Chapter 2
Inferentialism and Its Discontents

A perennial question in consideration of logic concerns where the rules of logic come from? This question is overflowing with sub-questions regarding, for example, what is meant by *the* rules of logic; whether or not we are concerned with the justification of rules, or the meaningfulness of certain rules, or both (these are often run together); how do we understand the relationship between logical rules and truth; what is the normative role of logical rules? In this chapter, I provide a brief overview of inferentialism in Sect. 2, before going on to identify a number of concerns frequently made against it. Section 3 develops the discussion of the circularity of justifying logical rules that I outlined in the introduction. These problems are compounded by the issues raised by paradoxical connectives such as "tonk", discussed in Sect. 4. One suggestion made in the literature is to combine the inferentialist and referentialist approaches in a "modest" form of inferentialism, which we discuss in Sect. 5, pointing out that this approach runs afoul of issues facing the full determination of meaning in terms of categoricity and compositionality. I finish, in Sect. 6 by discussing a more promising response that does not require semantic justification or rules, but looks to internal constraints on rules, namely that they are "harmonious" However, this, it is suggested, is still problematic from the point of view of circularity, and, moreover, it also does not circumvent the issues of categoricity and compositionality.

1 Inferentialism (A Brief Overview)

Inferentialism is an approach to the meaning of expressions that takes the agents' inferential activity to be the primary semantic notion. Rather than take reference, or denotations of expressions, to establish meaning, it is certain rules of inference (and inferences themselves) that do this job. This belongs to a wider tradition in philosophy of language, perhaps most clearly articulated in the work of Robert Brandom (e.g. [1]), who argues that our language use ought to be understood in terms of a "game of giving and asking for reasons" (GOGAR) [2]. Broadly speaking, then, inferentialism is a kind of conceptual role semantics, in which the meaning of a con-

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cept is determined by its conceptual role.[1] This, following the work of Wilfrid Sellars (e.g. [4]), and also the later Wittgenstein (e.g. [5]) may be seen as constructing an alternative position that is distinct from the much more standard truth-conditional semantics, or, more broadly, representationalism. It is not, however, the case that the inferentialist picture of meaning must be at odds with more traditional approaches to semantics, as we shall see later (Sect. 3). Rather, it is perfectly plausible that talk of "truth" and "reference" can be maintained. What is important is that these should not be considered to be *epistemically* or *explanatorily* prior. As such, as I show later in this chapter, one might maintain a "modest" form of inferentialism in which rules of inference may be said to determine (in some way) a referential semantics. More standard approaches to inferentialism have rather suggested that inferentialism brings with it a revision of standard semantic theories such that truth is displaced by other more epistemically tractable notions such as "proof" or "assertibility". [2] It is, on this view, central to an approach to meaning that it is "graspable" in agents' ordinary understanding, and for many authors, this has led to consideration of constructive approaches to logic (discussed further below in Sect. 4).[3]

What coalesces inferentialist approaches is their opposition to representationalist approaches to meaning that are based upon word-world relationships that are supposed to be established directly. For example, Jerry Fodor's [10–12] externalist approach to meaning takes it that the meaning of a concept is constituted, in the main, by a nomological relation between mind and world, where concept acquisition is a matter of 'getting nomologically locked to the property that the concept expresses' [12, p. 125]. So, for example, the content of a concept such as CUP is explained by its being "locked" to the property of "being a cup" Understanding the meaning of a concept has to do not with accepting a specific range of inferences (or even associated beliefs), rather it is simply a matter of having the ability to make accurate tokenings of one's concepts. It is plausible, then, that agents can be locked to the same properties whilst entertaining vastly different inferential patterns regarding the linguistic expression that tokens that concept. Without going into too much detail, consider that this view is fairly intuitively too coarse-grained. It seems, for example, that the expressions "water is clear" and "H_2O is clear" are distinct, whilst, on the locking view, the expressions have the same content. Relatedly, Georges Rey [13] has argued against this view with what he calls the "fortuitous locking problem" The kernel of his worry is this: if we pay no attention whatsoever to the inferential relations that agents associate with a given expression, then the nomic relationships in virtue of which they have those specific concepts (as well as any related propo-

[1] Block [3] provides an excellent overview of the collection of positions that fall under this view.

[2] Michael Dummett is perhaps the most well-known proponent of this account (e.g. [6–8]).

[3] Perhaps most famously, Dummett's "manifestation thesis" is used to argue for an anti-realist approach to logic in which the manifestation of our ability to verify a sentence is constitutive of agents' knowledge of that sentences meaning. This requirement, that an agent have some justification (such as a proof, or verification) of a statement, fairly quickly leads (according to Dummett) to a rejection of classically valid logical laws such as law of excluded middle (which is supposed to be valid even in the absence of verification of either disjunct). Further discussion of these issues is in [9].

Fig. 1 Matrices for conjunction

α	β	$\alpha \wedge \beta$
T	T	T
T	F	F
F	T	F
F	F	F

sitional attitudes and so on) will be largely opaque and subjectively inaccessible to those agents. In that case, it is possible that we might stand in a nomic relationship with a property that is, intuitively, not the subject matter of our conversations. As he puts it, '[i]insofar as one is moved by standard externalist claims about the reference of terms, one supposes that they refer to whatever real phenomenon the people in one's community are getting at in their uses of the terms' (p. 317); but, Rey goes on to say, the lockings must be fortuitous, otherwise, they would "not give rise to any meaning intuitions, they would not enter into cognitive deliberations, and, most importantly, they would not seem to figure in any cognitive psychological laws" (p. 318). The concern is that purely covariational relations would not seem to safeguard against agents being linked up with properties that are not those to which they think we refer. The account threatens to make the reference-fixing facts both opaque and implausibly separate from rational inquiry and agents' understanding.

Of course, these issues may well be adequately dealt with, but even so it is widely accepted (even by Fodor at times) that an inferentialist story is better suited for logical expressions. Say, for example, that we want to take a representationalist approach to the meaning of logical expressions. Then we might say that the meaning of a logical expression is the contribution that the expression makes to the truth-conditions of propositions in which it occurs, and where those truth-conditions are constraints on the way that the world must be if that proposition is true.[4] Then, the occurrence of $\wedge$ in the proposition expressed by the sentence "$\alpha \wedge \beta$" contributes to the truth-conditions of $(\alpha \wedge \beta)$ is its Boolean function as captured in the standard truth-table (Fig. 1).

We can then say that $\wedge$ has the semantic function of conjunction because its meaning is such that it requires the sentence to conform to the logical properties of conjunction. Even if it is possible to deal with obvious issues facing abstract entities, this view presents overwhelming problems when it comes to providing an epistemology of logical meanings. How, for example, could a token mental state come to have these specific logical properties as its content, and thus play a role in intentional psychology? Alone, they seem *prima facie* inadequate to type token mental states. The problem is one of determining how it is possible for mental transitions to come into contact with those (presumably abstract) truth-conditional contents. This is somewhat difficult since it does not seem possible, even in relatively simple cases, to determine every instance of the truth-preserving schema for conjunction, say, in advance of being able to know whether or not it is valid. Recall that an inference of

[4] Peacocke [14] makes a similar suggestion, though his position is more nuanced as discussed below.

the form: "$\alpha \wedge \beta \vdash \beta$" is supposed to be valid whenever β is a logical consequence of $\alpha \wedge \beta$, so that, when $\alpha \wedge \beta$ is true, β is true also. The inference, however, is typically thought to hold between speech-acts, judgements, assumptions, and so forth, whereas its validity is, on this story, grounded on propositional contents. Of course, on this view, in order for an agent to entertain a proposition involving conjunction, the agent must first grasp the truth-conditional content of conjunction. Then, famously, grasping conjunction will involve conjunction itself: otherwise, how could the agent antecedently grasp that truth-conditional content? If what we are after here is an account of the epistemological justification of conjunction, we are unlikely to be satisfied by any account which (a) utilises the concept in its explanation (b) presupposes antecedent knowledge of the proposition expressed by a sentence involving the logical term. As Mark Kalderon puts it;

> [One] cannot coherently claim that a speaker could come to entertain the content of [conjunction] by knowing the standard explanation since the standard explanation presupposes the antecedent intelligibility of [conjunction]. [15][5]

There is also a difficulty in explaining how agents such as children can make inferences involving conjunction without an ability to know its truth-conditional structure, since they would be disqualified from grasping the relevant logical content. Any such account would appear to be too conceptually demanding, since many people cannot state the truth-tables or other truth-functional explanation of the logical constants, whilst, (at least seeming to be) making logical inferences.[6]

The inferentialist approach to meaning, on the other hand, seems to be especially well-suited to logical expressions, suggesting that their meaning is constituted by a set of inferential rules, and where these rules have a substantial connection with ordinary reasoning practices with those expressions.

