## Contents

1 Overview of Adaptive Dynamic Programming .................. 1
   1.1 Introduction ........................................ 1
   1.2 Reinforcement Learning ............................... 3
   1.3 Adaptive Dynamic Programming ....................... 7
      1.3.1 Basic Forms of Adaptive Dynamic Programming . . 10
      1.3.2 Iterative Adaptive Dynamic Programming ........... 15
      1.3.3 ADP for Continuous-Time Systems ................ 18
      1.3.4 Remarks .................................... 21
   1.4 Related Books ...................................... 22
   1.5 About This Book .................................... 26
References ............................................... 27

Part I Discrete-Time Systems

2 Value Iteration ADP for Discrete-Time Nonlinear Systems ....... 37
   2.1 Introduction ........................................ 37
   2.2 Optimal Control of Nonlinear Systems
      Using General Value Iteration .......................... 38
      2.2.1 Convergence Analysis .......................... 40
      2.2.2 Neural Network Implementation .................. 48
      2.2.3 Generalization to Optimal Tracking Control ......... 52
      2.2.4 Optimal Control of Systems
          with Constrained Inputs . . . . . . . . . . . . . . . . . 56
      2.2.5 Simulation Studies ............................. 59
   2.3 Iterative θ-Adaptive Dynamic Programming Algorithm
      for Nonlinear Systems .................................. 67
      2.3.1 Convergence Analysis .......................... 69
      2.3.2 Optimality Analysis ............................ 77
      2.3.3 Summary of Iterative θ-ADP Algorithm .......... 80
      2.3.4 Simulation Studies ............................. 83
3 Finite Approximation Error-Based Value Iteration ADP

3.1 Introduction ........................................ 91
3.2 Iterative $\theta$-ADP Algorithm with Finite Approximation Errors ........................................ 92
    3.2.1 Properties of the Iterative ADP Algorithm with Finite Approximation Errors ......................... 93
    3.2.2 Neural Network Implementation .............................. 100
    3.2.3 Simulation Study ........................................ 104
3.3 Numerical Iterative $\theta$-Adaptive Dynamic Programming ..................... 107
    3.3.1 Derivation of the Numerical Iterative $\theta$-ADP Algorithm ................................... 107
    3.3.2 Properties of the Numerical Iterative $\theta$-ADP Algorithm ................................... 111
    3.3.3 Summary of the Numerical Iterative $\theta$-ADP Algorithm ................................... 120
    3.3.4 Simulation Study ...................................... 121
3.4 General Value Iteration ADP Algorithm with Finite Approximation Errors ................. 125
    3.4.1 Derivation and Properties of the GVI Algorithm with Finite Approximation Errors .................. 125
    3.4.2 Designs of Convergence Criteria with Finite Approximation Errors ................................... 133
    3.4.3 Simulation Study ...................................... 140
3.5 Conclusions ........................................ 147
References ............................................... 147

4 Policy Iteration for Optimal Control of Discrete-Time Nonlinear Systems .................. 151
4.1 Introduction ........................................ 151
4.2 Policy Iteration Algorithm ................................... 152
    4.2.1 Derivation of Policy Iteration Algorithm .............................. 153
    4.2.2 Properties of Policy Iteration Algorithm ..................... 154
    4.2.3 Initial Admissible Control Law ................................... 160
    4.2.4 Summary of Policy Iteration ADP Algorithm ................... 162
4.3 Numerical Simulation and Analysis ................................... 162
4.4 Conclusions ........................................ 173
References ............................................... 174
5 Generalized Policy Iteration ADP for Discrete-Time Nonlinear Systems ................................................. 177
  5.1 Introduction .................................................................................................................. 177
  5.2 Generalized Policy Iteration-Based Adaptive Dynamic Programming Algorithm .................. 177
     5.2.1 Derivation and Properties of the GPI Algorithm .................................................. 179
     5.2.2 GPI Algorithm and Relaxation of Initial Conditions .............................................. 188
     5.2.3 Simulation Studies .................................................................................................. 192
  5.3 Discrete-Time GPI with General Initial Value Functions .............................................. 199
     5.3.1 Derivation and Properties of the GPI Algorithm .................................................. 199
     5.3.2 Relaxations of the Convergence Criterion and Summary of the GPI Algorithm ........... 211
     5.3.3 Simulation Studies .................................................................................................. 215
  5.4 Conclusions ..................................................................................................................... 221
References ............................................................................................................................... 221

