Chapter 2
Distribution System Modelling

2.1 Introduction

As discussed in this chapter, an accurate modelling of the time-varying load models can help in determining the penetration of renewable DG in a distribution system precisely. In addition, the generation characteristics of intermittent renewable resources: wind and solar, which depend on meteorological conditions (e.g., weather and temperature), should be captured correctly in DG planning.

In this chapter, different time-varying voltage dependent load models are first defined. The generation characteristics of renewable DG (i.e., solar PV, wind and biomass) and BES are next presented. Finally, three different distribution test systems used throughout in this study are also described.

2.2 Load Modelling

The time-varying voltage-dependent load model or the time-varying load model is defined as a load model, which is dependent on the time and voltage. Accordingly, the voltage dependent load model in [1] which incorporates time-varying loads at period $t$ can be expressed as follows:

$$
P_k(t) = P_{ok}(t) \times V_k^{n_p}(t); \quad Q_k(t) = Q_{ok}(t) \times V_k^{n_q}(t)$$

(2.1)

where $P_k$ and $Q_k$ are respectively the active and reactive power injections at bus $k$, $P_{ok}$ and $Q_{ok}$ are respectively the active and reactive load at bus $k$ at nominal voltage; $V_k$ is the voltage at bus $k$; $n_p$ and $n_q$ are, respectively, the active and reactive load voltage exponents as given in Table 2.1 [1].
2.3 Generation Modelling

Renewable resources (i.e., solar, wind and biomass) and BES are considered in this book. They can be classified into two major categories: dispatchable and nondispatchable generation as far as their capability of energy delivery is concerned. DG units are considered as a dispatchable source, if its output power can be controlled at a fixed output automatically, typically by varying the rate of fuel consumption. This includes generation technologies such as biomass-based gas turbines and small hydro power plants. In contrast, DG units are considered as a nondispatchable source, if its output power cannot be automatically controlled and totally depends on weather conditions (e.g., wind speed and solar irradiance). Solar PV and wind turbines are examples of such generation technologies.

2.3.1 Biomass

Biogas produced from biomass raw materials or steam produced from the heat by burning biomass can be used to run steam or gas turbines, which are connected to synchronous machines. These machines can be dispatched according to the load demand curve. The reactive generation by the synchronous machines is limited by armature and field currents as illustrated in Fig. 2.1 [2].
2.3.2 Solar Irradiance

The probabilistic nature of solar irradiance can be described using the Beta Probability Density Function (PDF) [3]. This model has been employed in many PV studies such as [4–7]. Over each period (e.g., 1 h), the PDF for solar irradiance \( s \) can be expressed as follows:

\[
\begin{align*}
    f_b(s) &= \begin{cases} 
    \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} s^{(\alpha - 1)}(1 - s)^{\beta - 1} & \text{for } 0 \leq s \leq 1, \; \alpha, \beta \geq 0 \\
    0 & \text{otherwise}
    \end{cases}
\end{align*}
\]  

where \( f_b(s) \) is the Beta distribution function of \( s \), \( s \) is the random variable of solar irradiance (kW/m\(^2\)); \( \alpha \) and \( \beta \) are parameters of \( f_b(s) \), which are calculated using the mean (\( \mu \)) and standard deviation (\( \sigma \)) of \( s \) as follows:

\[
\beta = (1 - \mu) \left( \frac{\mu(1 + \mu)}{\sigma^2} - 1 \right); \quad \alpha = \frac{\mu \times \beta}{1 - \mu}.
\]

The output power from the PV module at solar irradiance \( s \), \( P_{PV_o}(s) \) can be expressed as follows [4–6]:

\[
P_{PV_o}(s) = N \times FF \times V_y \times I_y
\]

where

\[
FF = \frac{V_{MPP}}{V_{oc} \times I_{sc}}; \quad V_y = V_{oc} - K_v \times T_{cy}
\]

\[
I_y = s[I_{sc} + K_i \times (T_{cy} - 25)]; \quad T_{cy} = T_A + s\left( \frac{N_{OR} - 20}{0.8} \right).
\]

Here, \( N \) are the number of modules; \( T_{cy} \) and \( T_A \) are respectively cell and ambient temperatures (°C); \( K_i \) and \( K_v \) are respectively current and voltage temperature coefficients (A/°C and V/°C); \( N_{OR} \) is the nominal operating temperature of cell in °C; \( FF \) is fill factor; \( V_{oc} \) and \( I_{sc} \) are respectively the open circuit voltage (V) and short circuit current (A); \( V_{MPP} \) and \( I_{MPP} \) are respectively the voltage and current at maximum power point.