2 Dispositions and Circular Justifications

Unfortunately, however, there are also several problems facing attempts to justify the validity of inferential rules, which, together, may push us back towards the representationalist account. As we discussed in the introduction, whilst the inferentialist approach looks to offer a clear way in which to think of how we come to use certain logical rules, it does not yet provide an account of why *these* rules are justified. But, even asking questions about the latter is tricky, for a number of well-worn reasons that I briefly rehearse here. Following in Carroll's footsteps, Quine [17], for example, argues that if we think that rules originate in some form of convention, then a regress argument takes hold:

[5]Conjunction replaces existential quantification in the original.

[6]For extended discussion, see [16]. This issue is also raised by Dummett's (e.g. [6], p. 216–7) argument against grounding logical content in truth-conditional semantics due to concerns regarding the acquisition of the relevant concept.

> [...] Derivation of the truth of any specific statement from the general convention thus requires a logical inference, and this involves us in an infinite regress [...] logic is needed for inferring logic from the conventions. [17, p. 351]

A more generalized form of the problem that Quine identified is that it is difficult to hold together the idea that logical rules have normative force, with the ability to question the justification of those rules. Providing justification for basic logical rules brings with it the issues of circularity that we discussed in the introduction. One option, that we shall discuss in this section, is to say that this is an acceptable form of circularity since it is just a *pragmatic*, rather than a *justificatory* circularity. Dummett [7], for example, argues that the circularity is unproblematic since it is not a gross circularity as the validity of a rule like *modus ponens* does not need to be used as a premise in an explanation of its own validity:

> We have some argument that purports to arrive at the conclusion that such-and-such a logical law is valid; and the charge [...] is that at least one of the inferential steps in the argument must be taken in accordance with that law. We may call this a "pragmatic circularity" [...]. [7, p. 202]

As such, pragmatic circularity is judged, by Dummett, to be safe from the point of view of explaining the validity of modus ponens. But, as he goes on to say, this will do nothing to assuage concerns for somebody who genuinely doubts its validity[7]:

> If the justification is addressed to someone who genuinely doubts whether the law is valid, and is intended to persuade him that it is, it will fail of its purpose, since he will not accept the argument. [7, p. 202]

But, this misses the force of the problem, by serving only to highlight the way in which logical rules can not be put in question. Rather, pragmatic circularity clarifies the rather autocratic nature of logical rules insofar as the attempt to dispute those rules brings with it their infinite re-establishment.[8] That an argument involving pragmatic circularity uses a rule in providing an explanation of its validity is problematic since no justification for the rule itself has been provided. Moreover, as pointed out by Celluci [19], this only paves the way for appeals to the justification of clearly non-deductive rules such as the rule of abduction.

One common suggestion that is found in the literature that is supposed to uphold the "basicness" of rules such as modus ponens is that pragmatic circularity is not question begging because the correct use of a logical rule is grounded in agents' dispositions to use that rule (or otherwise some kind of implicit grasp of that rule) (e.g. [14, 15, 20–24]). There are a variety of different positions that orbit around a central idea: a circularity of justification for a specific set of rules is not problematic for a view on which even to think with a concept requires one to be disposed to infer according to those rules. For example, Boghossian [21, p. 230] thinks that an agent's

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[7] These points hold for any logical rule, but it is worth noting that modus ponens is not unrestrictedly valid in logics such as paraconsistent LP, see [18].

[8] It is also worth noting that pragmatic circularity quickly leads us to a form of epistemic circularity once we bring in talk of validity and truth.

dispositions to infer according to the classical natural deduction rules determines what that agent means by logical expressions such as “and”, and “if, then”:

> […] the logical constants mean what they do by virtue of figuring in certain inferences and / or sentences involving them and not in others. If some expressions mean what they do by virtue of figuring in certain inferences and sentences, then some inferences and sentences are constitutive of an expression’s meaning what it does, and others aren’t. [20, p. 353]

With these in place, Boghossian takes it that such an agent is justified in making inferences according to those rules, since those rules are understood to be stipulated to be valid in this way. Then, “and”, for example comes to have the meaning required for the validity of the inference rules defining conjunction.[9] Furthermore, according to Boghossian, it is necessary for a thinker to have those inferential dispositions to even have the relevant logical concept (and grasp the content of the expression); without those dispositions, a thinker could not even entertain beliefs involving that content.

On Christopher Peacocke’s [14, 23, 24] view, a thinker must find certain inferences “primitively compelling” [10] An inference is primitively compelling if a thinker finds it compelling, it is underived from other principles, and its correctness is not answerable to anything else for possession of a certain concept [25, p. 6]. Thus, in order for a thinker to have a logical concept (or to grasp a content in Peacocke’s terms), there are a substantial set of constraints, called possession conditions, that a thinker must meet, and these constraints also determine the meaning of that concept. These, again, are certain inferential practices to which that thinker must be disposed. Roughly speaking, Peacocke endorses a supervenience thesis, on which, A and B grasp the same content N iff they have the same set of fundamental inferential dispositions I (involving N). According to Peacocke, these inferential dispositions are also supposed to individuate content, constituting what is required of a thinker to grasp a given content, so the corollary also holds: A and B have a specific set of inferential dispositions I iff they grasp the same content N. Despite their differences, the general position can be distilled as follows:

(Supervenience): Where two thinkers A and B have attitudes involving logical expressions Nα and Nβ, and they are disposed to accept the validity of the same set of fundamental inferences I involving Nα and Nβ, then A and B grasp the same logical content.[11]

So, for example, on Peacocke’s view, the possession condition for “and” is that the thinker finds “primitively compelling” the standard introduction and elimination rules for conjunction. That is to say, for a thinker to grasp the content of conjunction, they must have a disposition to infer in accordance with the introduction and elimination rules as a matter of semantic competence. A thinker would not be ascribed beliefs or judgments involving the content of conjunction if they did not have those specific dispositions to infer.

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[9] Whilst similar, Mark Kalderon [15] emphasises agents’ dispositions to accept the validity of certain inferences.

[10] As with Boghossian, these inferences conform to the classical natural deduction rules.

[11] Kalderon [15] has a similar account.

