Surfing with Rod

Michael R. Fellows(✉)

Department of Informatics, University of Bergen,
P.O. Box 7803, 5020 Bergen, Norway
michael.fellows@uib.no

Abstract. I wish Rod a happy birthday. Rod has offered a good historical account of our early days of collaboration in his paper, "The Birth and Early Days of Parameterized Complexity" [D12]. One of the themes of our life-long collaboration has been a shared passion for surfing, and many of our best ideas were hammered out on surf trips. This contribution is best viewed as a gloss on that earlier account by Rod, taking as an organizing skeleton, our various surf trips and what we were thinking about, *with a particular emphasis on open problems and horizons that remain, and some reflections on the formation of our research community.*

1 Introduction

Happy birthday Rod! When I first heard about the festival, I was somewhat incredulous: "Why is he doing this? He is only 50!!" (I probably needed an egg.) Keep going, man! We've had a lot of fun!

This being a personal reminiscence about our collaboration, my first thought was to write a history of the development of the central ideas of parameterized complexity, recalling the historical context, which is a bit colorful, as many of the key ideas were developed on surf trips.

But Rod, you have already done a lot of that in the entertaining and informative article [D12] about how the field developed in its early years.

So Plan B was to focus on our surf trips as the narrative skeleton of a colorful history of (most of) the main ideas in PC. But that was not going to work as I found it impossible to collate the many surf adventures with the intellectual adventures.

So I have settled on Plan C: to discuss a small selection of key surfing adventures, and the ideas we were discussing then (so a limited historical window on the field) *and especially the open questions that remain.*

I think we have surfed together at least a hundred times at more than 25 different surf spots, some entirely and perhaps deservedly obscure (Red Rocks, Wairaka Rock), always talking about our parameterized complexity projects, latest ideas, poetry, literature, the latest goings-on of the math-for-kids projects, etc. Always another surf trip and new ideas. Long may this be so!

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2 About Surfing

What People Generally Don't Know About Surfing
What people who do not surf generally do not understand is how much time surfers spend driving around looking at the waves and not actually surfing.

This happened to me on my first trip to New Zealand. I had a lot of grant money and freedom thanks to the enlightened administration of Canada's NSERC at that time (early 1990's). NSERC was not in the business of herding the cats. There were essentially two principal rules:

(1) Thou shalt not spend your research money on snow tires for your car.
(2) Thou shalt not spend your research money on taking out the entire department to dinner at an all-you-can-eat place just because you have a research visitor.

I contributed a paper to a relatively obscure (to me, anyway) combinatorics conference ACCMCC in New Zealand. Rod has hilariously written about this adventure in his essay about the origins of parameterize complexity [D12]. I had all of my surf gear along: wetsuit, flippers, bodyboard. Down the stairs from the airplane, cattle class, and there came up down the other stairs from First Class, Ron Graham, and he said to me: "What are *you* doing here?!"

I was there to present some of my results on algorithmic aspects of the Graph Minor Theorem, and I was also there to surf.

I met Rod through a social pointer by Marston Conder. Rod and I immediately hit it off about the basic ideas of what we would now call FPT *versus* XP and agreed that this was a rich theme for mathematical investigation. After two bottles of Villa Maria (as has been recounted elsewhere), we had a rough plan about how to proceed, with this obviously natural research horizon. The tentative plan was to write a series of numbered papers (like Robertson and Seymour) and fold it into a small book. My recollection is that in our visionary state, the plan was something like:

(PC I) *basic complexity framework and results*
(PC II) *concrete complexity results*
(PC III) *applications.*

My recollection is that we got this far (tentatively) over the Villa Maria, without having the concrete mathematical results to fill in the three containers, even (PC I), although there was a start in the paper [AEFM89] and some then unpublished results building on that. The earlier work of Langston and me [FL87, FL88] laid the groundwork for (PC II) and (PC III). My recollection is that we had a confident vision that there were plenty of issues to support a general program.

I can't recall if we discussed surfing at that time. I think this shared passion turned up later in email or phone conversations. Or maybe Rod had prior commitments that weekend.

