

Chapter 2

Fluid Statics

Abstract This chapter deals with Fluid Statics. Starting from properties of fluids, fluid pressure under static conditions is explained. Pascal's law and its applications in Mariotte bottle and hydraulic lever are given. Buoyancy of airships, sea navigating ships are explained. Water storage, dams and sluice gates are next explained. Basic principles of thermodynamics, perfect gases, Vapor pressure and saturation pressure are explained next. The cavitation phenomenon that depends on vapor pressure is next explained. Heat energy aspects, internal energy, enthalpy, specific heats, adiabatic process, reversible process, entropy, Isentropic process are explained while dealing with mechanical energy and heat energy and work done on gases.

Keywords States of matter · Fluid pressure · Pascal's siphon · Mariotte bottle · Hydraulic lever · Buoyancy · Airships · Dams · Thermodynamics laws · Boyle's and Charles' laws for perfect gases · Vapor pressure · Saturation pressure and temperature · Cavitation · Internal energy · Enthalpy · Specific heats · Adiabatic process · Reversible process · Entropy · Isentropic process

In this chapter we consider the study of fluids at rest. Popularly fluid statics or hydrostatics is called Hydraulics and is practiced since ancient times for storing water and transportation using canals.

2.1 States of Matter

All matter takes three primary states

- Solid—it takes a definite shape and volume, e.g., iron, copper, stone ... The size and shape change only slightly under the application of external forces.
- Liquid—is one that takes the shape of its container with a definite volume, e.g., water, kerosene, mercury ... They can change their shape very readily. If the

volume of vessel is larger than that of the liquid put into it, there will be a free surface at the top of the liquid.

- Gas—e.g., air, carbon dioxide ... Differs from a liquid in that a gas has neither size nor shape. A quantity of gas placed in a container will completely fill that container. There is no free surface. The volume of the gas is the volume of the container.
- There is also a special state—Plasma in which the electrons move out of their designated orbits and even escape making the atoms positively charged.

Liquids and gases are classified as Fluids.

2.2 Pressure in Fluids at Rest

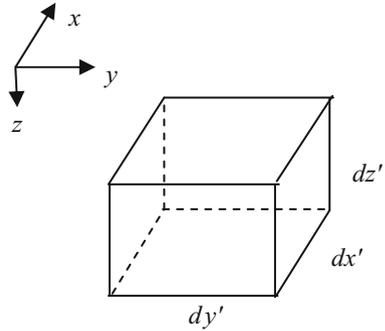
A fluid at rest cannot exert any permanent forces tangential to a boundary; therefore any force that it exerts on a boundary must be normal to the boundary. Such a force F is proportional to the area A on which it is exerted, and is called a *pressure*. We can imagine any surface in a fluid as dividing the fluid into parts pressing on each other, as if it were a thin material membrane, and so think of the pressure at any point in the fluid, not just at the boundaries. In order for any small element of the fluid to be in equilibrium, the pressure must be the same in all directions (or the element would move in the direction of least pressure), and if no other forces are acting on the body of the fluid, the pressure must be the same at all neighboring points.

Pascal's Law: The pressure will be the same throughout in a fluid at rest and is the same in any direction at a point. Due to the fundamental nature of fluids, a fluid cannot remain at rest under the presence of a shear stress. If a point in the fluid is thought of as an infinitesimally small cube, then it follows from the principles of equilibrium that the pressure on every side of this unit of fluid must be equal. If this were not the case, the fluid would move in the direction of the resulting force. Thus, the pressure on a fluid at rest is isotropic and allows fluids to transmit force through the length of pipes or tubes; i.e., a force applied to a fluid in a pipe is transmitted, via the fluid, to the other end of the pipe.

We define *pressure* in liquids as a force F that is applied to the surface of a fluid acting over an area A perpendicular to it, and then the average pressure P is F/A . In SI units it is force F in Newtons over an area one meter squared, i.e., N/m^2 , called Pascal, named after the famous scientist Pascal. A Pascal is abbreviated as Pa. The average pressure one Newton over one meter squared is very small, commonly we use Mega Pascals abbreviated as MPa, N/mm^2 , or even Giga (10^9) Pascals, GPa.

In a fluid at rest velocity $V = 0$, all frictional stresses vanish and the state of stress of the system is called *hydrostatic*. The hydrostatic pressure can be determined from a control volume (test volume) analysis of an infinitesimally small cube of fluid $dx' \times dy' \times dz'$ shown in Fig. 2.1.

Fig. 2.1 Infinitesimally small cube of fluid at depth z'



Gravity is an example of a *body force* that disturbs the equality of pressure in a fluid. The presence of the gravitational body force causes the pressure to increase with depth, according to the equation $dp = \rho g \times dh$, in order to support the water above. We call this relation the *barometric equation*, for when this equation is integrated, we find the variation of pressure with height or depth.

Let p be the hydrostatic pressure (Pa), ρ the fluid density (kg/m^3), g gravitational acceleration (m/s^2), A the test area (m^2), z the height (parallel to the direction of gravity) of the test area (m), and z_0 the height of the reference point of the pressure (m). Then the force acting on any such small cube of fluid is the weight of the fluid column above it; the hydrostatic pressure is obtained from

$$\begin{aligned}
 p(z) &= \frac{1}{A} \int_{z_0}^z \left\{ \int_A \rho(z') g dx' dy' \right\} dz' \\
 &= \int_{z_0}^z \rho(z') g dz'
 \end{aligned}
 \tag{2.1}$$

For incompressible liquids (unlike gases) the density is constant. The above simplifies to

$$p = \rho gh
 \tag{2.2}$$

where h is the height $z - z_0$ of the liquid column between the test volume and the zero reference point of pressure. The absolute pressure compared to vacuum is

$$p = \rho gh + p_{\text{atm}}
 \tag{2.3}$$

Worked Example 2.1:

Determine the maximum height of water that can be supported by atmospheric pressure or maximum height a suction pipe can raise the water. If it is mercury instead of water repeat the calculation.

The density of water is about 1000 kg/m^3 . The depth of water that corresponds to the normal sea-level atmospheric pressure 101325 Pa can be obtained as $\frac{101325}{1000 \times 9.84} = 10.297 \text{ m}$.

This is the maximum height that can be supported by atmospheric pressure; or a suction pump can raise the water to a maximum height of 10 m .

Now let's do the same thing with liquid mercury, whose density is 13.5951 times that of water. The height of the mercury column is then $\frac{10.297}{13.595} = 757 \text{ mm}$, i.e., 30 in Hg .

In practice, it is convenient to measure pressure differences by measuring the height of liquid columns, called *manometry*. The barometer is a familiar example of this, and atmospheric pressures are traditionally given in terms of the length of a mercury column. Corrections are necessary for temperature because mercury density depends on the temperature. Evangelista Torricelli (1608–1647) invented the mercury barometer in 1643, and brought the weight of the atmosphere to light.