Let us briefly review a number of criticisms of this view.[12] Timothy Williamson [29, Chaps. 4, 5] has perhaps been most vocal in providing reasons to think that any such account will fail to provide justification for the validity of inferential rules. Williamson argues convincingly, that there is no straightforward connection between understanding the meaning of a logical expression and assent to the validity of arguments in which it figures. The argument is two-pronged. First, Williamson argues that there are many cases where speakers of a language fail to be disposed to assent to instances of the rules that are taken to define logical connectives whilst yet being judged as linguistically competent. Call this the argument from dispositional discrepancy. Let us briefly flesh out this issue. Any approach that rests upon a principle, such as (Supervenience), provides conditions that a token cognitive state must meet, by metaphysical necessity, in order to type that state as a specific propositional attitude. In relation to belief-formation, these dispositions are manifested in fixing propositional attitudes relevant to a thinker's judgments. However, it does not seem acceptable to say that in order to grasp a content, every inference of introduction or elimination needs to be manifested in judgments. If that were so, then thinkers grasping a content would not be capable of making errors of judgments, or basic inferential mistakes; if they did, they could not be ascribed a judgment with that content. This seems too inflexible a requirement to deal with the practice of actual thinkers. One way that the problem might be dealt with is by allowing for performance errors. It may be possible, still, to introduce a fairly standard performance/competence distinction, in which a thinker's actual inferential practices do not tell against their competence, because of various performance errors, interference factors, memory limitations, distractions, and so on.[13] One might appeal, for example, to the idea that there are cases in which thinkers make mistakes regarding inferences of introduction rules, because those dispositions are masked. Though, for this to have traction, one has to be careful, since, it is required that whatever is constitutive of content ought to be manifested in thinker's actual dispositions in order to ascribe the correct content to that thinker.[14] Now, say, for example, that it is allowed that a thinker can be ascribed an attitude with a specific content in terms of a particular competence that thinker possesses, but that this competence is not performed in the thinker's judgments. Then, it becomes tricky to see how a thinker can be attributed with that content rather than some other content that does seem to be performed in their judgments.[15] In this regard, even in the simple case of conjunction, the inferentialist account seems to be on shaky ground. For example, there are exemplary failures of closure under conjunction. In cases such as the preface paradox (as well as complicated tautologies), routine failures to infer according to conjunction introduction without restriction occur. Regarding the

[12] For discussion and argument against these views, see [26–29].

[13] See also the discussion in [26] on these issues.

[14] Otherwise, it wouldn't seem possible to say that *by those dispositions* a specific content determined by a specified set of rules is being manifested, rather than some other content altogether. Whilst, for conjunction, this does not look to be a difference that makes a difference, since most accounts of conjunction agree, this is not the case for other logical constants such as negation, for example.

[15] In essence, this objection to dispositionalism is in Kripke [27, p. 30].

elimination rules, consider cases of the conjunction fallacy as in the infamous Linda problem [30]. In such cases, participants routinely and systematically fail to infer in accordance with the conjunction elimination rule; and rather, suggest that the probability of a conjunction is greater than one of its conjuncts. Whilst many philosophers have discussed the relevance of heuristics and biases literature to performance ability, the relevance to competency is not often recognized, for if the fallacy corresponded to performance error alone, there should be random failures with little correlation, but those failures are systematic and consistent:

> If each departure from normative responding represents a momentary processing lapse due to distraction, carelessness, or temporary confusion, then there is no reason to expect covariance among biases across tasks (or covariance among items within tasks, for that matter) because error variances should be uncorrelated. [31, p. 646]

The issue may be understood by means of the distinction between whether or not the dispositions in (Supervenience) are supposed to be read *descriptively*, or *normatively*.[16] On the latter view, the normativity would then be cashed out as inhering in the meanings of the logical constants, so that having a concept of a logical constant requires having a disposition to make certain inferences as normatively correct.[17] The problems raised above can be put like this: say that the specified set of inferences are supposed to be descriptive of actual use (e.g. [25]), then this does not seem to adequately map onto agents' ordinary reasoning; if they are supposed to be *normative* over use (e.g. [22]) then the flexibility of use of expressions would make it hard to see why we should think this offers a way of grounding the justification of rules (i.e. why *this* rule, rather than *that* rule?). It seems, for example, that Williamson is correct to say that thinkers routinely understand even simple logical contents such as conjunction, whilst nonetheless failing to instantiate the relevant inferential competency. But, conceiving of such inferential transitions as required to have thoughts involving a content (and, so also attribution of content to a thinker) would effectively entail routine content-failure. This seems both implausible and unwarranted, particularly given the significant literature regarding the flexibility regarding mental states that is nonetheless consistent with thinkers having the capacity for attitudes with the relevant content. Routine failures to infer according to a set of natural deduction rules does not provide good evidence for content-failure, in which case, the semantic content of logical expressions is not settled by the inferential rules thinkers are disposed to make.[18]

[16] Simplifying a little, Peacocke takes the former view, whilst Boghossian takes the latter.

[17] We may presume, then, that for Boghossian, elements of the problem posed by Carroll are immunised since, it can not be possible for Tortoise to have the concept of logical conditional without feeling some sort of normative "pull" to make the correct inference. In Chap. 3 I shall discuss alternative approaches to the normativity of rules.

[18] These problems are not assuaged by an appeal to implicit conceptions as in Peacocke [32] (but note that implicit conceptions are not supposed to be necessary for the simple cases discussed here). Briefly, an implicit conception is taken as a possession condition for a concept that influences and explains our judgments, but is less stringent than possession conditions. Nonetheless, for implicit conceptions to do any serious explanatory work, they must be guaranteed to be truth-preserving

Second, Williamson argues that there are cases where competent speakers of a language may be disposed to assent to the validity of deviant sets of inference rules. This may be the case, for example with material modus ponens, $\{\alpha, \alpha \supset \beta\} \therefore \beta$, which, whilst classically valid is paraconsistently invalid.[19] Williamson argues that such deviance should neither be taken as evidence that a paraconsistent logician fails to understand the meaning of the expression "if, then", nor as evidence that they are using the expression with a different meaning. Williamson points to Vann McGee's supposed counterexample to modus ponens, with the suggestion that modus ponens is not, therefore, a valid rule of inference. Williamson goes on to say that, supposing McGee is incorrect, it would still not be the case that we should say that McGee does not grasp the meaning of the expression "modus ponens" Generalising further, the suggestion is this: given that there are philosophical experts that deny that basic inference forms are valid, it cannot be the case that for an inference form to be meaning-determining, any expert speaker competent with the relevant expressions must be disposed to accept it. Call this the argument from deviance.[20]

Both arguments are taken by Williamson to give grounds to argue against the idea that there are such things as meaning-determining rules, and, as such, that inferentialism fails to provide a decent account of logical meaning. I think this is rather too quick, but nonetheless, it does seem to tell against any position that rests upon a principle that looks like (Supervenience).[21] The important moral, for our purposes, is that these accounts do little to dispel the concerns over circularity of justification as identified in our discussion of Carroll's short story. That is, even if we could find some way of adequately spelling out inferential rules (by means of thinkers' dispositions) in terms of introduction and elimination rules in natural deduction systems for classical logic, this will not provide an account of their justification on pain of circularity.[22] One response that the inferentialist might take here is to say that it is simply not possible to challenge the classical meaning of logical expressions such as "and" or "not" in the sense that they would fail to be genuinely meaningful. Boghossian [22] makes an argument of this form, suggesting that all that is required to immunise the inferentialist against objections from circularity is that "and" means what we all take it to mean. However, Glüer [36] has pressed an objection against this view, to the effect that this entails that we are required to know that the classical rules are

(Footnote 18 continued)
(according to Peacocke). The problem is that Peacocke stipulates as an additional premise that the implicit conception of "or" is: (α or β is true iff either α is true or β is true) [32, p. 46], so the regress is reinstated. Similar problems occur for Peacocke's [33] qualification that it is a fundamental "rule of reference" that determines meaning, since, there the constraints on grasping meaning are even more substantive than those discussed here.

[19] Loosely, a paraconsistent logic is just one in which the inference $\alpha, \neg\alpha \vdash \beta$ (where β is an arbitrary formula) is invalid.

[20] Note that, if, as Priest [34] suggests, a difference in truth-conditions is taken to be sufficient for a difference in meaning, then a representationalist account (such as Williamson's) will equally face difficulties regarding similarity of meaning for proponents of classical and paraconsistent logics.