I had my gear and a rental car, and I had a couple of days to drive around looking for surf. I had a map. Castle Point looked promising, so I drove over there, over the scenic hills, slowly through large herds of sheep on the backroads at several points along the way. Castle Point looked good on the map, but when I got there, I was surprised. On the one side of the point, the wind was ferocious — like in a movie about Lawrence of Arabia in a sandstorm — if you stood there for 15 min the skin would be sandblasted off your legs. Ouch! The surf was huge and totally chaotic and there was nobody there. It was all rocky and horrible.

On the other side of the point, sandy beaches, and the wind was a bit less strong and offshore, but there were no waves, or only just tiny ones, and absolutely nobody there. I don't know what I was expecting. So I just drove back to Palmerston North.

Thus ended my first surf trip to New Zealand. Just drove around and looked at stuff and never got wet. As surfers do, more often than you would think.

Makara Point and Courage

Makara Point is close to Wellington and pretty good when it is on, but it is in a weird location and requires special conditions. Rod and I were standing in his office talking about parameterized complexity. His office has a wonderful view out across the harbor, and all of the sudden Rod yells, "It's happening!" and out window one could see this slanting black wall of rain approaching from the east. "We have to get moving!"

Then the Chairman of the Department knocked on the door and asked Rod if he would be attending the meeting in the afternoon. Rod said, "Sorry, I have a prior engagement."

The engagement was that we needed to rush off to Makara Point right now! Because the way it works is that when one of the storm fronts from the Antarctic wave machine hits, there are super-winds from the west which build up this big chaotic swell that can make it into Makara Point (unsurfably chaotic). Then, when the front moves through, as we were seeing, the winds reverse direction, and blow offshore, cleaning off the swell, and for about 1-2 h during the reversal there are good surf conditions, which is why you have to get moving! After some hours, it is gale force again, offshore this time, and it is no good again, as the reef is a long paddle out with too much fetch.

One time we drove around to the north from Makara where there is another obscure point. It is really nasty, like a lot of the surf breaks in geologically young New Zealand, just a rocky long shoreline in the middle of nowhere (maybe some sheep farms). Rod claimed this was sometimes a pretty good spot. Me: "You *surfed* this?! What, with Craig?" Rod: "No, alone." Me: "You surfed this *alone*!?".[1]

Rod's answer was, *"Look, if you don't have physical courage, you will never do great mathematics."*

[1] Most surfers do not surf alone, because it is a statistically proven fact that if you do not surf alone, your chances of being taken by a shark are improved by at least fifty percent.

I buy that! This attitude has infused the research community of parameterized complexity that we have nurtured, to a significant extent. We have tutored many young researchers into surfing (meaning body-boarding, which you can have fun with in 15 min, and is arguably superior to stand-up surfing).[2]

Surf Sufism

Rod and I would probably agree on the following mystical tenets:

- Surfing conforms well with mathematical science, where research is generally *not* a team sport. If there is a multi-author paper in the mathematical sciences, it is generally "1 + 1 + 1" rather than "3". Biologists and Physicists should choose rugby as their courage mascot, since research there is typically done by surprisingly large teams. In the mathematical sciences, individual courage, tenacity (what is 25 duck dives ...), risk and imagination are primary, as in surfing. Things can sometimes be achieved relatively rapidly in Mathematical Science; in contrast, breakthrough results in Biology and Astronomy are generally achieved by determined slogs.
- Surfing also conforms well metaphorically with the relationship between pure and applied mathematics. The point of mathematical science is not to sit outside and wait for a big wave, and then catch it, and then ride it straight into the beach, to the applause of the whitewater. You can do that, and have fun, but it is not the main point. The main point is to catch the wave, drop in, harnessing the energy of the wave, and then pull in and go sideways (left or right) at high speed: a physical metaphor for applied mathematics driven by pure mathematics.
  I have always thought that Scottish Country Dancing was appropriate for a logician. Here we stick to surfing.

3 Some Selected Surf Trips and Associated Open Problems in Parameterized Complexity

I reminisce here about a few of our surf trips and locations, associated with some of my favorite open problems.