Worked Example 2.2:

A U-shaped manometer contains mercury of density $\rho_m = 13534 \text{ kg/m}^3$. The manometer is used to measure pressure p of a fluid in a tank. When connected to the tank, the manometer reads $h = 4.5 \text{ cm}$. Determine the pressure in the tank.

$$p = \rho_m g h = 13534 \times 9.80 \times 0.045 = 5968.5 \text{ Pa}$$

Vacuum: When pressure is referred to a vacuum, it is an *absolute* pressure, while a *gauge* pressure is referred to the atmospheric pressure. A negative gauge pressure is a (partial) vacuum. When a vacuum is stated to be so many mm, this means the pressure below the atmospheric pressure of about 757 mm Hg . A vacuum of 700 mm is the same thing as an absolute pressure of 57 mm Hg . The mm of mercury is called a *torr* named after Torricelli. 700 torr vacuum means a vacuum of 700 mm .

Pressure is sometimes stated in terms of the height of a fluid. If it is the same fluid whose pressure is being given, it is called *head*, and the factor connecting the head and the pressure is the weight density ρg . As an example 10.297 m head of water multiplied by $\rho g = 1000 \times 9.84$ gives one atmospheric pressure (101325 Pa). It can also be considered energy *availability*. Water with a pressure head of 10 m can furnish the same energy as an equal amount of water raised by 10 m . Note that water flowing in a pipe is subject to *head loss* because of friction.

Pascal's Siphon: Blaise Pascal (1623–1662) demonstrated that the siphon worked by atmospheric pressure as shown in Fig. 2.2. The two beakers containing mercury are connected by a three-way tube as shown, with the upper branch open to the atmosphere. As the large container is filled with water, pressure on the free surfaces of the mercury in the beakers pushes mercury into the tubes. When the state shown is reached, the beakers are connected by a mercury column, and the siphon starts, emptying the upper beaker and filling the lower.

Mariotte's Bottle: To deliver a constant rate of flow from a closed tank independent of the input pressure and volume of liquid, Edme Mariotte (1620–1684) bottle shown in Fig. 2.3 can be used.

Fig. 2.2 Pascal’s siphon

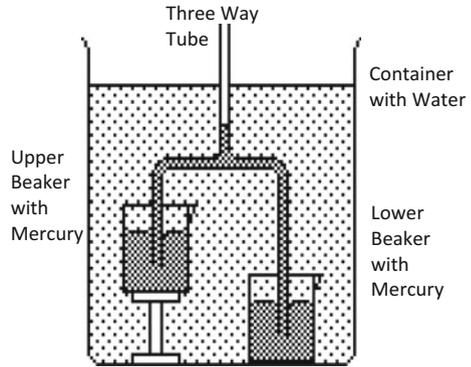
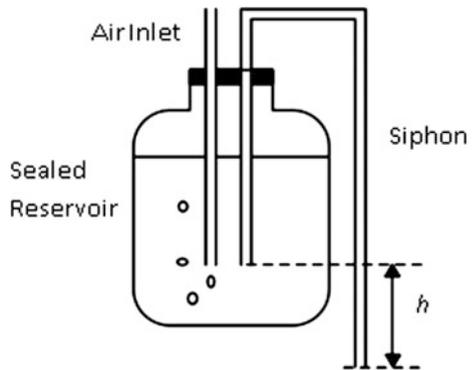


Fig. 2.3 Mariotte bottle for giving a constant output pressure



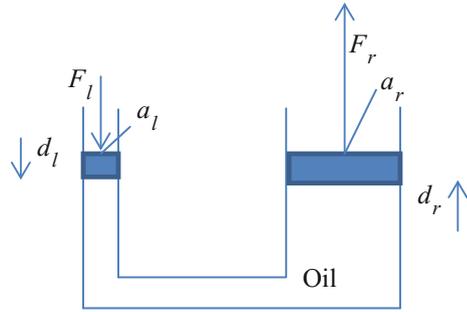
The sealed stoppered reservoir is supplied with an air inlet and a siphon. The pressure at the bottom of the air inlet is always the same as the pressure outside the reservoir, i.e. the atmospheric pressure. If it were greater, air would not enter. If the entrance to the siphon is at the same depth, then it will always supply the water at atmospheric pressure and will deliver a flow under constant head height, regardless of the changing water level within the reservoir.

The plug must seal the air space at the top very well. A partial vacuum is created in the air space by the fall of the water level exactly equal to the pressure difference between the surface and the end of the open tube connecting to the atmosphere. The pressure at this point is, therefore, maintained at atmospheric while water is delivered. The head available at the nozzle as shown is equal to h that gives a constant velocity $v = \sqrt{2gh}$.

Hydraulic Lever: Consider the device with a medium of oil as shown in Fig. 2.4. The left side piston of area a_l carries a small input force F_l and the right side piston of area a_r carries a large output force F_r .

In operation, when an external force F_l acts downward on the left-hand (or input) piston the incompressible oil in the device produces an upward force of magnitude F_r on the right-hand (or output) piston. Using Pascal’s law, we can write

Fig. 2.4 Hydraulic lever



$$F_r = F_l \frac{a_r}{a_l}$$

The above shows that the output force on the right piston F_r is greater than the input force F_l if $a_r > a_l$ as in Fig. 2.4.

If we move the left input piston downward by distance d_l , the output piston on the right side moves upward a distance d_r , keeping the same volume V of the incompressible liquid displaced at both pistons. Then

$$V = a_l d_l = a_r d_r \Rightarrow$$

$$d_r = \frac{a_l}{a_r} d_l$$

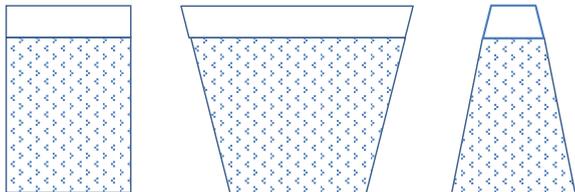
The output piston on the right side moves a smaller distance than the input piston moves. The work done on both the pistons remains same.

The advantage derived out of a hydraulic lever is same as that of mechanical lever of Archimedes. This principle is used in the Bramah hydraulic press, invented in 1785 by Joseph Bramah (1748–1814).

Forces on Submerged Surfaces: The hydrostatic pressure acting on the bottom of a vessel is determined solely by the height of the column of liquid as given in Eq. (2.2). What about the sides of a vessel? Simon Stevin (1548–1620) investigated this problem of hydro statics of fluids.

Figure 2.5 shows three water tanks, with one side vertical but the opposite side inclined inwards or outwards. The bottoms of the tanks are of equal sizes and equal heights, filled with water, the pressures at the bottoms are equal, so the vertical force on the bottom of each tank is the same. The tanks do not contain the same weight of water, yet the forces on their bottoms are equal!

Fig. 2.5 Stevin problem



The horizontal forces exerted by the water on the two sides must be equal and opposite, or the tank would scoot off. If the side is inclined outwards, then there must be a downwards vertical force equal to the weight of the water above it, and passing through the centroid of this water. If the side is inclined inwards, there must be an upwards vertical force equal to the weight of the ‘missing’ water above it. In both cases, the result is demanded by ordinary statics.

2.3 Buoyancy

The most widely known anecdote about Archimedes (287–212 BC) faced the question of the honesty of a goldsmith who made a crown for the King. The question arose as to how one could be certain none of the gold had been substituted by silver. Archimedes found the answer while taking his bath in the tub. He discovered that a body immersed in a fluid experiences a buoyant force equal to the weight of the fluid it displaces and is now known as Archimedes’ principle. He used this principle to solve the King’s problem. Expressed mathematically, the buoyancy force F is

$$F = \rho g V \quad (2.4)$$

where V is the volume of fluid displaced.