[21] That is, I accept that the arguments put pressure upon the attempt to justify certain inferential rules in terms of the dispositions of ordinary agents with expressions of natural languages.

[22] See also the discussion in [35].

valid on the meaning of "and" prior to knowing that "and" means what we take it to mean, that is, if "and" is to be understood as the content that makes those rules valid.

Without getting ahead of ourselves, we might also think that this calls into question a prescriptive and monological approach to inferentialism. For our present purposes, the issue is that a view such as Boghossian's commits an agent who understands the meaning of a logical expression to being disposed to make an inference in the correct linguistic context, as a kind of directive such as: "on assuming α, infer β!". Whereas, as suggested above, there are many cases of ordinary reasoning where agents do not infer β from α. Given this, we may well anticipate that any attempt to straightforwardly justify some specific formal inferential rules by appeal to ordinary reasoning will be tricky, to say the least.

3 Paradoxical Connectives

A slightly different, though equally well-trodden, problem that also pushes inferentialism towards semantic justification comes in the form of the paradoxical connective, "tonk" [37]. Tonk is a binary connective defined as follows:

$$\frac{\alpha}{\alpha Tonk \beta}\ (Tonk\text{-I}) \qquad\qquad \frac{\alpha Tonk \beta}{\beta}\ (Tonk\text{-E})$$

Tonk appears to have been provided with a definition, but successive application of the rules (as long as the logical structure is transitive) allows us to infer any conclusion from any premise:

$$\cfrac{\cfrac{\alpha}{\alpha Tonk \beta}\ (Tonk\text{-I})}{\beta}\ (Tonk\text{-E})$$

Something has gone awry with the definition. Tonk trivializes inferential practice since it allows a thinker to infer arbitrary propositions from any premise, and so it does not seem to express a meaning. It is, therefore, necessary to provide a way of discerning which rules of inference confer genuine meanings on a logical connective. The attempt to figure out exactly what has become a philosophical industry involving the various constraints one might place on a definition to avoid the problem.[23] A number of possibilities for ruling out such paradoxical connectives are available in the literature. For example, one suggestion [40] is that the rules must give a conservative extension whereby a logical constant introduced into the vocabulary should not allow for an inference not involving the new constant to be deducible that was not deducible before the constant was added (e.g. [6, p. 454]). Tonk clearly fails on this account, but, given however that conservative extension is relative to the underlying logical system, this does little to settle the problem of whether or not it expresses a genuine meaning. As Enoch and Schecter [28] put it:

[23] See, for example, [24, 38, 39].

> […] considerations of conservativeness only apply to a concept given some inferential background; whether a rule is a conservative extension can only be evaluated with respect to some presupposed derivability relation.

For example, as Priest [34] points out, adding the inferential rules for Boolean negation to classical logic is conservative, but it is not for intuitionist logic. There are also well-known occasions where conservative extension fails, such as for second-order logic, which are nonetheless taken to be valid.[24] The issue with such paradoxical rules is that we are pushed towards thinking that proof-theoretic validity may well serve to give us an epistemological grip on certain inferences according to a proof-theory, but these may well bring with them unwanted consequences, potentially trivializing the derivability relation over a language. The lesson, according to Prior [37], is that a semantic justification of logical implications must be given in advance of their inferential characterization, since it is entirely plausible that a set of rules will fail to be valid. If true, this would not, necessarily, mean that we must leave the ambit of inferentialist approaches behind, since, as we now discuss, there are "modest" forms of inferentialism which are perfectly compatible with certain kinds of representationalism.

4 Modest Inferentialism and Its Problems

The kind of thought described above leads us back to considerations regarding the semantical characterisation of consequence with which we began. However, it is possible, as suggested above, to maintain an inferentialist approach whilst combining this with a truth-conditional semantics. For example, there is a suggestion, perhaps most strongly argued for by Christopher Peacocke [25, 32, 42, 43] (taking up a response originating in [38, 39]), which suggests that certain rules may fail because they are not sound with respect to truth-conditional semantics. Whilst the details of these earlier responses have an air of too-swiftly leaving aside motivations for the proof-theoretic approach, Peacocke attempts to resolve this issue by ensuring that no assumptions are made regarding the underlying semantics other than their interaction with the relevant inference rules. What results is a *modest* form of inferentialism where a specified set of inference rules determine the truth-conditional content of logical constants. Let us momentarily set aside our worries regarding dispositions and primitive compulsions, so that we may discuss this as a possible response to problems of circularity and paradoxical connectives.[25] This is to combine the benefits of inferentialist and representationalist approaches so that, roughly, a specified

[24] For example, [41] discusses the way in which second-order quantifiers may lead to systems that are not conservative extensions of e.g. Peano arithmetic.

[25] It is plausible, for example, to hold a form of modest inferentialism in which a set of formal rules determine truth-conditional semantics without taking a stand upon how those rules are related to our reasoning practices, e.g. [44–46]. Note, though, that, whilst this may be thought to deal with paradoxical connectives, it would not dispel our concerns with the circularity of rule justification.

set of natural deduction rules are taken to determine the truth-conditional content of logical constants. On Peacocke's [24, 25] view, this is put in terms of "reading-off" valuational semantics from inference rules that a thinker finds "primitively compelling". The innovation here is that only inferences that are truth-preserving can be genuinely content-determining. This constraint, as applied to tonk, shows that there is no binary function on truth-values that validates both its introduction and elimination rules, and, hence, there are no coherent semantics for tonk. The introduction rule requires that "α tonk β" is true when α is true and β is false, but the elimination rule requires that when α is true and β is false, "α tonk β" is false. Hence, there is no coherent semantic assignment of truth-values for tonk since there is no truth-function that makes tonk-inferences truth-preserving. Then, as Boghossian [47] has it; there can be no determinate way the world has to be, if "α tonk β" is to come out true. Resultantly, this not a "pure" inferentialism, but rather a modest form of inferentialism, in which truth-conditional semantics still plays a role in ensuring that our meanings do not misfire. According to Peacocke, this is practically forced on us by considerations regarding tonk-like connectives:

> [w]hat I have argued equally implies that the roles that determine genuine warrant cannot be picked out unless we rely on reference and truth. Even if mere warranted assertibility were the aim of judgement, we would still need to say which sets of rules determine genuine meanings and which do not. Unless this theorist has some new resource for ruling out spurious meanings, he will need to rely on considerations having to do with reference, semantic value and truth; and then his conceptual role theory is no longer pure. [48, p. 389]

It is not quite clear in Peacocke's work, for example, how exactly a set of inferential rules determine truth-conditional content. He does offer, however, a "determination theory" invoked in [23], with the general requirement that:

> The given rules of inference, together with an account of how the contribution to truth-conditions made by a logical constant is determined from those rules of inference, fixes the correct contribution to the truth-conditions of sentences containing the constant. [49, p. 172]

The trouble with Peacocke's account, however, is that we are given little more than a means of checking the credibility of a definition (in his preferred classical natural deduction setting) to see if there is a corresponding classical truth-function. Peacocke takes the existence of a meaningful definition to depend upon the existence of an associable Boolean truth-function. For example, to check that, the classical inference rules define conjunction, we need only look around for a binary-function making those rules truth-preserving under all assignments [25, p. 18ff]. Take, for example, the classical sequent rules for conjunction, where, following Gentzen's suggestion, the commas on the left of $\vdash$ are interpreted as "and", and those on the right as "or":

$$\frac{\Gamma, \alpha, \beta \vdash \Delta}{\Gamma, \alpha \wedge \beta \vdash \Delta}\ \wedge\text{-L} \qquad\qquad \frac{\Gamma \vdash \Delta, \alpha \qquad \Gamma \vdash \Delta, \beta}{\Gamma \vdash \Delta, \alpha \wedge \beta}\ \wedge\text{-R}$$

We can read these as telling us the conditions under which it is legitimate to derive a conjunction from its conjuncts (and a conjunct from a conjunction). But, we can equally read the rules as placing constraints upon when a conjunction may be judged to be true or false, given the truth or falsity of its conjuncts. On the latter, we can extract, from the rules, the classical truth-function for $\wedge$:

$$f^{\wedge}(x, y) = \begin{cases} 1 & if\ x = 1\ and\ y = 1 \\ 0 & otherwise \end{cases}$$

Peacocke fails to offer any formal account of how the determination can be made precise or generalisable (see for example, [25]). As such, let us consider a formal framework that does this, if only in order to point out a number of problems with the position.

