedly Connected to the Rail

The presumption for deduction of boundary conditions

- Voltage in the point of contact line connected to the rail is zero.
- Current is not passing through on the opposite end of contact line, which is isolated.

Under presumption that connection between contact line and rail is localized to origin of length coordinate \( x \) and length of contact line is \( l \) and then \( U_{20} = I_{2l} = 0 \).

For voltage and current in any location of section closure, we obtain:

\[
U_{2x} = -Z_{2v} \cdot k_1 \cdot \gamma \cdot I_1 \cdot x = -j\omega M_{12} \cdot I_1 \cdot x
\]

\[
I_{2x} = -k_2 \frac{U_1}{Z_{2v}} \cdot \gamma (l - x) = -j\omega C_{12} \cdot U_1 (l - x)
\]

The findings for line one-sidedly connected to the rail

- Voltage, which is induced in the contact line of the second track, is directly proportional to the length of the one-sided connected section and then directly proportional to the current that passes through the operating contact line of the second track.
- Current, which passes through the point of connection of the second track of the contact line to the rail, is obtained only from the mutual capacity of both lines and the voltage of the operating line.

In operation, this voltage occurs in the place where the worker places the second ground rod (i.e. on the opposite end of the work section closure from where the ground rod is already placed). In the case of higher currents in the neighboring
operating section, this voltage can reach dangerous values. Current, values of which are deduced from Eq. (10), passes through the ground wire of the first rod and it has no practical meaning from the viewpoint of its negligible magnitude.

6.3 Contact Line Two-Sidedly Connected with Rail

The boundary conditions for this case arise from the circuit diagram, so that voltage at both ends of examined line is zero (i.e. in the place of both earth rods) and then $U_{20} = U_{2x} = 0$. By using Eqs. (5), (6) and the usual simplification of exponential function, we obtain the following:

\[
U_{2x} = k_2 \cdot U_1 \left( 1 - \frac{2 \cdot \gamma \cdot \ell}{2 - \gamma \cdot \ell} \right) = 0 \quad (11)
\]

\[
I_{2x} = -j \omega C_1 \cdot U_1 \cdot \frac{\ell - 2x}{2 - \gamma \cdot \ell} - k_1 \cdot I_1 \quad (12)
\]

The findings for line double-connection to the rail:

- The voltage is zero in the entire length of section between the ground rods at the points of track closure.
- Current, which passes through this part of the contact line, consists of two components: inductive and capacitive.
- Inductive components depend only on $k_1$ and are directly proportional to the current of the operating track. It depends neither on the length of the section nor location of the point at track closure.
- Capacitive component depends on voltage of contact line of operating track. It is zero in the center of the examined section (i.e. in the middle of the ground rods) and increases linearly to both ground rods but with different polarity.
- Inductive components are applied significantly in the operation, whereas capacitive components have no impact (i.e. its values are insignificant). Inductive components can reach the high values that require very good condition of the ground wire cable at the ground rods.

6.4 Findings for Contact Line Power Closure

For numerical values of the above-mentioned equations, it is necessary to use numerical values of a particular constant of the contact line. For this reason, results from measuring are utilized [23]. The value for impedance of one track of contact line was measured $Z_2 = 0.222 \div 0.485j = 0.534\angle65.5^\circ \ \Omega \text{km}^{-1}$. The reactance of
mutual coupling of both contact lines is \( Z_{12} = 0.133\, \Omega\, \text{km}^{-1} \). This value is given by Eq. (9) inducted voltage at single-sided connection to the line.

The measured voltage is \( U_2 = 0.133\, \text{V A}^{-1}\, \text{km}^{-1} \). The capacity of the contact line to the ground was measured \( C_2 = 0.0155\, \mu\text{F km}^{-1} \). The capacity between the contact lines, which was calculated by roles of potentials between conductors, is \( C_{12} = 0.0136\, \mu\text{F km}^{-1} \). It is possible to calculate other coefficients from numerical values by their definition equations \( \gamma = 2.21 \times 10^{-3} \), \( Z_{2v} = 242\, \Omega\, \text{km}^{-1} \), \( k_1 = 0.250 \) and \( k_2 = 0.467 \).

Voltages and currents for three circuit diagrams were calculated by equations:

- Voltage for isolated contact line by Eq. (7), when the voltage in the operating track is 25 kV is \( U_2 = 11.7\, \text{kV} \). This value does not depend on length of section closure and in its any possible place. Internal resistance of this source is given by capacity \( C_{12} \) and its value is hundreds, \( \text{k}\Omega \) which decreases with length of section.

- Inducted voltage for one-sided contact line with rail at the opened end (i.e. the high value at this point), which is calculated by Eq. (9), is \( U_2 = 0.133\, \text{V A}^{-1}\, \text{km}^{-1} \). Current passes through the point of connection (i.e. contact line and rail), which is calculated by Eq. (10) at 25 kV in the operating track, is \( I_2 = 0.107\, \text{A km}^{-1} \). This value is negligible.

- Voltage for the two-sided contact line with rail is calculated by Eq. (11). Voltage in any possible place of operating contact line is zero. Induced component of current passing through to the point of connection of the contact line closure with rail by (12) is \( I_2 = 0.250\, \text{A A}^{-1} \). The capacity component of this current is

7 Conclusion

New technologies in rail, including devices for infrastructure and transport, have meant improved safety, comfort, and speed of services. It has also brought new problems for the AC 25 kV 50 Hz traction power supply system with a specific filter-compensation device in traction substations in the Czech Republic. These problems have resulted in some specific behavior issues during operation modes. For these reasons, research activities during last several years in engineering departments at the University of Pardubice have been focused on studying the behavior of the traction system during faults or selected operation states, for example, transient effects and coupling to surrounding elements. These states can be studied in particular by mathematical simulation without building physical models in the given range, with partial verification of results by real measurements. The main findings and the results are explored in each part of this article.
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