From the above we note that for any immersed object the volume of submerged portion equals the volume of fluid it displaces. For example if half of a 1000 cc container is submerged 500 cc volume of fluid is displaced, regardless of the container’s contents. If we fully submerge the same container one 1000 cc of liquid will be displaced. If we put this container under water there will be gravitational force besides it will be pushed upwards. This upwards force is the buoyancy force equal to the weight of the fluid displaced.

The weight of an object in water is less than the weight of object in air, because of the buoyancy force. The buoyancy force being the weight of the fluid displaced by an object, it is the density of the fluid ρ multiplied by the submerged volume V multiplied by the acceleration due gravity g given in (2.4). Therefore the buoyancy force is greater on a submerged body with greater volume compared with an object of same weight with smaller volume, i.e., with larger density.

Let a body’s weight be 10 kg in a vacuum. Let this body displace 3 kg water when lowered into water. It then weighs $10 - 3 = 7$ kg. Therefore it is easier to lift the 10 kg body as if it is only 7 kg. Archimedes’ principle can be reformulated as follows:

Apparent immersed weight $w_{ai} =$ body weight $w -$ weight of displaced fluid w_{df} or

$$w_{ai} = w - w_{df} \quad (2.5)$$

Dividing both sides by w_{df}

$$\frac{w_{df} + w_{ai}}{w_{df}} = \frac{w}{w_{df}} \quad (2.6)$$

Equation (2.6) can be rewritten with the left-hand side in densities as

$$\begin{aligned} \frac{\text{body density}}{\text{fluid density}} &= \frac{\text{weight}}{\text{weight of displaced fluid}} \\ \frac{\text{body density}}{\text{fluid density}} &= \frac{\text{weight}}{\text{weight} - \text{apparent immersed weight}} \end{aligned} \quad (2.7)$$

Archimedes realized that a substitution of gold by a cheaper metal cannot be detected by simply weighing the crown, since the crown can be made to weigh the original weight of the gold supplied. Therefore finding the density of the crown would give the answer. Archimedes therefore measured the volume by the amount of water that ran off when the crown was immersed in a vessel filled to the brim and compared the result with pure gold. The two being not the same it was deduced that the crown was adulterated.

Equation (2.7) allows the calculation of density of the immersed object relative to the density of the fluid without measuring any volumes.

Worked Example 2.3:

Consider the case of a 1 m long circular wood of area 1000 cm^2 (60 kg weight) having a density of 600 kg/m^3 . If you drop this wood into water, buoyancy will keep it afloat. Determine the cross-sectional area under water.

Density of water is 1000 kg/m^3 .

Weight of water displaced will be 60 kg.

Volume of water displaced is $60/1000 = 0.06 \text{ m}^3$

Area of 1 m long water is 0.06 m^2

Cross-sectional area of wood under water is 600 out of 1000 cm^2 .

In the case of a ship, its weight is balanced by pressure forces from the surrounding water, allowing it to float like the log of wood above. If more cargo is loaded onto the ship, it would sink more into the water, displacing more water. In turn the ship receives a higher buoyancy force to balance the increased weight.

In general the horizontal and vertical (buoyancy as above) components of the hydrostatic force acting on a submerged surface are given by the following:

$$\begin{aligned} F_H &= p_c A \\ F_V &= \rho g V \end{aligned} \quad (2.8)$$

where

p_c is the pressure at the center of the vertical projection of the submerged surface

A is the area of the same vertical projection of the surface

- ρ is the density of the fluid
- g is the acceleration due to gravity
- V is the volume of fluid directly above the curved surface

2.3.1 Application of Buoyancy Principle to the Stability of a Ship

In Fig. 2.6 G is the center of gravity of the ship and its weight W acts through G. The force exerted by the water on the bottom of the ship when it is upright acts through the center of gravity B of the displaced volume V , i.e., the center of buoyancy. $\rho gV = W$; G is above B since the CG of the ship will be higher than the CG of the displaced water; this can lead to the ship to capsize.

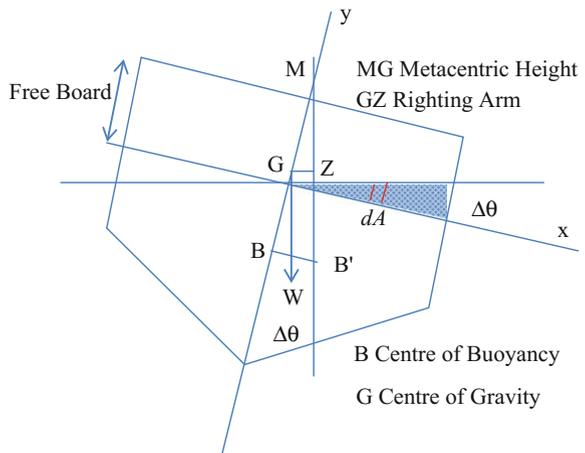
When the ship heels by an angle $\Delta\theta$, a wedge-shaped volume of water is added on the right, and an equal volume is removed on the left, so that V remains constant. The center of buoyancy is then moved to the right to point B'.

Taking moments of the volumes about the y-axis

$$V \times (BB') = V \times (0) + \text{Moment of the shaded volume} - \text{Moment of the equal compensating volume.} \tag{2.9}$$

In the displaced volume shown shaded to the right, let dA be the elemental of area in the $y = 0$ plane; the volume of this element is $x \times \Delta\theta \times dA$. It can be seen that the volume to the left of $x = 0$ is same but negative. The moment of this elemental area is

Fig. 2.6 Ship stability from buoyancy



$$x^2 \times \Delta\theta \times dA = I \times \Delta\theta \quad (2.10)$$

where $I = \int x^2 dA$ is the moment of inertia of the water-level area of the ship. (Note that the contributions from $x > 0$ and $x < 0$ are both positive.)

$$\begin{aligned} \therefore V \times (BB') &= I\Delta\theta \\ \Rightarrow \frac{BB'}{\Delta\theta} &= \frac{I}{V} \end{aligned} \quad (2.11)$$

From Fig. 2.6 $\frac{BB'}{\Delta\theta} = MB$ and $MB = MG + GB$. This gives

$$MG = \frac{I}{V} - BG \quad (2.12)$$

where M is metacenter and MG is the metacentric height from the center of gravity.

When the ship is displaced to the position shown in Fig. 2.6, a restoring moment given by the weight W times the GZ ($= MG \times \Delta\theta$), acts making it stable. GZ is called the *righting arm*.

When a ship rolls it has an inertia torque given by the product of its polar mass moment of inertia and angular acceleration. The restoring torque and the inertia torque keep the dynamic equilibrium. The frequency of vibration depends on the restoring torque; for the same inertia, if the restoring torque is small, i.e., its angular stiffness is small and the natural frequency is small or long rolling period. A small GM means a small restoring torque, and so a long roll period.

A ship with a small GM is said to be *tender*; a passenger ship may have a roll period of 28 s making it comfortable. A ship with a large GM and a short roll period is called *stiff*. Metacentric heights GM are typically 1–2 m.