First, let us provide a generalised and suitably abstract definition of a "logic" which we will think of a structure of entailment, rather than a consequence operation, since we are primarily interested in an inferentialist account.[26]

Definition 1 Let S be a set of well-formed formulae in some propositional language $\mathcal{L}$, and $\vdash$ is a binary relation $P(S) \times P(S)$ (where $P(S)$ is the set of all finite subsets of S). Then, let a "logic" L, (as entailment structure) be just an ordered pair, $(S, \vdash_L)$. We say that L is normal whenever $\vdash_L$ is reflexive, transitive, monotonic and finitary over each element of S.

Entailment structures can be restricted in different ways, which we shall think of in terms of sequents in a logic.[27]

Definition 2 A sequent is an ordered pair, $\langle \Gamma, \Delta \rangle$ where Γ, Δ are finite (possibly empty) sequences of formulas of S. Say that a right-asymmetric sequent (Γ, α) is restricted to at most a single formula on the right; a left-asymmetric sequent (α, Γ) is restricted to at most a single formula on the left, and a symmetric sequent (Γ, Δ) has no such restrictions. A sequent rule $\mathcal{R}$ in any logic L is an ordered pair consisting of a finite sequence of sequent premises and a sequent conclusion $\mathcal{R} = \langle \{SEQ^P\}, SEQ^C \rangle$, and, in case the list of premises is empty, the instance of a rule is called an axiom.

In this way, we may think of a specific logic to be determined by a proof structure (set of axioms and sequent rules), where any collection of sequents $\mathcal{S}$ that is closed under standard structural rules determines a finitary, normal, logic. For example, it is the case that $\Gamma \vdash \alpha$ iff for some finite $\Gamma_0 \subseteq \Gamma$, we have $(\Gamma_0 \vdash \alpha) \in \mathcal{S}$.[28]

There are a number of ways in which we can think of truth-conditional semantics as being determined by a set of rules, but, to my mind at least, the most obvious from an inferentialist point of view is to construct it from chains of entailments in a logic. In this setting, we are only interested in standard, right-asymmetric proof-structures, so we can give a simple definition in terms of closed and consistent theories of a logic.

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[26] For similar approaches to logic as abstract structures, see [50–52].

[27] On the relationship between sequent calculus and natural deduction frameworks, see [53, 54].

[28] For symmetric sequents, this is $\Gamma \dashv\vdash \Delta$ iff for some finite$\Gamma_0 \subseteq \Gamma$, $\Delta_0 \subseteq \Delta$ we have $(\Gamma_0 \dashv\vdash \Delta_0) \in \mathcal{S}$. See also [50, p. 113]. Note that I use $\dashv\vdash$ rather than typical $\vdash$ for symmetric sequents to highlight that they can be read in both directions as discussed in the following chapter.

Definition 3 (Closure) Say that a theory is some subset, $\Sigma \subseteq S$ which is $\vdash_L$-closed, for some L, when, for all $\Sigma \vdash_L \alpha, \alpha \in \Sigma$.

Then, let us define completeness and consistency for a logic L.

Definition 4 A logic L is consistent when there is no formula α, such that $\emptyset \vdash_L \alpha$ and $\alpha \vdash_L \emptyset$.

We also require a general definition of a maximal theory for a logic. The obvious thing to do would be to follow a standard Lindenbaum-Asser construction, but we do not necessarily want to prejudice the construction towards any specific set of proof-theoretic constraints on L, so we do not want to employ classical negation. To deal with this, we utilise what Béziau (originally developed in [55]) terms a relatively maximal theory.[29]

Definition 5 For a theory Γ, a formula α, and a relation $\vdash_L$, say that Γ is relatively maximal with respect to α in L when $\Gamma \nvdash_L \alpha$, and, for all $\beta \notin \Gamma$, $\Gamma, \beta \vdash_L \alpha$.

The first clause ensures that Γ avoids the formula α, and the second ensures maximality. Since, when Γ is closed, we have the equivalence $\Gamma \vdash_L \alpha$ iff $\alpha \in \Gamma$, any closed set Γ obeying the above clauses is a relatively maximal theory. Then, any proper superset of Γ will be trivial, such that, for any $\alpha \notin \Gamma$, $\Gamma \cup \{\alpha\}$ is trivial. This allows us to state the following theorem.

Theorem 6 *For any finite normal logic L, given a formula β, and a theory Γ (where $\Gamma, \beta \in S$) such that $\Gamma \nvdash \beta$, Γ can be extended to Γ' (where $\Gamma \subseteq \Gamma'$ and Γ' is relatively maximal with respect to β (in L) so that for no proper superset Γ'' of Γ' do we have $\Gamma'' \nvdash \beta$.*

Here is a sketch of a proof.

Proof Take the enumerable formulas of S, $\{\alpha_1, \alpha_2 ... \alpha_i, \alpha_{i+1}\}$, and a theory $\langle\Gamma\rangle$ (which is closed under $\vdash$, and $\Gamma \subseteq S$), where $\Gamma \nvdash \beta$. Then, for some formula $\alpha_i \in S$, we know that either $\Gamma \cup \{\alpha_i\} \nvdash \beta$, or $\Gamma \nvdash \beta \cup \{\alpha_i\}$. When the former is the case, we can extend Γ to $\Gamma \cup \{\alpha_i\}$. In other words, by induction on the enumerable formulas of S, we can build up the relatively maximal theory Γ', which avoids each $\alpha_i \in S \backslash \Gamma'$ through the construction of a chain:

i. $\langle\Gamma_n\rangle = \langle\Gamma\rangle$

ii. $\langle\Gamma_{n+1}\rangle = \begin{cases} \langle\Gamma_n \cup \{\alpha_{i+1}\}\rangle \text{ if } (\Gamma_n \cup \{\alpha_{i+1}\} \nvdash \beta) \\ \langle\Gamma_{n+1} = \Gamma_n\rangle \text{ otherwise.} \end{cases}$

The limit of this construction is:

$$\langle\Gamma'\rangle = \bigcup_{n \in \prime} \Gamma_n$$

It is simple to see that Γ' is relatively maximal *w.r.t* β in L, since, on the above chain, we have $\Gamma_n \nvdash \beta$, and, so $\Gamma' \nvdash \beta$. If not, there must be some finite subset $\Gamma \subseteq \Gamma'$,

[29]Thanks to an anonymous referee for pointing me towards the original source.

for which $\Gamma \models \beta$ (by finiteness). This means that, for some $\Gamma_{n+i} \supseteq \Gamma$, $\Gamma_{n+i} \vdash \beta$ (by$\mathbb{M}$), which contradicts the definition of Γ_{n+i}, (where $\Gamma_{n+i} \nvdash \beta$). Then, take a formula α_i, where $\alpha_i \notin \Gamma'$. By definition, $\Gamma_{n+1} \subseteq \Gamma'$, so $\alpha_i \notin \Gamma_{n+1}$. Then, we know that $\Gamma_{n+1} = \Gamma_n$, and $\Gamma_n, \alpha_i \vdash \beta$. By monotonicity, and the fact that $\Gamma_n \subseteq \Gamma'$, we have $\Gamma', \alpha_i \vdash \beta$. □

In [56], it is proven that a standard semantics for a logic, L, may be constructed in this way by simply taking the characteristic function of relatively maximal theories for L as follows: given a sequent $\alpha_1, ..., \alpha_n \models \beta$ in a logic L, and a relatively maximal theory Γ' of L, we say that Γ' satisfies this sequent iff, whenever $\Gamma' \models \alpha$, for each $\alpha_1, ..., \alpha_n$, $\Gamma' \models \beta$.

