The hybridization of the European car fleet should be an attempt for a positive contribution to meeting regional CO₂ and emission (CO, NOx, particles, and total hydrocarbons) targets, respectively. This has led to enormous research and development efforts in academia and industry to make the necessary technology mature for series production in large numbers.

The scope of the book is dedicated to the optimization of passenger hybrid vehicles. Recent results in the area of optimal control and hybrid vehicle design (powertrain architecture and size of the components) are presented.

The author’s intention is to provide a complete overview to the field of optimal control of hybrid vehicles by studying one of the key elements that have a significant impact on the performance: energy management. The book is written from a mathematical viewpoint but takes care of the mixed audience. Much effort has been put into balancing the level of the presentation of topics of control, optimization, and automotive technology. Theoretical results in the field of applied optimal control are stated and commented to provide the reader with more insight but the proofs—with some exceptions—are omitted.

The prerequisites for this book are as follows: the reader should be familiar with dynamic systems in general and their representation in the state space in particular, as covered in standard undergraduate control courses. Especially, the reader should have already gained some experience in modeling and simulating mechanical and electrical systems. Furthermore, the reader should have a fairly sound understanding of differential calculus and some rudimentary understanding of functional analysis, optimization, and optimal control theory.

Intended Readership

The book has been written for master students, researchers, and practitioners in the fields of control engineering, automotive technology, and applied mathematics that are interested in techniques that provide the minimum energy consumption under
further restrictions by taking advantage of the control freedoms provided by hybridization. It is intendedly written from a practical point of view to be attractive to:

- students from various disciplines who envisage a career in control in the automotive industry. They find a transfer of theory to applications;
- applied mathematicians to find some nonstandard algorithms for solving large-scale optimal control problems;
- engineers being involved in specifying, developing, or calibrating energy management systems to get an introduction into a mathematical field of optimization and control which is not easily accessible and some hints how to model the system appropriately;
- researchers who are interested to see how energy management problems are specified and solved in industry; and
- managers or decision makers to get an inspiration of the potential of mathematical tools in this field.

A part of the topics has been taught at the University of Rostock and at the Ruhr University of Bochum and numerous final year students have been mentored under our responsibility during the past 5 years. The comments received from the students have been beneficial in the selection and preparation of the book’s topics.

What are the Contributions of This Book

Energy management problems in practice can be large which means simply that the number of controls, states, and time horizon is large. This enforces hard conditions that have to be satisfied by good optimization candidates. This book proposes to solve such problems as hybrid optimal control problems.

Many problems encountered in practical hybrid vehicle applications seem on the first view not easily accessible for mathematical optimization theory, mainly arising from the difficulty that discrete decisions appear in the problem formulations, which causes considerable difficulties to many optimizers. Reconsidering of the underlying systems as hybrid systems and the control problem as hybrid optimal control problems can simplify the way to find a solution. This book supports this methodology and gives a selection of real-world problems, which are tackled as optimal control problems of hybrid systems.

We decided not to rely on the use of a specific third-party nonlinear programming solver but to induce modifications to well-known SQP algorithms in order to improve convergence and numerical stability. We list some well-proven algorithms in detail to give the readers a deeper insight into relevant implementation aspects, which are indispensable for the assessment of third-party nonlinear programming software packages or for writing one’s own software code. A major contribution is the presentation of a sparse SQP framework based on sparse quasi-Newton updates to solve discretized optimal control problems for many controls, states, and discretization points. A new, very efficient, and robust sparse SQP algorithm will be
presented that decomposes the Hessian update mechanism into many subproblems of small dimension but with less numerical deficiencies.

To the authors’ best knowledge, there is almost no book in the market which covers a complete spectrum of relevant stages to obtain optimal energy management (in a mathematical sense) including a discussion of theoretical aspects, a comprehensive treatment of algorithm implementation, and diverse application scenarios. The first obstacle for practitioners is the fact that many information and algorithms are widely scattered between various disciplines, which generates an initial barrier to enter this field. Many important algorithms, e.g., from the mathematical field of graph theory, are not easily accessible either to engineers or to applied mathematicians. It is therefore a time-consuming and demanding task to develop efficient algorithms for large hybrid vehicle problems. Our intention is, however, not to give blueprints for all possible problems (this is by far not possible) but to encourage the reader to use the provided information in this book including cited literature, proposed algorithms, etc., as basic kit for solving their own problems.

**What is Not Covered in This Book**

The book deals exclusively with time-invariant process descriptions which means that the process parameters remain constant over the complete time. Process types with time-varying parameters, e.g., battery aging, are not covered in this book. However, the proposed algorithms can serve as an initial tool set to be adapted to this problem class.

**Structure of the Book**

The book is modular structured and organized into six parts:

- Part I—Theory and Formulations;
- Part II—Methods for Optimal Control;
- Part III—Numerical Implementations;
- Part IV—Modeling of Hybrid Vehicles for Control;
- Part V—Applications; and
- Part VI—Appendix.

Part I of the book can be skipped if they wish to continue directly with the description of methods for obtaining numerical solutions of optimal control problems.

Chapter 1 discusses the challenges of designing and calibrating hybrid vehicles nowadays and motivates the usage of optimal control theory. It gives a general problem statement as an orientation for the following chapters and discusses the
most important control strategy of hybrid vehicles—energy management—and their algorithmic challenges.

Part I—Theory and Formulations. In Chap. 2, the theory of nonlinear programming is reviewed. The widely used sequential quadratic programming is presented for the solution of constrained nonlinear minimization problems, which is fundamental in our optimization framework to the solution of optimal control problems. A compact treatment of sensitivity analysis is presented as a tool for studying parameter changes of a system.