6 Error Bounds of Adaptive Dynamic Programming Algorithms .............................................. 223
  6.1 Introduction ..................................................................................................................... 223
  6.2 Error Bounds of ADP Algorithms for Undiscounted Optimal Control Problems ................. 224
     6.2.1 Problem Formulation ............................................................................................... 224
     6.2.2 Approximate Value Iteration ................................................................................... 226
     6.2.3 Approximate Policy Iteration ................................................................................... 231
     6.2.4 Approximate Optimistic Policy Iteration ................................................................. 237
     6.2.5 Neural Network Implementation .............................................................................. 241
     6.2.6 Simulation Study ....................................................................................................... 243
  6.3 Error Bounds of Q-Function for Discounted Optimal Control Problems ................................ 247
     6.3.1 Problem Formulation ............................................................................................... 247
     6.3.2 Policy Iteration Under Ideal Conditions ................................................................... 249
     6.3.3 Error Bound for Approximate Policy Iteration ....................................................... 254
     6.3.4 Neural Network Implementation .............................................................................. 257
     6.3.5 Simulation Study ....................................................................................................... 259
  6.4 Conclusions ..................................................................................................................... 262
References ............................................................................................................................... 263

Part II Continuous-Time Systems

7 Online Optimal Control of Continuous-Time Affine Nonlinear Systems .............................................. 267
  7.1 Introduction ..................................................................................................................... 267
  7.2 Online Optimal Control of Partially Unknown Affine Nonlinear Systems ............................. 267
     7.2.1 Identifier–Critic Architecture for Solving HJB Equation ........................................ 269
7.2.2 Stability Analysis of Closed-Loop System ........... 281
7.2.3 Simulation Study .................................. 286
7.3 Online Optimal Control of Affine Nonlinear Systems
with Constrained Inputs ................................. 291
7.3.1 Solving HJB Equation via Critic Architecture ........ 294
7.3.2 Stability Analysis of Closed-Loop System
with Constrained Inputs ................................. 298
7.3.3 Simulation Study .................................. 302
7.4 Conclusions ........................................ 305
References ............................................... 306

8 Optimal Control of Unknown Continuous-Time Nonaffine
Nonlinear Systems ........................................ 309
8.1 Introduction ........................................ 309
8.2 Optimal Control of Unknown Nonaffine Nonlinear Systems
with Constrained Inputs ................................. 310
8.2.1 Identifier Design via Dynamic Neural Networks .... 311
8.2.2 Actor–Critic Architecture
for Solving HJB Equation ............................... 316
8.2.3 Stability Analysis of Closed-Loop System ........... 318
8.2.4 Simulation Study .................................. 323
8.3 Optimal Output Regulation of Unknown Nonaffine Nonlinear
Systems ........................................... 327
8.3.1 Neural Network Observer ....................... 328
8.3.2 Observer-Based Optimal Control Scheme
Using Critic Network ................................ 333
8.3.3 Stability Analysis of Closed-Loop System ........... 337
8.3.4 Simulation Study .................................. 340
8.4 Conclusions ........................................ 343
References ............................................... 343

9 Robust and Optimal Guaranteed Cost Control
of Continuous-Time Nonlinear Systems ................. 345
9.1 Introduction ........................................ 345
9.2 Robust Control of Uncertain Nonlinear Systems ....... 346
9.2.1 Equivalence Analysis and Problem Transformation . 348
9.2.2 Online Algorithm and Neural Network
Implementation ........................................... 350
9.2.3 Stability Analysis of Closed-Loop System ........... 353
9.2.4 Simulation Study .................................. 356
9.3 Optimal Guaranteed Cost Control of Uncertain Nonlinear
Systems ........................................... 360
9.3.1 Optimal Guaranteed Cost Controller Design ........ 362
9.3.2 Online Solution of Transformed Optimal Control
Problem ............................................... 368
9.3.3 Stability Analysis of Closed-Loop System ........... 373
9.3.4 Simulation Studies .................................. 378
9.4 Conclusions ........................................ 383
References ............................................... 384

10 Decentralized Control of Continuous-Time Interconnected
Nonlinear Systems ........................................ 387
10.1 Introduction ........................................ 387
10.2 Decentralized Control of Interconnected Nonlinear Systems .... 388
   10.2.1 Decentralized Stabilization via Optimal Control
         Approach ........................................ 389
   10.2.2 Optimal Controller Design of Isolated Subsystems .... 394
   10.2.3 Generalization to Model-Free
         Decentralized Control .......................... 400
   10.2.4 Simulation Studies ............................. 404
10.3 Conclusions ........................................ 414
References ............................................... 414