The Great North Island Road Trip

Thanks to the miminalist (and quite sensible) rules of NSERC governing Canadian scientists in the early 1990's, described above, we were able to do this wonderful thing: I came over to New Zealand, stayed at Rod's house when we were in Wellington, rented a car (thanks to NSERC) and then we made our

[2] Stand-up surfers look down on body-boarders, but they shouldn't! Body-boarders can routinely catch waves that stand-up guys can't, because bodyboarders have stronger sprinting ability, and the stability to handle a lip-launch as the wave breaks "late" (which involves flying through the air a bit). One could go on about this. There are lots of ways to have fun in the waves. When I first met Rod, he was a hand-gunner, which is a rare practice (except in Queensland) closely related to plain old body-surfing.

way around the North Island on our research expedition. Besides mathematical discussions that led to the framing of the core definitions of parameterized complexity and some of the fundamental theorems, we enjoyed surfing at famous surf spots, and stops at famous wineries, and reading poetry out loud in the car. The science road-trip of a lifetime!

On that adventure we engaged the four major themes of the research community we have nurtured in parameterized complexity:

(1) The mathematical frontiers.
(2) The connection between physical adventure and intellectual adventure.
(3) Poetry, love of literature and story-telling.[3]
But there was also another theme that we discussed on the road trip that I was obsessed about at the time:
(4) Theoretical computer science (and mathematical sciences generally) for ten-year-olds.

Rod thought this fourth cultural component was a waste of time. He had put some effort into "math education" and found it time-consuming and discouraging. Geoff Whittle of VUW followed in his tracks and put a lot of effort into math-teacher-training and I think came to the same conclusion. They gave blood at the Red Cross of mathematics education and moved on.

The whole area is a fiasco, for sure. It is comical how it marches in circles, decade after decade. But Frances Rosamond and I have established an island of joy in the wasteland. We have the advantage of focusing on the fundamental mathematics of computer science, which is mostly miles from the shopkeeper arithmetic that dominates the official math curriculum and the standardized exams. We consciously operate as anarchists and skeptics with respect to curriculum. We take our stuff to elementary schools for fun. We can (and do) go, with no prior preparation, into an event with 90 ten-year-olds, tomorrow morning, for three hours and have fun with them, engaging them with the fundamental mathematics of computing (without computers). It's too easy, and joyful. Theoretical computer science offers a lot of great material.

Our principal technical objective on that trip was to try to sort out the $W[1,t]$ dilemma, as described in [D12] (where the reader can find the definitions). The bigger theme that was coming into focus at the time had two principal components:

(1) Understanding the fundamental tower of parameterized intractability classes.
(2) Worrying about whether those definitions had sufficient traction with the obvious, well-known, naturally parameterized problems of "computing practice". That is, how "natural" were these classes?

[3] This is a mainstay of the community. There is a sort of informal award system for the best young researchers in the field, which is to receive a copy of what we call "Mr. Opinion" — this is the book: *The New Guide to Modern World Literature* by Martin Seymour-Smith.

We wanted to give a single homogeneous definition of the $W[t]$ classes: FPT reduction to classes of weft t circuits of bounded small-gate depth. Our initial investigations were mathematically elegant for at least for $t \geq 2$, but $t = 1$ remained frustrating: it seemed that maybe it splintered into a messy hierarchy of $W[1, t]$ classes. Not pretty. Eventually (I'm not sure we achieved this on the road trip) we proved that the nice clean perspective that worked for for $t \geq 2$ did indeed work for $t = 1$, but required special arguments.[4]

With respect to (2), we were beginning to get paranoid about the possibility that various natural problems (e.g., Irredundant Set) might belong to some $W[1.5]$ degree that we didn't know how to define. Those paranoias have receded since our primordial surf trip. The $W[t]$ degree structure seems to be extremely crisp and neat. I honestly think we have spent more time over the years sweating about (2) than about sharks!