With this in place, we have a formal construction for modest inferentialism, with which we can highlight two problems that occur for it. The first issue is the well-known "categoricity" problem [50, 57–62], which has the result that the standard inferential rules for classical logic fail to rule out non-standard interpretations, so they do not suffice to determine the meaning of logical constants. In particular, that framework is easily shown to be sound and complete with respect to *both* the classical semantic model, and a model in which every formula is interpreted "true". It is also the case, as pointed out in [58], that a valuation can be defined such that $\alpha \vee \neg\alpha$ comes out as true, whilst each disjunct is false, whilst respecting the standard rules of classical propositional logic. This is also problematic from the point of view of compositionality, which is an issue levelled at modest inferentialism, perhaps most clearly in [11, 12], that it fails to conform to certain intuitively correct constraints upon the compositionality of language. In brief, the claim there is that compositionality requires that the content of complex concepts is fully derivable from the content of their constituent concepts. But, it is simple to show that the rules defining disjunction, for example, do not ensure that the truth-values of the sub-formulas for a formula $\alpha \vee \beta$ always determine the truth-value of that formula.[30]

Take the rules ordinarily used to define classical negation (in a right-asymmetric entailment structure):

$$\frac{\Gamma, \neg\alpha \models \beta \qquad \Gamma, \neg\alpha \models \neg\beta}{\Gamma \models \alpha}\,(Reductio) \qquad \frac{\Gamma \models \alpha \qquad \Gamma \models \neg\alpha}{\Gamma \models \beta}\,(EFQ)$$

The issue is that there are possible relatively maximal theories where both α and $\neg\alpha$ are in Γ', and which are not ruled out by the rules used above (essentially, this occurs when every formula is in Γ'). Now, take $\vee$ as example. Sticking with the natural deduction form for a moment, we schematise $R^{\vee}$ as follows:

$$\frac{\alpha}{\alpha \vee \beta}\vee I^{\alpha} \qquad\qquad \frac{\beta}{\alpha \vee \beta}\vee I^{\beta}$$

$$\frac{\alpha \vee \beta \qquad \begin{matrix}[A^u]\\ \sigma\end{matrix} \qquad \begin{matrix}[\beta^v]\\ \sigma\end{matrix}}{\sigma}\vee E^{u,v}$$

[30] See [46, 63].

The issue arises for cases in which α and β are not in Γ' for the rule $E^{u,v}$. In this case, we can not ensure that $\alpha \vee \beta$ is not in Γ' since we are able only to conditionally infer σ from $\alpha \vee \beta$, given independent sub-derivations to σ (which will not figure in the immediate sub-formulas of complex formulas involving $\vee$). In other words, what we do not have in schematic form is all of the relevant information encoded within the immediate sub-formulas involved in the derivation. What $E^{u,v}$ tells us is just that, if there are proofs available from α to σ and β to σ, then we have proofs of $(\alpha \supset \sigma)$, and $(\beta \supset \sigma)$. With these, and the disjunction elimination rule we can then show only $(\alpha \vee \beta), (\alpha \supset \sigma), (\beta \supset \sigma) \vdash \sigma$. With this in mind, we may rewrite the rules in sequent form (again, call this $R^{\vee}$):

$$\frac{\Gamma \vdash \alpha \vee \beta \qquad \Gamma, \alpha \vdash \sigma \qquad \Gamma, \beta \vdash \sigma}{\Gamma \vdash \sigma}\ \vee\text{-E} \qquad \frac{\Gamma \vdash \alpha}{\Gamma \vdash \alpha \vee \beta}\ \vee\text{-I}\alpha$$

$$\frac{\Gamma \vdash \beta}{\Gamma \vdash \alpha \vee \beta}\ \vee\text{-I}\beta$$

However, this won't fix matters, since, again, we have a situation in which there are relatively maximal theories agreeing on the immediate sub-formulas, α, β, but not on the formula $\alpha \vee \beta$ itself. Again, the issues arise with the elimination rule, which only gives us something along the lines of: if we can infer σ from α, and we can infer σ from β, and $\alpha \vee \beta$ is in Γ' then σ is in Γ'. If we consider the derivations of σ from α, β, however, we require for $\Gamma, \alpha \vdash \sigma$ only that $\sigma \notin \Gamma'$ when $\alpha \in \Gamma'$. Then, for $\Gamma, \alpha \vdash \sigma$ (and equally, $\Gamma, \beta \vdash \sigma$), we need either that $\alpha \notin \Gamma'$, or $\sigma \in \Gamma'$. This provides a counterexample to compositionality for $R^{\vee}$ since we need only find an pair of relatively maximal theories Γ^1, Γ^2, that are identical for α, β, whilst $(\alpha \vee \beta) \in \Gamma^1$ and $(\alpha \vee \beta) \notin \Gamma^2$. This is possible when $\alpha, \beta \notin \Gamma^1$ and $\alpha, \beta \notin \Gamma^2$, whilst $\sigma \notin \Gamma^1$ and $\sigma \in \Gamma^2$.[31] For example, if $\alpha = \sigma$, then both of the conditional premises are satisfied by Γ^1, and, in order for Γ^1 to satisfy the rule it must be the case that the major premise is satisfied, and so $(\alpha \vee \beta) \in \Gamma^1$. Hence, there are relatively maximal theories that are equivalent on the sub-formulas but that do not agree on the formula itself, and so $R^{\vee}$ is not compositional. Things becomes particularly interesting when we bring both issues together by considering formulas involving negation such as $\alpha \vee \neg\alpha$, since this may be satisfied by some relatively maximal theory, Γ' whilst neither α nor $\neg\alpha$ are.

We began this section by considering a possible way of dealing with tonk-like issues by appealing to some form of truth-conditional semantics to settle the issue of which rules determine "genuine" meanings, and which do not. Problems of categoricity and compositionality suggest that this is unlikely to provide an adequate response.[32] Resultantly, whilst issues of circularity and paradoxical connectives suggest that an inferentialist account of logical rules inevitably requires semantic justi-

[31] This example is discussed in detail in [44, 46].

[32] In the following chapter I discuss alternative accounts for which these issues do not arise, though others do.

fication, a modest form of inferentialism provides little hope since it fails to provide an account of how rules completely determine meaning, and even where it does, these meanings do not obey intuitive constraints upon compositionality. We may be tempted, then, to jettison inferentialism altogether, but, given the problems (raised above and in the introduction) with the representationalist approach, this looks to provide meagre consolation. In the following section, we turn to an alternative, stringently inferentialist, suggestion, in which it is argued that internal features of rules suffice to determine genuine logical meanings, and so may be thought of as "self-justifying".