11 Learning Algorithms for Differential Games
of Continuous-Time Systems ................................ 417
11.1 Introduction ........................................ 417
11.2 Integral Policy Iteration for Two-Player Zero-Sum Games .... 418
   11.2.1 Derivation of Integral Policy Iteration .............. 420
   11.2.2 Convergence Analysis .......................... 423
   11.2.3 Neural Network Implementation .................. 425
   11.2.4 Simulation Studies ............................. 428
11.3 Iterative Adaptive Dynamic Programming for Multi-player
Zero-Sum Games ........................................ 431
   11.3.1 Derivation of the Iterative ADP Algorithm ......... 433
   11.3.2 Properties ................................... 438
   11.3.3 Neural Network Implementation .................. 444
   11.3.4 Simulation Studies ............................. 451
11.4 Synchronous Approximate Optimal Learning for Multi-player
Nonzero-Sum Games ..................................... 459
   11.4.1 Derivation and Convergence Analysis .............. 460
   11.4.2 Neural Network Implementation .................. 464
   11.4.3 Simulation Study .............................. 473
11.5 Conclusions ........................................ 478
References ............................................... 478
Part III Applications

12 Adaptive Dynamic Programming for Optimal Residential Energy Management ........................................... 483
  12.1 Introduction ........................................ 483
  12.2 A Self-learning Scheme for Residential Energy System Control and Management .................... 484
    12.2.1 The ADHDP Method ................................ 488
    12.2.2 A Self-learning Scheme for Residential Energy System ........................................ 489
    12.2.3 Simulation Study .................................. 492
  12.3 A Novel Dual Iterative Q-Learning Method for Optimal Battery Management ............................. 496
    12.3.1 Problem Formulation ........................... 496
    12.3.2 Dual Iterative Q-Learning Algorithm ............... 497
    12.3.3 Neural Network Implementation .................. 503
    12.3.4 Numerical Analysis ................................ 506
  12.4 Multi-battery Optimal Coordination Control for Residential Energy Systems .......................... 513
    12.4.1 Distributed Iterative ADP Algorithm ............... 515
    12.4.2 Numerical Analysis ................................ 527
  12.5 Conclusions ........................................ 533
  References ............................................... 533

13 Adaptive Dynamic Programming for Optimal Control of Coal Gasification Process ............................ 537
  13.1 Introduction ........................................ 537
  13.2 Data-Based Modeling and Properties ......................... 538
    13.2.1 Description of Coal Gasification Process and Control Systems ............................ 538
    13.2.2 Data-Based Process Modeling and Properties .......... 540
  13.3 Design and Implementation of Optimal Tracking Control ................................................ 546
    13.3.1 Optimal Tracking Controller Design by Iterative ADP Algorithm Under System and Iteration Errors .......................... 546
    13.3.2 Neural Network Implementation ........................ 554
  13.4 Numerical Analysis ................................... 557
  13.5 Conclusions ........................................ 568
  References ............................................... 569

14 Data-Based Neuro-Optimal Temperature Control of Water Gas Shift Reaction .......................... 571
  14.1 Introduction ........................................ 571
  14.2 System Description and Data-Based Modeling .......................................................... 572
    14.2.1 Water Gas Shift Reaction ................................ 572
    14.2.2 Data-Based Modeling and Properties .................. 573
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.3  Design of Neuro-Optimal Temperature Controller</td>
<td>575</td>
</tr>
<tr>
<td>14.3.1 System Transformation</td>
<td>575</td>
</tr>
<tr>
<td>14.3.2 Derivation of Stable Iterative ADP Algorithm</td>
<td>576</td>
</tr>
<tr>
<td>14.3.3 Properties of Stable Iterative ADP Algorithm</td>
<td>578</td>
</tr>
<tr>
<td>with Approximation Errors and Disturbances</td>
<td></td>
</tr>
<tr>
<td>14.4  Neural Network Implementation for the Optimal Tracking Control</td>
<td>582</td>
</tr>
<tr>
<td>Scheme</td>
<td></td>
</tr>
<tr>
<td>14.5  Numerical Analysis</td>
<td>585</td>
</tr>
<tr>
<td>14.6  Conclusions</td>
<td>589</td>
</tr>
<tr>
<td>References</td>
<td>589</td>
</tr>
<tr>
<td>Index</td>
<td>591</td>
</tr>
</tbody>
</table>
Adaptive Dynamic Programming with Applications in Optimal Control
Liu, D.; Wei, Q.; Wang, D.; Yang, X.; Li, H.
2017, XXX, 594 p. 203 illus., 175 illus. in color., Hardcover
ISBN: 978-3-319-50813-9