In Chap. 3, a general definition for hybrid and switched systems is introduced. Some important formulations for hybrid optimal control problems of dynamic processes described by systems of ordinary differential equations are discussed. The focus is on switched systems, a subclass of hybrid systems that switch between subsystems only in response to a command. This subclass already covers a great range of technical problems.

Chapter 4 discusses the Pontryagin’s minimum principle. This important result is briefly approached from the classical calculus of variation. The Hamilton–Jacobi–Bellman method is discussed as an alternative approach to gain first-order necessary conditions for optimality. It is shown that both approaches correspond to each other under restrictive assumptions. The original Pontryagin’s minimum principle for continuous optimal control problems is not suitable for hybrid optimal control problems. However, a quite natural reformulation of the hybrid optimal control problem admits the classical theory for deduction of first-order necessary conditions in the sense of Pontryagin. The charm of this methodology is its comprehensible derivation.

Part II—Methods for Optimal Control. This part starts the important topic of discretizations, since all numerical procedures rely on numerical integration schemes. In Chap. 5, the famous Runge–Kutta discretizations is presented. The determination of the Runge–Kutta order is briefly discussed and order conditions up to the fourth order are given including the additional conditions for solving optimal control problems. Regarding optimal control problems only explicit and implicit Runge–Kutta discretizations which satisfy additional conditions for the adjoint differential equations are discussed.

In Chap. 6, the Hamilton–Jacobi–Bellman principle is used to introduce the dynamic programming algorithm. Dynamic programming is an appealing approach for the solution of optimal control problems in many situations. The theoretical foundation is relatively easy to understand compared with the much more involved indirect methods. The general algorithm can be stated in a simple form and is easy to apply to continuous optimal control problems and with some minor reformulations, it is also well suited for switched optimal control problems.

In Chap. 7, indirect methods to solve optimal control problems are discussed. Indirect methods rely on first-order necessary conditions, summarized in Pontryagin’s minimum principle, and attempt to generate control and state trajectories, which satisfy these conditions. An extension of the indirect shooting method for switched and hybrid systems that yields a solution for systems of low complexity is presented as well.
In Chap. 8, the algorithmic development for optimal control problems of switched systems is considered on the aspect of *First Discretize, then Optimize*. These methods are commonly known as direct methods. Direct methods transform the original problem via a discretization of the control and the state functions on a time grid to a nonlinear constrained optimization problem. This procedure is known as direct transcription of an optimal control problem and refers to the method of approximating the infinite-dimensional problem by a finite-dimensional one and to solve it with nonlinear programming algorithms.

Part III—**Numerical Implementations.** Optimal control problems formulated with direct transcription methods lead to large-scale nonlinear programming problems. One suitable framework for the solution of this type of optimization problems is sequential quadratic programming. But for an efficient implementation of the SQP algorithm it is crucial to take into account the particular properties and structure of the objective and constraint functions. This leads to Karush–Kuhn–Tucker (KKT) matrices, which occur in the subproblems to be sparse. Chapter 9 deals with techniques for the determination of the structure of the involved matrices, the calculation of the numerical derivatives, and the implementation of a sparse Quasi-Newton update.

Part IV—**Modeling of Hybrid Vehicles for Control.** In Chap. 10, the main hybrid vehicle configurations are presented including all relevant mechatronic subsystems. Models are derived with respect of optimization which imposes additional restriction in terms of complexity and smoothness. Several powertrain models of different depth for parallel and power-split hybrid vehicle configurations are formulated as hybrid systems. The easiest model includes representations of the electrical and the mechanical subsystem, whereas the most complex model also incorporates a detailed thermodynamic model of the internal combustion engine and the exhaust system as well as an emissions model.

Part V—**Applications.** In Chap. 11, the calibration process for hybrid vehicles is treated, which can be a cumbersome task if no systematic calibration approach is applied. Therefore, the fuel optimal operation of hybrid vehicles is formulated as switched optimal control problems and solved using dynamic programming, indirect shooting, and direct solution methods. Control parameters are derived for the calibration process as well as for the development of new functional approaches for improving the vehicle’s performance. The step of transferring the solution of optimal control problems into calibration parameters for the electronic control unit is non-trivial. It is shown that look-up tables for rule-based energy managements can be derived directly from the solution that reduces significantly the time required for a high-quality calibration.

In Chap. 12, the theory of optimal control also suggests new functional approaches. Predictive energy management for minimizing wheel-to-meters energy losses are new functional candidates. Three different predictive control strategies are discussed for battery electric vehicles, full hybrid vehicles, and plug-in hybrid vehicles, which make the solution of a (switched) optimal control problem amenable for real-time implementation in the electronic control unit. This is only possible, if a profile of the driving route is a priori known. A prediction based on data
from modern navigation systems is made to obtain an estimation of this profile. It is demonstrated that predictive control strategies for energy management can significantly achieve fuel saving in real-world test drives.

Engineers aiming to find efficient hybrid powertrain configurations can benefit greatly from the seamless interaction of multi-objective optimization and optimal control methods. In Chap. 13, the simultaneous optimization of design parameters and energy management for a fixed parallel hybrid powertrain structure is discussed.

Gifhorn, Germany

September 2016

Thomas J. Böhme

Benjamin Frank
Hybrid Systems, Optimal Control and Hybrid Vehicles 
Theory, Methods and Applications
Böhme, Th.J.; Frank, B.
2017, XXXIII, 530 p. 143 illus., 113 illus. in color.,
Hardcover
ISBN: 978-3-319-51315-7