5 An Alternative in Harmony?

Rather than look to soundness *w.r.t* a semantic model as a deciding factor for the consideration of logical rules, a fairly common alternative response is to consider internal, syntactic, constraints such as conservativeness (e.g. [40]); inversion principles (e.g. [64]); harmoniousness (e.g. [7, 65]). Whilst these approaches have significant differences, they share the view that introduction and elimination rules in natural deduction calculi (or left and right rules in sequent calculi) are in balance with each other. Gentzen [66], for example, puts this as follows:

> The introductions represent, as it were, the "definitions" of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions. This fact may be expressed as follows: In eliminating a symbol, we may use the formula with whose terminal symbol we are dealing only in the sense afforded it by the introduction of that symbol. (p. 80)

This is developed by the idea of an inversion principle, in which a rule is considered invertible if (broadly speaking) its conclusion implies its premises. Importantly, this constraint requires that anything "extraneous" to a proof can be eliminated through normalization (or cut-elimination) such that Gentzen's *Hauptsatz* can be proved for the logical system.[33] Dummett (e.g. [7]) uses the term "harmony" to describe this constraint upon genuine logical rules, suggesting that, for rules to be meaningful requires that the grounds for making an assertion should be in harmony with the consequences that we are entitled to draw from it. For example, in [7], Dummett argues that it is possible to think of certain basic logical rules as self-justifying by means of introduction rules for a logical connective providing what is called the *canonical* way of introducing an expression involving a logical connective. For example, a canonical argument for $\alpha \wedge \beta$ must end with an application of $\wedge$-introduction (i.e. an inference from α, β to $\alpha \wedge \beta$). Then, say we want to know if the inference from $\alpha \wedge \beta$ to β is valid, all we need do is look for a canonical argument for β, which we already have to hand since we have a canonical argument for the premise, $\alpha \wedge \beta$.

[33] As developed by the so-called Curry-Howard correspondence between proofs and programs (e.g. [67]) the inversion principle has computational significance, which we discuss in more detail in Chap. 6.

Thus, according to Dummett, the inference from $\alpha \wedge \beta$ to β must be valid because it is possible to take the argument for the premise $\alpha \wedge \beta$, and transform it into a canonical argument for β. This balance is given a fairly intuitive spin by Dummett [7], and Tennant [68], the latter of which puts it as follows:

> In the natural-deduction setting the constitutive balance can be described as follows. The introduction rule states the conditions under which a conclusion with that operator dominant can be inferred. It describes, if you like, the obligations that have to be met by the speaker in order to be justified in asserting the conclusion in question. The corresponding elimination rule states what the listener is entitled to infer from the speaker's assertion. (p. 628)

So, bluntly, the balance internal to logical rules is also supposed to track central features of agents' use of logical expressions. Moreover, this restriction to harmonious rules is supposed logical meanings with a form of "self-justification" as Tennant [68] has it:

> […] a logical word's being governed by harmonious rules is what confers upon it its precise logical sense. The rules have to be in place for the word to mean what it does. There is no independent access to the meaning of the word, with respect to which one could then raise the question whether the rules governing it 'respect' that sense, or properly transmit truth to and from sentences involving it. (p. 629)

So, perhaps, in this way, the justification of logical rules may be achieved by imposing restrictions on the admissibility of certain rules. Whilst tonk falls by the wayside since it is clearly non-harmonious, this is tricky, since, analogous to the justification of non-deductive rules raised above, so too there seem to be harmonious, yet non-deductive rules. Then, harmony, by itself, does not seem adequate to demarcate those rules which are deductive from those that are not. Perhaps most famous, are the concepts "flurg" and "aqua", which have introduction and elimination rules as follows:

$$\frac{\text{x is water}}{\text{x is aqua}}\ \text{Aqua-I} \qquad \frac{\text{x is aqua}}{\text{x is } H_2O}\ \text{Aqua-E}$$

and;

$$\frac{\text{x is an elliptical equation}}{\text{x is flurg}}\ \text{Flurg-I}$$

$$\frac{\text{x is flurg}}{\text{x can be correlated with a modular form}}\ \text{Flurg-E}$$

As Enoch and Schecter [28] put it;

> Simply introducing the terms "aqua" and "flurg" with their associated rules of inference is insufficient for being justified in employing those rules. Had it been, science and mathematics would have been much easier than they actually are.[34]

Steven Read [70] has also defined a harmonious, yet inconsistent pair of introduction of elimination rules for the operator •. In brief, he defines the introduction rule for • as:

[34] Though see (e.g. [69]) for discussion of possible responses.

$$\dfrac{\begin{matrix}[\bullet\alpha] \\ \alpha\end{matrix}}{\bullet\alpha}\ \bullet\text{-I}$$

which, yields the clearly harmonious elimination rule:

$$\dfrac{[\bullet\alpha] \qquad [\bullet\alpha]}{\alpha}\ \bullet\text{-E}$$

But, as Read points out, the pair quickly lead to triviality (as long as standard structural rules exist in the system), since we can prove α for any α:

$$\dfrac{\dfrac{\dfrac{\bullet\alpha \qquad \bullet\alpha}{\alpha}\ \bullet\text{-E}}{\bullet\alpha}\ \bullet\text{-I} \qquad \dfrac{\dfrac{\bullet\alpha \qquad \bullet\alpha}{\alpha}\ \bullet\text{-E}}{\bullet\alpha}\ \bullet\text{-I}}{\alpha}\ \bullet\text{-E}$$

As such, Read argues that harmony is insufficient to ensure non-triviality. Furthermore, as with conservative extension, harmoniousness is arguably dependent upon the underlying structure of the proof system employed. So, as Priest [34] points out, harmony does little to justify the structure itself. Therefore, it looks like, whilst harmony may be a decent requirement on a rule, it is insufficient for ensuring the validity of that rule.

Even so, let us momentarily accept that some form of harmony does indeed establish a condition on the admissibility of logical rules. Even then, it is difficult to see how this approach fares much better when it comes to avoiding the circularity identified above. For example, even given harmony, we are surely still warranted in asking why, for example, the set of introduction rules that were considered to define a meaningful assertion are correct to begin with. Otherwise, we would also require a way of accounting for the way in which agents correctly grasp a logical rule, and these will lead us back to the circularity arguments with which we began. Unless this can be shown to bottom out somewhere, regress follows from the fact that the justification of a logical rule relies on correct application of an initial rule (presumably an introduction rule).[35]

In the context of discussions of balance and harmony, it is often suggested that these (amongst other factors) should cause us to consider constructive, rather than classical, logics. In the terminology introduced above, this is usually to take the sequents in right-asymmetric form to define intuitionistic, rather than classical, logic. We will examine the account in detail in the following chapter, but, in brief, in this setting, the validity of a proof is not dependent upon an interpretation (even one developed in terms of relatively maximal theories), but rather on local constraints such as canonicity and harmony (e.g. [71]). This provides additional reasons for thinking that cut-elimination procedures (and harmony) are important in considerations of validity, since, according to Dummett's (e.g. [7]) "fundamental assumption"'if we have a valid argument for a complex statement, we can construct a valid argument for it which finishes with an application of one of the introduction rules governing its principal operator' (p. 254). This way of thinking about valid proofs is not appropriate

[35]We will return to a discussion regarding the reasons for considering harmony beyond the typical attempt to show that certain rules are self-justifying in the following chapter, by considering the bifurcation of proofs as "objects" and proofs as "acts".