2.3.2 Balloons and Airships

The buoyancy principle is a perfect application to be applied to balloons. The balloon or airship needs to be filled with a gas lighter than air so that the buoyancy from air (as compared to water in the case of ship) can make the balloon float. The first hot air balloon ascended first in 1783 with a paper envelope made by Joseph-Michel Montgolfier (1740–1810) and Jacques-Étienne Montgolfier (1745–1799). They used a fire balloon; initially they used crumpled paper and lit under the bottom of the box to observe the levitation. Their first demonstration is illustrated in Fig. 2.7.

Such “fire balloons” were then replaced with hydrogen-filled balloons, and then with balloons filled with coal gas, which was easier to obtain and did not diffuse through the envelope quite as rapidly. Methane would be a good filler, with a density 0.55 that of air.

Fig. 2.7 Montgolfier brothers' public demonstration in 1783.
[https://upload.wikimedia.org/wikipedia/commons/6/6b/Early_flight_02562u_\(2\).jpg](https://upload.wikimedia.org/wikipedia/commons/6/6b/Early_flight_02562u_(2).jpg)



Balloons are naturally stable, since the center of buoyancy is above the center of gravity in all practical balloons. Natural rubber balloons with hydrogen have been used for meteorological observations; Helium is a good substitute for hydrogen as it is not inflammable. Submarines are other examples that utilize Archimedes and Pascal principles.

2.3.3 *Hydrostatics of Dam*

A dam or retainer wall is constructed to store water. This stored water at a height can be used to convert hydraulic head to electricity. One of the largest hydroelectric power plants in the world shown in Fig. 2.8 has a capacity of 12,600 MW and located on the border between Brazil and Paraguay on the Parana River. Three Gorges Dam in China has the highest generating capacity of 22,500 MW.

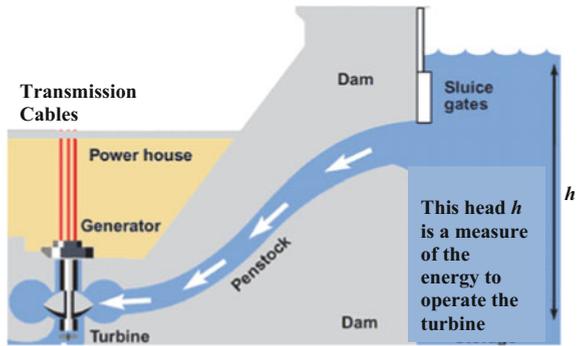
The electric power is generated from the head of the water which is led through penstocks through the height of the dam to a hydraulic turbine which drives a generator, see Fig. 2.9. This is enabled through sluice gates which allow the flow to take place and thus produce power. The sluice gate design is based simply on principles of hydrostatics.

Consider the dam of area A at an angle θ as shown in Fig. 2.10. The axis y is taken along the dam downwards with x axis perpendicular as shown. Taking the pressure



Fig. 2.8 Dam on the Parana River on the border between Brazil and Paraguay. http://3.bp.blogspot.com/_AqMeNIWSpns/SIDiw3INZ2I/AAAAAAAAArI/SEKeTqC90ew/s320/Itaipu+dam+wallpapers,+7+wonders+of+the+world+images.jpg

Fig. 2.9 Sluice gates allow water from reservoir to penstocks



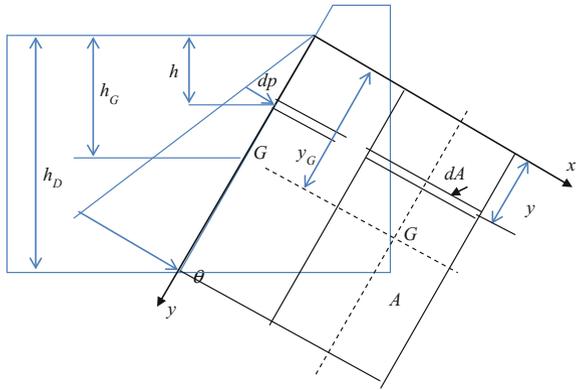
acting on the surface to be zero, i.e., disregarding the atmospheric pressure, the pressure dp at a depth h or a distance y along the dam can be written from (2.2) as

$$dp = \rho g y \sin \theta \tag{2.13}$$

The force dF on an element of area dA is

$$dF = \rho g y \sin \theta \times dA \tag{2.14}$$

Fig. 2.10 Hydrostatic forces on a gate



The total force F on the underwater area of the dam wall A is:

$$\begin{aligned}
 F &= \int_A dF = \rho g \sin \theta \int_A y dA \\
 &= \rho g \sin \theta \times y_G A = \rho g h_G A
 \end{aligned}
 \tag{2.15}$$

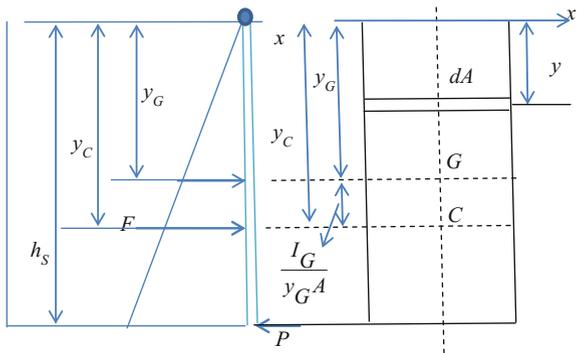
This force F above is the product of the pressure at the centroid G and the underwater area A of the dam.

Worked Example 2.4:

Consider now a rectangular sluice gate similar to the retaining wall but vertical and hinged at the top water level as shown in Fig. 2.11.

The hydrostatic force acting normal to the gate has to be countered by a force P at the bottom such that the torque about the hinge of the hydrostatic forces is equal and opposite to the reaction P times the height of the sluice gate h_s .

Fig. 2.11 Sluice gate forces



The force F acting on the whole plane of the gate from Eq. (2.15) is $\rho g y_G A$. The force acting on the elemental area dA is $\rho g y \times dA$, the moment of this force about the hinge axis xx is $\rho g y^2 dA$ and the total moment on the gate is

$$M_{xx} = \int \rho g y^2 dA = I_{xx} \rho g \quad (2.16)$$

where $I_{xx} = \int y^2 dA$ is the second moment of area of the sluice gate about xx axis.

Let C be the center of pressure as shown in Fig. 2.11, where the net hydrostatic load can be considered to act, then

$$M_{xx} = I_{xx} \rho g = F y_C \quad (2.17)$$

Using parallel axis theorem $I_{xx} = I_{GG} + A y_G^2$ the above becomes

$$(I_{GG} + A y_G^2) \rho g = F y_C \quad (2.18)$$

$$\begin{aligned} (I_{GG} + A y_G^2) &= A y_G y_C \\ y_C &= \frac{(I_{GG} + A y_G^2)}{A y_G} = y_G + \frac{I_{GG}}{A y_G} \end{aligned} \quad (2.19)$$

The second moment of area about GG axis is $I_{GG} = \frac{1}{12} b h_s^3 = \frac{1}{12} A h_s^2$ we have

$$y_C = y_G + \frac{1}{12} \frac{h_s^2}{y_G} \quad (2.20)$$

$$\begin{aligned} \rho g y_G A \left(y_G + \frac{1}{12} \frac{h_s^2}{y_G} \right) &= P h_s \Rightarrow \\ P &= \frac{\rho g y_G A}{h_s} \left(y_G + \frac{1}{12} \frac{h_s^2}{y_G} \right) \end{aligned}$$

The above derivations help us in design of dams, penstocks, gates etc. In a similar way this formulation will also be useful in the design of valves in pipelines.