2.3.3 Wind Speed

The probabilistic nature of the wind speed can be described using the Weibull PDF [5, 8]. Over each period (e.g., 1 h), the PDF for wind speed \( v \) can be expressed as follows:
\[ f_w(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp \left[ -\left( \frac{v}{c} \right)^{k} \right] \] (2.4)

where \( f_w(v) \) is the Weibull distribution function of \( v \), \( v \) is the random variable of wind speed (m/s); \( k = 2 \) is the shape index, \( c \approx 1.128 v_m \) is the Rayleigh scale index; \( v_m \) is the mean value of wind speed that is calculated using the historical data for each time period. The output power at wind speed \( v \) \( (P_{w_o}) \) can be expressed as follows [9]:

\[
P_{w_o}(v) = \begin{cases} 
0 & 0 \leq v \leq v_{ci} \\
a_0 + a_1 v + a_2 v^2 + a_3 v^3 & v_{ci} \leq v \leq v_r \\
P_{\text{rated}} & v_r \leq v \leq v_{co} \\
0 & v_{co} \leq v 
\end{cases}
\] (2.5)

where \( v_{ci} \), \( v_r \), and \( v_{co} \) are cut in, rated and cut out speed of the wind turbine, respectively; \( P_{\text{rated}} \) is the rating of wind turbine; \( a_0, a_1, a_2 \) and \( a_3 \) are the coefficients calculated using any standard curve fitting technique such as ‘polyfit’ routine in Matlab [9].

### 2.3.4 Battery Energy Storage

The BES unit is assumed to be connected to an AC system via bidirectional DC/AC converters that can be dispatched in all four quadrants. It can operate at any desired power factor (lagging/leading) to charge or discharge active power. In other words, the BES unit can operate as a load during charging periods and a generator during discharging periods. It can inject or absorb reactive power as well.

### 2.3.5 DG Penetration Level

The DG penetration level in percent is defined as the ratio of the total energy generated by a DG unit divided by the total energy consumed by a network.

### 2.3.6 Generation Criteria

As reported in Sect. 2.5, most of the DG units are normally designed to operate at unity power factor under the recommendation of the standard IEEE 1547, [10]. This study assumes that inverter-based PV units are allowed to inject or absorb reactive power in compliance with the new German grid code [11]. Biomass gas turbines are
modelled as synchronous machines. Wind farms are modelled as Doubly-Fed Induction Generators (DFIGs) or full converter synchronous machines. Such machines are also capable of controlling reactive power while delivering active power [12]. The relationship between the active and reactive power of a DG unit \( P_{DG} \) and \( Q_{DG} \) can be expressed as [13]:

\[
Q_{DG} = aP_{DG}
\]  

(2.6)

where, \( a = \pm \tan(\cos^{-1}(pf_{DG})) \); \( a \) is positive for the DG unit supplying reactive power and negative for the DG unit consuming reactive power; and \( pf_{DG} \) is the operating power factor of the DG unit.

2.4 Test System Modelling

Three test distribution systems with varying sizes and complexities have been used in this book for showcasing the benefits of intelligent or smart integration of renewable energy sources. These test systems are widely used in the literature of distribution system planning and operation. Single line diagrams of the test systems along with pertinent basic information for DG planning are presented below.

2.4.1 33-Bus Test System

Figure 2.2 shows a single line diagram of the 12.66 kV, 33-bus test radial distribution system. It has one feeder with four different laterals, 32 branches and a total peak load of 3715 kW and 2300 kVAr. The total loss of the base case system is

![Fig. 2.2 The 33-bus test distribution system](image)
211.20 kW. The complete load data are given in Table A.1 (Appendix A) [14]. This system has been used in Chaps. 3, 4, 5 and 6.

### 2.4.2 69-Bus One Feeder Test System

A single line diagram of the 12.66 kV, 69-bus test radial distribution system is shown in Fig. 2.3. It has one feeder with eight laterals, 68 branches, a total peak load of 3800 kW and 2690 kVAr and its corresponding loss of 224.93 kW. Its complete load data are provided in Table A.2 (Appendix A) [15]. This system has been used in Chap. 3.

### 2.4.3 69-Bus Four Feeder Test System

Figure 2.4 shows a single line diagram of the 11 kV, 69-bus test radial distribution system, which is fed by a 6 MVA 33/11 kV transformer [16]. This system consists of four feeders with 68 branches, a total peak load of 4.47 MW and 3.06 MVAr and its corresponding loss of 227.53 kW. The complete load data of this system are provided in Table A.3 (Appendix A) [16]. This system has been used in Chap. 7.

![Diagram](image-url)

**Fig. 2.3** The 69-bus one feeder test distribution system
2.5 Conclusions

This chapter provided a definition of time-varying voltage-dependent load models and modelling of renewable energy resources. Three test distribution systems used in the book were also explained in this chapter.

References


Fig. 2.4 The 69-bus four feeders test distribution system [16]
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