The concept of volatility refers to any phenomenon presenting features of instability, unpredictability, and a likeliness to change frequently, often without apparent or cogent reason; in a word, a phenomenon that exhibits random variations. Therefore, it is an essential element of almost all branches of science, and the measurement of its impact and effects is of paramount importance. This book mainly focuses on the measurement of the statistical parameter which Bachelier (1900) called “nervosité” (the coefficient of nervousness) of a market price and which nowadays is referred as variance or volatility in the context of financial applications. Nevertheless, many of the methods and results presented here could be applied to other disciplines (from turbulence to chemistry, from physics to computer science and even medicine).

Ideally, we start from the book chapter “Volatility Estimation by Fourier Expansion” by Malliavin and Thalmaier (2006) and follow the rapid development of the Fourier-Malliavin estimation theory over the last decade. The purpose of this book is to give a picture of the state of the art concerning this theory and to suggest new directions for its application in the study of financial markets. We aim to give the interested reader a clear, comprehensive, and self-contained book on the use of the Fourier-Malliavin technique for volatility estimation, providing the theoretical and numerical tools needed to understand and apply the methodology to real cases. Specifically, readers are given examples and instruments to implement this methodology in various financial settings, and some new applications to real data are proposed. Detailed bibliographic references are pointed out to permit a study in depth. This book will appeal to the financial econometrics and quantitative finance community and, in particular, to PhD students, researchers, and practitioners in these fields.

Chapter 1 briefly introduces the main elements, namely, various concepts of volatility, the peculiar characteristics of market (high-frequency) data, and the Fourier analysis for financial time series. In Chapter 2, the reader is introduced to the basic idea underlying the Fourier-Malliavin method, and some intuitions on the method are anticipated. Chapter 3 mainly focuses on estimating integrated volatility and cross-volatility on a fixed time horizon, e.g. a day, while in Chapter 4, the Fourier estimation of instantaneous volatility is studied. In Chapter 5, the efficiency
of the estimation method is analyzed when the observed asset prices are contami-
nated by market microstructure noise effects, as it happens when high-frequency
data are employed. Chapter 6 gives some examples of the potential of the Fourier
method to deal with the real-time use of the volatility estimates. The essentials of
the mathematical background are presented in Appendix A, which enables the non-
expert reader to follow the theory presented in the book. Furthermore, Appendix B
provides a collection of MATLAB® codes useful for reproducing the numerical
results contained in the book.

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