2.4 Basics of Thermodynamics

Thermodynamics is the scientific treatment of mechanical actions or relations of heat. The basic principles of thermodynamics are the laws governing the energy from one form to another. Among the many consequences of these laws are relationships between the properties of the matter and the effects of changes in pressure and temperature. Thermodynamics is based on observations of common experience that have been formulated into thermodynamic laws.

We first define a system, e.g., a fixed mass of gas, a block of ice or the hot plate used for cooking. The region outside the system is called the surroundings. An open

system may exchange mass, heat and work with the surroundings. A closed system may exchange heat and work but not mass with the surroundings.

It is a common experience for us to talk of a body when we touch it that it is hot, e.g., boiling water or cold, e.g., a block of ice. This is a primitive way of expressing the condition of a body, as it is a relative measure of a system with respect to human body. Thermodynamics can refine this primitive way of expression by a precise concept of temperature.

Let's construct a mercury-in-glass thermometer that has its pressure held constant. This thermometer forms a system by itself. It can be used to measure the temperature of another system, the mercury changes in its volume under constant pressure until an equilibrium state is reached with the system, whose temperature is being measured. Bodies in thermal equilibrium are said to have the same temperature.

2.4.1 Zeroth Law

Consider a large body of water in thermal contact through a wall with large body of another fluid, say alcohol, both in the state of equilibrium that is they both have the same temperature. When a mercury-in-glass thermometer is used to measure the temperature of either of these fluids, it will register the same volume of the mercury. This fact of experience is commonly known as Zeroth law of Thermodynamics, namely, *If two bodies A and B are separately in thermal equilibrium with a third body C, then A and B are in equilibrium with each other.*

2.4.2 Hydrostatics of Gases

A fluid medium can be considered to consist of particles, a collection of molecules, atoms, ions, and electrons etc., which are more or less in random motion. As these particles are all under an electronic structure a force field exists in the space between them. Such a force field is called intermolecular force field. If the fluid medium is a gas and that its particles sufficiently separated such that the intermolecular forces are negligible, it is called a *perfect gas*. For temperatures and pressures normally encountered in the compressible flows of our common interest, the particles in the medium are generally apart by a distance of ten molecular diameters or more. Hence we will be concerned in our compressible flow studies with such gases that can be considered as perfect in nature. The density of a gas changes with pressure; for example the atmospheric pressure changes with the height so also the density.

Boyle's-Charles' law for gases states that

$$pv = RT \quad (2.21)$$

where p is the pressure of a gas, v the specific volume, T the absolute temperature and R the gas constant. This is the equation of state for a perfect or ideal gas, and $v = \frac{1}{\rho}$. The value of R varies for different gases, for air it is 287.1. Gases at considerably higher temperatures than the liquefied temperature can be considered to be a perfect gas.

Worked Example 2.5:

The pressure inside a room with dimensions $6 \times 8 \times 6$ m contains air at a pressure 0.1 MPa and temperature 22 °C. Determine the density, specific gravity, and mass of the air treating air as ideal gas with a gas constant 287 Pa m³/kg °K.

The density is determined from $\rho = \frac{p}{RT} = \frac{0.1 \times 10^6}{287 \times 295} = 1.1811$ kg/m³.

Dividing the density with density of water, specific gravity of air therefore is 0.0011811.

Mass of air in the room is $6 \times 8 \times 6 \times 1.1811 = 340$ kg.

2.4.3 Vapor Pressure

There is one-to-one correspondence between temperatures and pressures. At a given pressure, the temperature at which a pure substance changes phase is called the *saturation temperature* T_{sat} . Similarly, at a given temperature, the pressure at which a pure substance changes phase is called the *saturation pressure* P_{sat} . Table 2.1 gives Saturation (or vapor) pressure of water at various temperatures.

Table 2.1 Saturation (or vapor) pressure of water as a function of temperatures

Temperature Pressure	
$T, ^\circ\text{C}$	P_{sat} kPa
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

At an absolute pressure of 1 standard atmosphere, 101.325 kPa, the saturation temperature of water is 100 °C. Conversely, at a temperature of 100 °C, the saturation pressure of water is 1 atm. The *vapor pressure* P_v of a pure substance is defined as the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature. This is a property of the pure substance, and turns out to be identical to the saturation pressure P_{sat} of the liquid.

It should be noted that vapor pressure and partial pressure are different. *Partial pressure* is defined as the pressure of a gas or vapor in a mixture with other gases. As an example atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor. The partial pressure of water vapor constitutes a small fraction less than 3 % of the atmospheric pressure which is mostly nitrogen and oxygen. The partial pressure of a vapor must be less than or equal to the vapor pressure if there is no liquid present. However, when both vapor and liquid are present and the system is in phase equilibrium, the partial pressure of the vapor must equal the vapor pressure, and the system is said to be saturated.

Worked Example 2.6:

A bucket of water is left at 25 °C left in a room with dry air at 1 atm. The vapor pressure of water at 25 °C is 2.4 kPa. When will the evaporation stop?

The water will begin evaporating and continue evaporating until all the water evaporates away if there is not enough water to establish phase equilibrium in the room.

Or the evaporation stops when the partial pressure of the water vapor in the room rises to 2.4 kPa at which point phase equilibrium is established.

Worked Example 2.7:

At what temperature does water boil in a pressure cooker operating at 3 atmospheres pressure? Also at what temperature does water boil at an elevation of 2000 m.

Vapor pressure increases with temperature as given in Table 2.1. Therefore, a substance at higher temperatures boils at higher pressures. Let us use linear interpolation from Table 2.1. Two atmospheres pressure is $304 - 101.3 = 202.7$ kPa. From Table 2.1 we find that a 50 °C rise at 100 °C raises the saturated pressure from 101.3 to 475.8 kPa, i.e., 374.5 kPa. For a rise of 202.7 kPa, the temperature rise is $\frac{202.7}{374.5} \times 50 = 27$ °C. Therefore water boils at 127 °C in the pressure cooker. The correct value is 134 °C.

At 2000 m elevation the atmospheric pressure is only 81 kPa (0.8 atm). Using linear interpolation as above, $\frac{(101.3-81)}{(101.3-12.35)} \times 50 = 11.4$ °C giving the boiling temperature equal to 88.6 °C. The correct value is 93 °C.

Cavitation: When the liquid pressure in a flow drops below the vapor pressure *cavitation* occurs resulting in unplanned vaporization. In Impeller tip regions on suction sides of the pumps the pressure can fall below vapor pressure of the flow medium. For example, water at 15 °C will flash into vapor and form bubbles

forming cavities when the pressure drops below its saturation pressure 1.71 kPa. The cavitation bubbles collapse as they are swept away from the low pressure regions, generating extremely high-pressure waves. Cavitation causes poor performance and even the erosion of impeller blades, is called cavitation. The pressure spikes resulting from the large number of bubbles collapsing near a solid surface over a long period of time causes erosion, surface pitting, fatigue failure, and the eventual destruction of the components or machinery.