On this surf trip we spent a lot of time discussing the basic definitions that would frame this new field. We spent a lot of time discussing possible "parameterized analogs" of landmark theorems of classical (one-dimensional) complexity theory. We spent some time discussing the still-open problem of upward or downward collapse of the $W[t]$ degrees: if, for example, $W[3] = W[4]$ does this imply either: (1) $W[t] = W[3]$ for all $t \geq 3$ (upward collapse), or (2) $W[2] = W[3] = W[4]$ (downward collapse)?

My favorite open problem from this era (as it developed over the years) is:

OPEN: Is $W[t] = W^*[t]$ for $t \geq 3$?

Our original definition of the $W[t]$ classes was aimed at making membership arguments easy: t-bounded circuit weft plus constant-bounded small-gate depth. But for many natural parameterized problems, the translation of the problem into parameterized Hamming weight Circuit Satisfiability requires small-gate depth that increases with the parameter.

Relaxing the constant small gate depth requirement to a connection k' dependent only on the parameter value k gives the broader class $W^*[t]$. Two of our most important technical results (with Udayan Taylor, and Ken Regan, respectively) are that $W^*[1] = W[1]$ [DFT96] and $W^*[2] = W[2]$ [DFR98, DFR98b].

What happens for $t \geq 3$? Our proofs that $W^*[1] = W[1]$ and that $W^*[2] = W[2]$ are quite different. They do not seem to be provable in a unified sweep of argumentation. The fact that the cases of $t = 1$ and $t = 2$ have been settled suggests that this open problem might be approachable.

Sombrio

We have surfed this wonderful break (there are two, actually) over and over — a beautiful walk through the woods, way out on the west coast of Vancouver Island, a bit of a drive from Victoria (like 2 h). Once we even walked down to the beach through a light blanket of snow. Rod said, "Crikey, this is cold!" This is saying a lot, as the sea temperatures around Wellington are far from tropical!

[4] This is not unheard-of in mathematics, that smaller "dimensions" require special approaches.

Everytime we have gone to Sombrio, we have had in tow wonderful new mathematical ideas. On one expedition to Sombrio in 1992, we had along my new PhD student at the time, Michael Hallett. Rod had told me, "Mike, it is your sacred duty to teach Hallett to surf!" I had borrowed a wetsuit for Hallett from my neighbor, Don Beckner, but it was this super-thick wetsuit, the kind of thing you wear with oxygen tanks doing slow-motion gathering of abalone in the North Pacific.

At the time, Hallett and I had just proved (with Hans Bodlaender) that BANDWIDTH is hard for $W[t]$ for all t (eventually a STOC paper) [BFH94].

So that is a complexity *lower bound*. But what about an upper bound? I had tasked Hallett with showing that BANDWIDTH is in $W[P]$. But he came to my office with a very interesting response:

> It cannot be in $W[P]$! $W[P]$ is basically $k \log n$ bits of nondeterminism plus P-time verification. But look! (using modern parlance) BANDWIDTH is AND-compositional. If I have one graph G_1 on n_1 vertices and BANDWIDTH is in $W[P]$ then $k \log n_1$ bits of nondeterministic information are sufficient for a polynomial-time verification. But look! What if I am concerned with G_1 ... G_m and I take G to be the disjoint union of the G_i? Then you are asking for a small amount of information to P-time verify that *all* of these graphs have bandwidth at most k, and that is unreasonable.

This is essentially the intuition behind the lower bounds methods for kernelization [BDFH08]. The central issue is, "too much information compression."

We got Hallett out in the water, with his massive-amount-of-rubber wetsuit. This was all new to him. Sombrio's main break involves taking off a moderate distance in front of a large rock. It is imperative to not hit the rock. A wave came along and Hallett gamely paddled into it, and Rod and I were shouting, "Don't hit the rock!" The next thing we saw was Hallett's bodyboard popping into the air quite dramatically, but not attached to Hallett! What happened? Apparently, he ended up (so floatable!) on his back, looking up at the sky, being washed into the beach on the white-water, like the ginger-bread man.

