Preface

This book is based on lectures given originally at Reading University and more recently at Oxford as part of the Continuing Education program of Oxford University in England. In a sense it is a sequel to *Gems of Geometry* (now in its second edition) and which is also based on lectures given at Reading and Oxford.

Having given the geometry lectures a few times, it was suggested that I should develop another course based on aspects of numbers. I was extremely busy at the time updating my book on the Ada programming language so settled on a short course of just five lectures entitled Nice and Noughty (sic) Numbers. However, as the time approached I was still too busy and so the course was kindly given by Aruna Hardy. The next year, the pressure of life abated and I was able to gave a full ten lecture course.

I have since given the course a number of times at both Reading and Oxford and the notes eventually matured into this book. The lectures divide into two kinds: some address the theme of simple number theoretic topics such as prime numbers and cryptography, whereas others address the theme of daily needs and pleasures such as keeping track of time and enjoying music.

There are ten basic lectures. The first lecture is entitled Measures. Measuring and counting various things are fundamental human activities. In earlier times one might have been interested in measuring the size of a field and counting cows; these days one might be more likely to measure the size of a garden and count money. The lecture starts by asking students what is their favourite number and why; this not only breaks the ice but lays the foundation for many topics such as what would be a good base for counting. That takes us into prime numbers, divisibility, factors, and perfect numbers; the lecture concludes with a survey of weights and measures and currencies.

The second lecture leads on from perfect numbers into Amicable numbers and provides a good opportunity to introduce modular arithmetic. This is followed by Probability and is more light-hearted and ends with applications to games such as craps and poker. We then return to the numerical theme by considering Fractions of various forms including the amazing Egyptian system of unit fractions. This is followed by a lecture on Time including the calendar and sunshine.

There are then two more lectures on the numerical theme covering Notations of various forms and Prime numbers and two lectures of a more fun nature covering Bell ringing and the evolution of Musical scales.

The final lecture looks at the topic of cryptography which has become so important with electronic communication. And for final light relief it concludes by looking at the gaits of animals and two popular puzzles.

There are also a number of appendices. Some provide additional material on topics such as Pascal's triangle and polydivisibility, whereas a few cover material from a short course called Puzzles and Pastimes which I have also given at Reading and Oxford. Thus the appendix on Groups draws on examples from Bell ringing and also leads into the amazing topic of Rubik's cube from the puzzles course. The final appendix considers the musical keyboards designed by Mersenne who is better known for his numbers; it concludes with a discussion of the Tonnetz schema for revealing harmony and the strange relation between music and the topology of the torus.

The main lectures contain some exercises (harder ones are marked with asterisks) but answers are not provided since I anticipate giving the course again.

An important issue when writing a book is to consider who might enjoy reading it. The mathematical background required is not hard (just a bit of simple algebra really) and is the kind of material anyone who studied a scientific subject to the age of perhaps 16 would have encountered. One group of potential readers is therefore young people with a zest for knowledge (I would have been delighted to have had such a book when I was 16). Another group as shown by students on courses includes those of maturer years who might like to know more about topics that they enjoyed when younger.

Students attending the courses are from various backgrounds – of all ages and sexes. Some have little technical background but revel in activities such as throwing dice, ringing handbells, and attempting to simulate the gaits of camels; others have serious technical experience and enjoy perhaps a nostalgic trip visiting some familiar topics and also meeting new ones.

I have made no attempt to avoid using mathematical notation wherever it is appropriate (readers can skip hard bits if they are weary). I am saddened that many popular mathematical books strive to avoid mathematics because some publisher once said that every time an equation is added, the sales divide by two. But I have aimed to provide various tables and illustrations to enliven the text.

I must now thank all those who have helped me in this task. First, a big thank you to my wife, Bobby, who helped with typesetting and photography, to my daughter Janet, who provided much background material, and to my daughter Helen for advice on design, and to David Godwin for information on the noble art of bell-ringing. Thanks also to the (anonymous) reviewers for their input and to Pascal Leroy who was a great help in finding a number of errors and suggesting many improvements and to Tucker Taft for assistance on analysing the performance of Ackermann's function and to Ahlan Marriott for the gift of a corkscrew with a message in base 4.

Thanks also to colleagues in Continuing Education at Oxford and especially to Aruna Hardy for digging me out of a hole by giving the first short numbers course and to Julian Gallop and Iryna Schlackow for some inspiring examples of probability and numerical puzzles.

I am grateful to many students on my courses for their feedback and encouragement. I must especially thank: Felix Lam for introducing me to the life of John William Colenso who wrote fascinating books on Arithmetic and became

Bishop of Natal; Susan Vaughan for help with the presentation of the strange material on music; Rita Sawrey-Woodwards for taking the tricky photographs of measuring devices and other artifacts; and Felicity Wood for introducing me to the Maria Theresa thaler. Susan Vaughan also introduced me to Geoff Chew at Royal Holloway who kindly brought me up to date on the world of music theory.

I am also grateful to the authors of the many books that I have read and enjoyed. I cannot mention them all here but I must mention a few. Two old favourites are *An Introduction to Probability Theory* by Feller and *On the Sensations of Tone* by Helmholtz. Regarding the ancient world, *Mathematics in the Time of the Pharaohs* by Richard Gillings is excellent. For more traditional number theory topics, I am grateful to my daughter for lending me her copies of two books entitled *Elementary Number Theory* by Kenneth Rosen and David Burton. Another wonderful book is *Makers of Mathematics* by Stuart Hollingdale.

For general fun, *The Penguin Dictionary of Curious and Interesting Number*s by David Wells is vital and for puzzles the classic *Amusements in Mathematics* by Henry Dudeney is hard to beat. And I must thank David Singmaster for giving me a copy of his intriguing little book on the Rubik cube.

Probably the first book to trigger my interest in numbers was *Arithmetic* by D A Young. This was aimed at accountants in the old days when everything was worked out with a pencil (my father was an accountant). But among basic stuff were little advanced bits in small print which dabbled in odd topics such as finding a cube root, continued fractions, and recurring decimal fractions. I am sure I spent hours browsing through this when still in short trousers!

Finally, and most important of all, I must thank Dorothy Mazlum, Sabrina Hoecklin, and all others at Springer who made this book actually happen.

I hope that all those who read or browse through it will find something to enjoy. I enjoyed writing it and learnt a great deal in the process.

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