2.4.4 Internal Energy

The subject of Thermodynamics itself does not define the concepts of energy or work; they are adopted from the subject of Mechanics. The principle of conservation of energy is also taken as axiomatic.

Now, consider an isolated system (no exchange with surroundings) with a finite volume of gas, formed from any part of a given system. It will have a definite amount of energy trapped in the molecules of the system in the form of kinetic energy and potential energy. This trapped energy is called *internal energy*.

The internal energy per unit mass is defined as *specific internal energy* denoted by e and it is a state variable depending only on the initial and final states of a system undergoing a change.

2.4.5 Enthalpy

For a given system *enthalpy* is defined as the sum of internal energy and the system's volume multiplied by pressure exerted by the system on the surroundings.

Enthalpy per unit mass is defined as *specific enthalpy*, h , which is written as

$$h = e + pv \quad (2.22)$$

Specific enthalpy is a commonly used quantity and it repeatedly appears in a discussion of thermodynamics. Enthalpy depends only on the initial and final states of a system undergoing a change, therefore a state variable in contrast to heat and work which are path functions.

The specific internal energy and specific enthalpy for a perfect gas are functions of temperature only, i.e.,

$$\begin{aligned} e &= e(T) \\ h &= h(T) \end{aligned} \quad (2.23)$$

2.4.6 Specific Heats

The *specific heat* is defined as the amount of heat required to raise unit mass of a material by one degree, i.e., $\frac{\text{J}}{\text{kg}^\circ\text{K}}$. As we are concerned with gases here, there are two different ways by which we can perform the heating operation, one at constant volume and the other at constant pressure.

Consider a system of gas with its mass enclosed at a constant volume, any heat added is used solely to raise the gas temperature, hence the heat added increases the internal energy of the gas, e . The specific heat under these conditions is defined by *specific heat at constant volume* and denoted by c_v . The change in the specific internal energy then is written as

$$de = c_v dT \quad (2.24)$$

For a calorically perfect gas the c_v is independent of temperature T and therefore

$$e = c_v T \quad (2.25)$$

For air under standard conditions, $c_v = 718 \frac{\text{J}}{\text{kg}^\circ\text{K}}$.

In a similar manner, now consider a system of gas which is allowed to expand and maintain a constant pressure due to any imposed change. Application of heat to such a system raises the temperature and in order to keep the pressure same, its volume also changes simultaneously. The change in the volume of the gas results in mechanical work on the surroundings, for example the piston carrying a constant load with gas in a cylinder. Thus it is necessary to supply the heat required to increase the temperature of the gas by one degree K as in the case of constant volume before, in addition the amount of heat equivalent to the mechanical work done against the surroundings. The total heat required is the *specific heat under constant pressure*, denoted by c_p . Now by definition of specific enthalpy in (2.22)

$$dh = c_p dT \quad (2.26)$$

For a calorically perfect gas the c_p is independent of temperature T and therefore

$$h = c_p T \quad (2.27)$$

For air under standard conditions, $c_p = 1005 \frac{\text{J}}{\text{kg}^\circ\text{K}}$.

For most of the compressible flow problems, the temperatures normally encountered are moderate and the gases can be considered as calorically perfect and therefore the specific heats are taken constant. The *specific heat ratio* is denoted by γ

$$\gamma = \frac{c_p}{c_v} \tag{2.28}$$

For a perfect gas

$$pv = RT \Rightarrow h - e = RT \Rightarrow c_p T - c_v T = RT$$

$$\therefore c_p - c_v = R \tag{2.29}$$

Eliminating c_v in (2.28), we get

$$c_p = \frac{\gamma R}{\gamma - 1} \tag{2.30a}$$

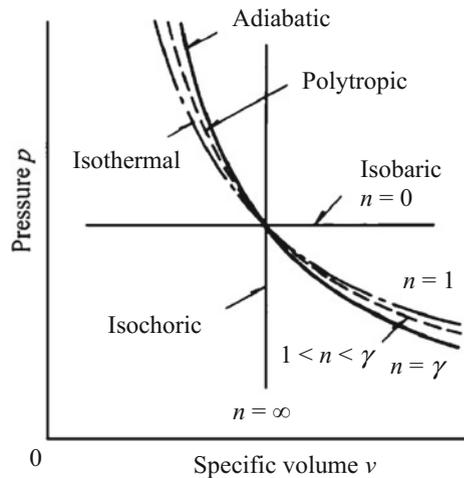
Similarly

$$c_v = \frac{R}{\gamma - 1} \tag{2.30b}$$

2.4.7 Polytropic Form for Pressure-Specific Volume Relation

In general the pressure and specific volume relation is expressed in a polytropic form $pv^n = \text{constant}$ and the exponent n for different state processes can be represented as in Fig. 2.12.

Fig. 2.12 Polytropic Exponent n in state relation for gases



In Fig. 2.12, γ is ratio of specific heats c_p and c_v . Therefore for gases it is not possible to integrate the pressure simply as in the case of a liquid. Referring to the sea level ($z = 0$) pressure p and density ρ with a subscript 0, we can write

$$\frac{p}{\rho^n} = \frac{p_0}{\rho_0^n} \quad (2.31)$$

We can write the expression for the density in the following form

$$\rho^n = \frac{p}{p_0} \rho_0^n \Rightarrow \rho = \rho_0 \left(\frac{p}{p_0} \right)^{\frac{1}{n}} \quad (2.32)$$

From Eq. (2.1) with the sea level as datum, we can write

$$\frac{dp}{dz} = -\rho g \Rightarrow dz = -\frac{dp}{\rho g} \quad (2.33)$$

Substituting for ρ from (2.32) we have

$$\begin{aligned} dz &= -\frac{dp}{\rho g} = -\frac{1}{g} \frac{p_0^{\frac{1}{n}}}{\rho_0} p^{-\frac{1}{n}} dp \\ &= -\frac{1}{g} \frac{p_0}{\rho_0} \left(\frac{p_0}{p} \right)^{\frac{1}{n}} d \left(\frac{p}{p_0} \right) \end{aligned} \quad (2.34)$$

From sea level as datum, integrating

$$z = \int_0^z dz = \frac{1}{g} \frac{n}{n-1} \frac{p_0}{\rho_0} \left[1 - \left(\frac{p}{p_0} \right)^{\frac{n-1}{n}} \right] \quad (2.35)$$

Rearranging the above, we get the pressure at z in terms of sea level value

$$\frac{p(z)}{p_0} = \left(1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \right)^{\frac{n}{n-1}} \quad (2.36)$$

From (2.32), we can get a similar relation for the density

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0} \right)^{\frac{1}{n}} = \left(1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \right)^{\frac{1}{n-1}} \quad (2.37)$$

For absolute temperature T_0 at sea level and T at the point of height z , we can write

$$\begin{aligned} \frac{p}{\rho T} &= \frac{p_0}{\rho_0 T_0} = R \\ \Rightarrow \\ \frac{T}{T_0} &= \frac{p}{p_0} \frac{\rho_0}{\rho} \end{aligned} \quad (2.38)$$

With the help of (2.36) and (2.37)

$$\frac{T}{T_0} = \frac{p}{p_0} \frac{\rho_0}{\rho} = \frac{\left(1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z\right)^{\frac{n}{n-1}}}{\left(1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z\right)^{\frac{1}{n-1}}} = \left(1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z\right) \quad (2.39)$$

From the above, we can obtain a relation for temperature as a function altitude given by

$$\begin{aligned} \frac{dT}{dz} &= -\frac{n-1}{n} \frac{\rho_0 g}{p_0} T_0 \\ &= -\frac{n-1}{n} \frac{g}{R} \end{aligned} \quad (2.40)$$

The sea level values in general are $p_0 = 101.325$ kPa, $T_0 = 288.5$ K and $\rho_0 = 1.225$ kg/m³. The polytropic exponent n then is 2.235. One can show that the temperature decreases by 0.65 °C every 100 m of height in the troposphere up to approximately 1 km high, but is constant at -50.5 °C from 1 to 10 km height.