to classical logic because canonicity requires the disjunction property to hold: a proof of a disjunction must be given in (or reduced to) canonical form, which requires that it is also possible to provide a proof of one of the disjuncts. A lack of disjunction property is problematic from the *p.o.v* of constructing a proof-theoretic semantics, since we require an ability to determine, in a fine-grained manner, the manner in which a proof is "valid" Moreover, if we accept that harmony is a necessary condition on logical rules then standard rules for classical negation may fall by the wayside with tonk (e.g [7, 65]). For example, the classical rule for negation elimination;

$$\frac{\neg\neg\alpha}{\alpha}\ \neg\text{-E}$$

is not in harmony with the rule of negation introduction[36]:

$$\frac{\begin{array}{c}[\alpha]\\ 0\end{array}}{\neg\alpha}\ \neg\text{-I}$$

whilst the intuitionistic rule for negation elimination is in harmony with this introduction rule[37]:

$$\frac{\alpha \qquad \neg\alpha}{0}\ \neg\text{-E}$$

In [72] it is argued that the problem of categoricity is avoided by this manoeuvre. In this context, the problematic situations would be a scenario in which every statement is provable is compatible with the inferential rules, or it is possible for $\alpha \vee \neg\alpha$ to be provable, whilst neither α nor $\neg\alpha$ are.[38] The reason that these situations can not arise, according to [72], is that there can be no canonical proof of 0, since there exists only the null introduction rule for 0 and the elimination rule is just the absurdity rule (e.g. [74]):

$$\frac{0}{\alpha}\ 0\text{-E}$$

As such, this is rules out a scenario in which there exists a proof of both α and $\neg\alpha$ (where $\neg\alpha$ is equivalent to $\alpha \Rightarrow 0$). But, as pointed out in [75], the self-justification of this rule makes use of the rule for 0-elimination, unlike other rules for the connectives. The sticking point is that:'one also needs to regard, controversially, the rule of 0-elimination as justified on the basis of a non-existent rule of 0-introduction, taken as

[36] Note that I use 0 in place of the usual $\bot$ throughout.

[37] The issue of intuitionistic negation, and also of proof-systems that are classical for which the negation rules are harmonious will be discussed in the following chapter.

[38] We might, instead, take an approach analogous to the "modest" inferentialism discussed for the classical logician above. Since Kripke-style semantics are typically taken to be standard for intuitionistic logic, this would take the idea that the sequent rules determine valuations in a Kripke-model. But, in this context, as discussed in detail in [46], problems of categoricity (for negation) and compositionality (for disjunction) are not assuaged. The latter is perhaps most surprising given the canonicity requirement (that a proof of "$\alpha \vee \beta$" requires a canonical proof of "$\alpha \vee \beta$" from either α or β). However, as [72] point out, this may be put down to an insistence, by Dummett, for example, that Kripke semantics do not adequately specify the semantics of connectives; they'are not to be thought of as giving the full picture of the way in which the intuitionistic logical constants are given meaning: that can only be done directly in terms of the notion of a construction and of a construction's being recognized as a proof of a statement'. [73, p. 287].

saying that there is no canonical proof of 0' [75]. So, it is difficult to maintain the requirement that the rules defining 0 are harmonious.[39] An alternative introduction rule, suggested by Dummett [7], and discussed in [75], is:

$$\frac{\alpha, \beta, \sigma \ldots}{0}\text{ 0-I}$$

The idea being that the premises of the rule include every atomic sentence of $\mathcal{L}$, and, since 0-E effectively allows us to infer any atomic sentence, the rules are in harmony. This is not so simple, however, since this alternative rule does, in fact, allow for situations in which proofs for α and $\neg\alpha$ may exist (specifically, where there exists a proof for each atomic formula of the language). This is similar to the argument made in [77], where it is pointed out that, whilst the 0-elimination rule only tells us that anything may be inferred from 0, this does not ensure that 0 has the meaning of false.[40] For example, it is possible to consider a language in which all atoms are true, and in which case 0 will be true rather than false, and in which case the 0-elimination rule does not determine the (intended) meaning of 0. Of course, this means that $\neg\alpha$, defined as $\alpha \Rightarrow 0$, must also be true by vacuous discharge, and so, in this language we would have both α and $\neg\alpha$ true. Of course, we could just stipulate that 0 "means" false, so that these situations can not arise, but this would not be in keeping with the idea that we are defining the behaviour of 0 by means of a set of rules for negation. So, again, the rules that are supposed to determine the meaning of negation, intuitionistically, do not do so.

6 Soundness and Completeness

For all of the aspirations of inferentialist approaches to meaning, the above, albeit fairly brief, survey, suggests that matters are not so simple. Indeed, we may well be attracted to attempts to mitigate problems facing justification of rules by means of the semantic justification of logical implication, by which we then provide justification for certain rules, i.e. those which preserve truth in all semantic models. But, as we argued in the introduction, and also in §2, this is no salve whatsoever. Nevertheless, the feeling that this is the correct way of construing logic is deeply held. For example, Read [79] forcefully argues this:

> What is good about the notion of proof-theoretic validity is that it recognises that what rules one adopts determines the meaning of the logical terms involved and commits one to accepting certain inferences as valid. What is bad is to infer from this that those inferences really are valid. Proof-theoretic validity serves an epistemological function to reveal how those inferences result from the meaning-determining rules alone. But it cannot serve the metaphysical function of actually making those inferences valid. Validity is truth-preservation, and proof must respect that fact.

[39] A detailed discussion can be found in [76].

[40] The argument is rehearsed in [78].

Typically, model-theoretic consequence is thought to delineate the correctness of inference because it is (in some sense) reducible to the categorical notion of truth-preservation. In rough:

β is derivable from $\alpha_1, \alpha_2 \ldots \alpha_n$ whenever β is a logical consequence of $\alpha_1, \alpha_2 \ldots \alpha_n$..

In the Bolzano-Tarski tradition, this is typically thought of as follows. If Γ is a theory of $\mathcal{L}$, and V is a semantic structure (typically thought of as a model) for $\mathcal{L}$, then V is a model of Γ: $V \models \Gamma$ if, for every $\alpha \in \Gamma$, $V \models \alpha$. And, α is a logical consequence of Γ ($\Gamma \models \alpha$) if, for every model V of Γ, $V \models \alpha$. Inferential derivability, on the other hand, is usually understood as derivability in a formal system (sequent calculus; natural deduction; Hilbert system), where $\Gamma \vdash_L B$ is valid in a formal system L, whenever B can be derived from Γ by means of the axioms and inference rules of L.

As is instilled in any introductory logic course, soundness and completeness ensure that the proof-theory is both correct (according to the semantics), and that it is also strong enough to prove any valid entailment. So, the story goes, the entailment structure gives us epistemic access to validity in the form of proofs, derivable sequents, and so on. But, we require the semantic structure to tell us where the counter-models are, lest we attempt an invalid proof. So, typically, when considering the relationship between an entailment structure and a semantic model, counter-models play the role of ruling out invalid formulae. For example, take Γ^+ as a set of sentential theorems, V as some model, with $\vdash$ a derivability relation, and $\models$ a model-satisfaction relation. Then, we ordinarily require that, for every formula α: Either $\exists\Gamma^+(\Gamma^+ \vdash \alpha)$ or, $\exists V(V \not\models \alpha)$.[41] That is to appeal to the idea that a sequent $\Gamma \vdash \alpha$ is invalid iff some model V makes all the formulas of Γ true, whilst making α false. It is precisely this appeal to counter-models that is not in keeping with a proof-theoretic approach to semantics, but it is also what it looks as though is required given the inability of the right-asymmetric entailment structures to rule out inadmissible cases. So-called paradoxical connectives such as "tonk" [37] provide grist to the mill for this order of priority. Given this, there is a residual feeling that we are not able to provide an account of "real" validity limited to entailment structures alone.[42] Nonetheless, as we have seen, neither a standard proof-theoretic, nor semantic, account of logical rules gives us an adequate solution to the problem of their justification. Furthermore, attempts to bring the two together in so called "modest" versions of inferentialism expose significant fissures between entailment structures and semantics that are easily missed by focusing upon soundness and completeness results alone. Problems facing proof-theoretic validity of rules pushes us toward a semantic justification in terms of logical consequence; in turn, this pushes us back towards the problems originally identified with inferentialist approaches to the validity of logical rules. We are in a tight circle indeed.

[41] This is just a form of Gödel's completeness theorem.

[42] As pointed out in the introduction, more broadly, these are linked to the issues raised by Chinese-room style arguments.

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