Rod and I have had several expeditions to Sombrio, and they mathematically connect. On a different expedition than the one related above, we were driving out to Sombrio with Neal Koblitz (of cryptography, elliptic curve and number theory fame). At the time, Rod and I were excited about the notion of kernelization. We had worked out the following definition:

Definition. A parameterized decision problem Π with input (x, k) with $|x| = n$ and parameter k is *kernelizable* if and only if there is a polynomial-time transformation of (x, k) to (x', k') (where k' depends only on k) such that:

(1) (x, k) is a yes-instance of Π if and only if (x', k') is a yes-instance of Π,
(2) $k' \leq k$
(3) $|(x', k')| \leq g(k)$ for some function $g(k)$.

For about 2 days Rod and I thought that the kernelizable parameterized problems might be an *interesting proper subset* of FPT. We were talking about it in the car on the way to Sombrio with Neal. About half-way there, Neal spoke up and said; "But can't you just ..." (with the trivial proof that FPT is the same as kernelizable). This was included in [DFS98] as a lemma. It is also implicit in [CCDF97]. It is often currently attributed to "folklore" but this is not accurate. It is a truly foundational observation, and sets up a whole new game, the kernelization races and the lower bounds program for kernelization. Rod and I thought about it for five seconds and said something like, "Oh, right." It should perhaps be called "Neal's Lemma".

I had another wonderful graduate student at the time: Michael Dinneen. We were working on theory and implementation of algorithms for computing the finite obstruction sets of minor ideals. So a very natural question is the following:

Question: For genus 0, there are two obstructions in the minor order: $K_{3,3}$ and K_5. How many minor-minimal obstructions characterize genus g? Could the size of the minor order obstruction set for genus g be bounded by a polynomial in g?

Theorem (Dinneen) [D97]. Not unless $coNP \subseteq NP/poly$.

Proof. Given (G, k) we must determine whether G does *not* have genus k. This is a $co - NP$ complete problem, since Graph Genus is NP-complete. Our polynomial-sized advice string for the inclusion question, which needs to be polynomial-sized in the input size n of G, consists of the obstructions to genus k that have size at most n (any larger obstructions are irrelevant). Given this polynomial-sized advice, which consists of the relevant obstructions $H_1 ... H_m$ and access to an NP machine, to answer the question we guess how one of the H_i lives in G as a folio and this can be verified in polynomial time. *QED*

OPEN: For basically the same intuitive reasons, can we prove that Bandwidth is not in $W[P]$ unless $coNP \subseteq NP/poly$?

Of course, these early results informed the investigation of kernelization lower bounds.

Newcastle

After serious consideration, Rod and I agreed that the University of Newcastle, Australia, is probably the best tradeoff available on the planet for a mathematical sciences researcher who loves surfing. Any wave / wind conditions, there is always something on: Newcastle Main Break, Flatrock, Kauri Hole, Nobbies Reef, The Spot, The Spit, The Wedge, The Harbor ...

What Rod and I were obsessing about the last time he came around to Newcastle is actually an elegant and fundamental "applied" problem. I give you a linear error-correcting code over $GF[2]$ presented as a generator matrix. Is the minimum Hamming distance for decoding at most k?

This is polynomial-time equivalent to the graph problem:
Even Set
Instance: $G = (V, E), k$
Question: Does G have a non-empty vertex subset $V' \subseteq V$ of size at most k such that for every vertex $v \in V$, $|N[v] \cap V'|$ is even.

This is kind of a "parity variation" on Dominating Set, but equivalent to a really fundamental algorithmic problem in Coding Theory.

We were obsessing about this in Newcastle, and long afterwards, and have spent hundreds of hours on the problem with many seductive false starts building on our results in the paper with Geoff Whittle and Alex Vardy [DFVW99]. I don't think we work on it anymore. It remains a challenge for the younger and smarter. Our (weak) conjecture is that it is hard for $W[1]$.

4 Horizons

We're still kick'in Rod! (With flippers, on bodyboards.) I'm still looking forward to a parameterized complexity workshop at Raymond's surf-camp at G-land, Java. That is a truly amazing place to surf and think. We have been talking about the possibility for years. When are we going to do it? Happy Birthday, Rod!

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