2.4.8 First Law of Thermodynamics

Consider a closed system; say a fixed mass of gas. Its internal energy can be altered only by an exchange of energy from its surroundings through the boundary. Such an exchange can take place in three different ways by

1. mass transfer
2. heat transfer or
3. work exchange

For a closed system no mass transfer can take place. Therefore its state can be changed either by heat addition or by work done on the system by its surroundings. This is the First Law of Thermodynamics.

Let the amount of incremental heat added be δq and the work done on the system be δw , then the change in the specific internal energy de is

$$de = \delta q + \delta w \quad (2.41)$$

Since e is a state variable, it is an exact differential.

2.4.9 Adiabatic Process

A special case of the First Law is when no heat is added or taken away. The process is then called an *adiabatic process*. For an adiabatic process, therefore, $\delta q = 0$. Then the change in internal energy is simply equal to the work done by the surroundings of the system.

$$de = \delta w \tag{2.42}$$

This case is similar to solids when the work done by external forces is equal to the internal energy, called *strain energy*.

2.4.10 Irreversible Process

Any process that occurs in nature follows the 1st Law of Thermodynamics, however many processes that satisfy the first law may never occur. There is ample evidence that processes prefer to proceed in one direction.

James Prescott Joule conducted an experiment in 1845 to find the Mechanical Equivalent of Heat. In his experiment he used a falling weight, in which gravity does the mechanical work, to spin a paddle-wheel in an insulated (adiabatically) barrel of water which increased the temperature. The total effect of the experiment was to increase the internal energy of the water and to lower the weight, keeping the surroundings unchanged. He estimated a mechanical equivalent of 4.41 J/cal (Nm/cal).

This process cannot be reversed, i.e., water cannot be restored to original state and raise the weight to the original height without making additional changes to the surroundings. Hence the process is *irreversible* (Fig. 2.13).

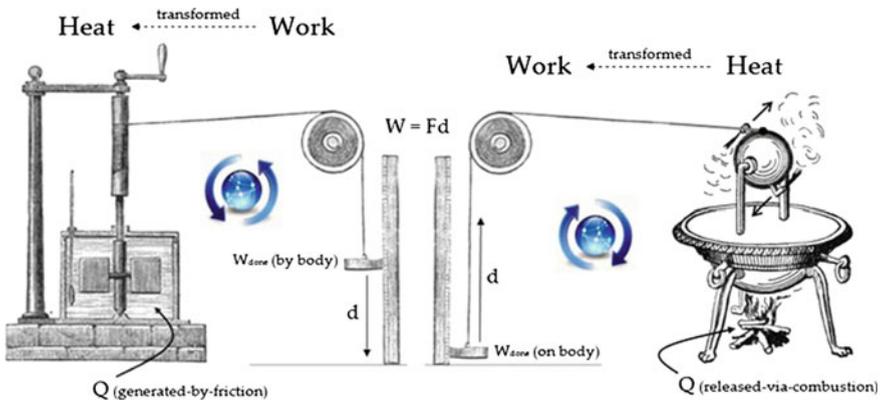


Fig. 2.13 Joule’s apparatus to find mechanical equivalent of heat and demonstrate the irreversibility. <http://image.wikifoundry.com/image/1/f04TM0t9WI4ADVjQGYm8MA78738/GW666H312>

2.4.11 Reversible Process

Thermodynamics makes use of an idealization, called *Reversible Process*. This is a limiting case of the natural or irreversible process. Reversible process is one in which no dissipative phenomenon (e.g., effects of viscosity) occur. A reversible process may be imagined to proceed through a succession of equilibrium states and may be reversed by an infinitesimal change in the external conditions.

Consider as an example a gas cylinder fitted with a frictionless piston. Let the piston be moved slowly, such that the pressure gradients are absent and that the gas is in equilibrium at all the times. The pressure difference between the gas in the cylinder and the external surroundings need be infinitesimal only at any given time to cause a motion of the frictionless piston. Under such restrictive conditions of a reversible process, we can write

$$\delta w = -pdv \quad (2.43)$$

Hence the first law for a reversible process can be written as

$$de = \delta q - pdv \quad (2.44)$$

2.4.12 Entropy and Second Law of Thermodynamics

The discussion on irreversible process in Sect. 2.4.10 led to the II Law of Thermodynamics, which is essentially a statement on the preferred direction of a given process.

Consider a block of ice that is brought into contact with a hot iron plate. It is a common knowledge that ice melts from the heat of the iron plate and the plate cools; hat transfer takes place from the hotter iron plate to cooler ice block.

The First Law of Thermodynamics does not define this process in a unique manner; instead it can allow a process, though physically not possible, wherein the ice block becomes cooler further by giving away heat to hot iron plate, making it hotter. The nature has a preference to this process in which heat flows from a hotter body to a cooler body.

Rudolf Clausius laid the foundation for the second law of thermodynamics in 1850, which states “It is not possible that at the end of a cycle of changes, heat has been transferred from a colder to a hotter body without providing some other effect”. Though Clausius statement explains the fact that some processes allowed by the First Law cannot occur in nature, it is not a mathematical statement.

2.4.13 Entropy

For understanding the mathematical consequences of the II Law and ascertaining the proper direction of a given process, a state variable *entropy* s is defined as follows.

Incremental amount of heat δq_{rev} added reversibly to a system is expressed as

$$\delta q_{\text{rev}} = Tds \quad (2.45)$$

For an irreversible process, where δq is the heat added, the above becomes

$$\begin{aligned} \delta q &= T(ds - ds_{\text{irrev}}) \Rightarrow \\ ds &= \frac{\delta q}{T} + ds_{\text{irrev}} \end{aligned} \quad (2.46)$$

ds_{irrev} is the entropy generated due to any irreversible process phenomenon, e.g., viscous effects occurring in the system. The dissipative phenomenon always increases the entropy, i.e.,

$$ds_{\text{irrev}} \geq 0 \quad (2.47)$$

For a reversible process, $ds_{\text{rev}} = 0$ therefore Eq. (2.46) leads to

$$ds \geq \frac{\delta q}{T} \quad (2.48)$$

For an adiabatic process, the above simplifies to

$$ds \geq 0 \quad (2.49)$$

The above two equations are mathematical expressions of the Second Law.

