Preface

Number theory is one of the oldest mathematical disciplines. Often enough number theoretical problems are easy to understand but difficult to solve, typically only by using advanced methods from other mathematical disciplines; a prominent example is Fermat's last theorem. This is probably one of the reasons why number theory is considered to be such an attractive field that has intrigued mathematicians and math lovers for over 2000 years.

The subdiscipline called *Diophantine analysis* may be defined as the combination of the theories of Diophantine approximation and Diophantine equations. In both areas, the nature of numbers plays a central role. For instance, a celebrated theorem of Klaus Roth (worth a Fields Medal) states that, roughly speaking, algebraic numbers cannot be approximated by rationals too well (see Theorem 1.4.21 in Chapter "Linear Forms in Logarithms"). As a consequence of Roth's theorem, one can show that certain cubic equations have only finitely many integer solutions, e.g.

$$aX^3 + bY^3 = c$$

with arbitrary, but fixed nonzero integers a, b, c.

In July 2014, the number theory group of the Department of Mathematics at Würzburg University organised an international summer school on Diophantine analysis. In the frame of this event, about fifty participants, mostly Ph.D. students from all over the world, but also a few local participants, and even undergraduate students, learned in three courses about different topics from Diophantine analysis; a fourth course gave in addition some historical background of some aspects of the early research in this direction.

- Sanda Bujačić (University of Rijeka, Croatia) lectured on *Linear Forms in Logarithms*. Starting with some classical Diophantine approximation theorems, her course focuses on Alan Baker's celebrated results from 1966 on effective lower bounds for the absolute value of a nonzero linear form in logarithms of algebraic numbers (another Fields Medal). His pathbreaking approach goes beyond the classical results and allows many interesting applications. In the course notes, which is joint work with Alan Filipin (University of Zagreb, Croatia), it is shown how to solve the system of two simultaneous Pellian equations (a classical application due to Baker and Davenport), how to find all repdigit Fibonacci numbers (a theorem by Florian Luca from 2000), how to determine the bound of the number of perfect powers in a binary recurrence sequence, etc.

- In the course of Simon Kristensen (Aarhus University, Denmark) on *Metric Diophantine Approximation—From Continued Fractions to Fractals*, the classical Khintchine theorems on metric Diophantine approximation are considered (by studying continued fractions by means of a dynamical system). After a crash course in fractal geometry, the notes outline the major topics of recent research as, e.g., Schmidt's game, the question whether the Cantor middle third set contains an algebraic irrational, and a discussion of badly approximable numbers. One of the highlights is the proof that the set of badly approximable β for which the pair (α, β) satisfies the Littlewood conjecture has Hausdorff dimension one; here, α may even be substituted by any countable set of badly approximable numbers (which is a new result due to Haynes, Jensen, and Kristensen).
- Tapani Matala-aho (University of Oulu, Finland) examines in *A Geometric Face of Diophantine Analysis* the so-called geometry of numbers. Building on the notions of convex sets and lattices as well as Hermann Minkowski's fundamental theorems, classical Diophantine inequalities are deduced. Also, some Diophantine inequalities over complex numbers are discussed. Then, he presents some variations of Siegel's lemma over rational and imaginary quadratic fields supplementing Enrico Bombieri's works. Moreover, building on Wolfgang Schmidt's work, it is proved that the heights of a rational subspace and its orthogonal complement are equal (by the use of Grassmann algebras). His lectures end with a proof of the Bombieri–Vaaler version of Siegel's lemma.
- In her course *Historical Face of Number Theory(ists) at the Turn of the 19th Century*, Nicola Oswald (University of Würzburg, Germany) describes the lives and mathematical works of the famous Adolf Hurwitz and his unknown elder brother Julius around the turn of the nineteenth/twentieth century. A careful discussion of the mathematical diaries of Adolf Hurwitz (or at least some of its aspects) provides an understanding of his mathematics on behalf of historical documents; a particular emphasis is put on his relation to David Hilbert. Moreover, Julius Hurwitz' work on complex continued fractions is investigated and further analysed with modern tools from ergodic theory.

This volume presents the lecture notes of these four summer school courses (some of them with additional material). Each of these notes serves as an essentially self-contained introduction. (Of course, a background in number theory might be useful.) Altogether, the reader gets a thorough impression of Diophantine analysis by its central results, relevant applications, and big open problems. The notes are complemented with many references and an extensive register which makes it easy to navigate through the book.

The authors and the editor are grateful to the anonymous referees for their valuable suggestions and remarks that have improved the book. They also thank Springer for making the collection of lecture notes a book and, in particular, Clemens Heine for his encouragement.

When turning the pages, it is impressive to see how one can approach frontiers of current research in this direction of number theory quickly by only elementary and basic analytic methods. We wish to take much pleasure in reading.

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