The II Law tells us the direction in which a process can take place. Specifically it tells us that a process in which generation of entropy is not positive, i.e., the entropy cannot increase, cannot take place even if it satisfies the I Law. The process should yield a net increase in the entropy from one state to another state. In the example of ice block and hot iron plate, we find that there is a net increase in the entropy only when ice receives heat from the hot plate, or

$$ds = \frac{\delta q}{T_{\text{ice}}} + \frac{-\delta q}{T_{\text{plate}}} > 0$$

On the other hand, if we conceive a process by which ice block gives away its heat, thus lowering its temperature further, and the hot plate receives heat,

increasing its temperature further, i.e., heat transfer takes place from colder to a hooter body, the entropy change is negative, or

$$ds = \frac{-\delta q}{T_{\text{ice}}} + \frac{\delta q}{T_{\text{plate}}} < 0$$

and the II Law prohibits this direction of the process.

2.4.14 Entropy Calculation for Any Process

Let us assume that heat is added in a reversible manner, Eq. (2.44); using (2.45), we can write

$$Tds = de + pdv \quad (2.50)$$

From the definition of enthalpy, Eq. (2.22)

$$dh = de + pdv + vdp$$

Therefore Eq. (2.50) becomes

$$Tds = dh - vdp \quad (2.50a)$$

Equations (2.50) and (2.50a) are essentially the same, they are alternate forms of the first law expressed in terms of entropy.

Consider now a perfect gas and according to the definitions of specific heats given in (2.24) and (2.26), we write the above (2.50) and (2.50a) as

$$ds = c_v \frac{dT}{T} + \frac{pdv}{T} \quad (2.51)$$

$$ds = c_p \frac{dT}{T} - \frac{vdp}{T} \quad (2.51a)$$

Equation (2.51) can be further written using $\frac{p}{T} = \frac{R}{v}$ from the perfect gas equation

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v} \quad (2.52)$$

Let a thermodynamic process take place with initial and end states represented by 1 and 2 respectively, then

$$s_2 - s_1 = c_v \int_{T_1}^{T_2} \frac{dT}{T} + T \int_{v_1}^{v_2} \frac{dv}{v} \Rightarrow \quad (2.53)$$

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

In a similar manner (2.51a) gives

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \quad (2.54)$$

We note from the above two equations that the entropy is a function of two thermodynamic variables, either p and T or v and T .

2.4.15 Isentropic Process

We now consider a third thermodynamic process without any change in the entropy of the system, called *isentropic process*. For a reversible process as given in Eq. (2.44), the quantity of heat transferred is directly proportional to the system's entropy change. Systems which are thermally insulated from their surroundings undergo processes without any heat transfer and such processes are denoted as adiabatic as given in Sect. 2.4.9. Thus, an isentropic process is one which is both reversible and adiabatic, so that $ds = 0$. For an isentropic process, Eq. (2.54) gives

$$c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 0 \Rightarrow \quad (2.55)$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\frac{c_p}{R}}$$

Using Eq. (2.30a)

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \quad (2.56)$$

Similarly (2.53) gives

$$\frac{v_2}{v_1} = \left(\frac{T_2}{T_1} \right)^{-\frac{1}{\gamma-1}} \quad (2.57)$$

By definition of specific volume

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} \quad (2.58)$$

Equation (2.57) can then be written as

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}} \quad (2.59)$$

Therefore, for an isentropic process

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1} \right)^\gamma = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} \quad (2.60)$$

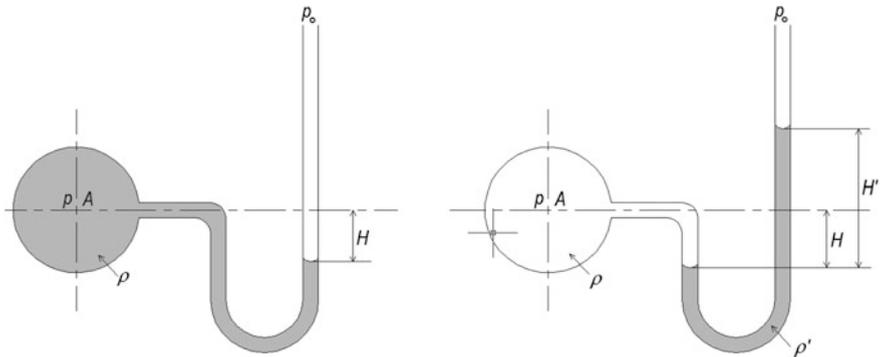
The above relation is essentially a combination of the first law of thermodynamics and the definition of entropy and hence it is basically an energy relation for an isentropic process. Though an isentropic process seems to be highly restrictive that it has to be adiabatic and reversible, it is of immense use in practical compressible flows.

As an example consider the flow between two adjacent blades in a turbomachine, which is like the flow through nozzle. In the regions adjacent to the walls of the nozzles, a thin boundary layer is formed, within which, the viscosity, thermal conduction and diffusion are predominant. Entropy increases substantially within such a boundary layer. Barring this thin boundary layer, the entire flow field is adiabatic and reversible, i.e., isentropic in nature. Hence the importance of isentropic process in the compressible flow problems.

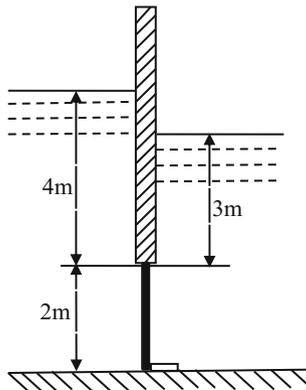
Exercises 2

1. Derive a relation for hydrostatic pressure of a given liquid with density ρ as a function of its height above the datum. Use acceleration due to gravity g . Show that the atmospheric pressure at sea level is 757 mm of mercury. The density of mercury is 13595.1 kg/m³.
2. What is absolute pressure and how do you relate it to gauge pressure. How do you express vacuum? Express a vacuum of 600 torr as an absolute pressure.
3. Determine the head of water equivalent to one atmospheric pressure 101325 Pa.
4. Determine the pressure at the bottom of a sea which is 6000 m deep.
5. Describe how a Pascal siphon works? Using this principle describe an apparatus for giving a constant output pressure.
6. Describe a Bramah hydraulic press and derive the relation between input and output pressures, similar to a mechanical lever.
7. Three water tanks of same height have side walls inclined from vertical at 130°, 90° and 75° respectively. If they contain a liquid of same height, show that the forces at the bottom are all same.

8. Determine the pressure at point P in the figures shown. p_0 is atmospheric pressure.



9. A barometer at a location gives a reading 750 mm Hg. If the density of mercury is $13,600 \text{ kg/m}^3$ determine the atmospheric pressure at this location.
10. The gage pressure in a liquid at a depth of 3 m is 28 kPa. What will be the gage pressure in the same liquid at a depth of 12 m?
11. The vacuum gage reading on a tank shows 28 kPa at a location where the barometric reading is 750 mm Hg. What is the absolute pressure in the tank? The density of mercury is $13,500 \text{ kg/m}^3$.
12. It was decided to use the basic barometer in measuring the height of a building. At the top of the building the barometer read 720 mm Hg and at the bottom it is 750 mm Hg. What is the height of the building? The density of air is taken an average value in this height which is 1.15 kg/m^3 .
13. Determine the force acting on the lower stay of the water gate 2 m high and 1 m wide shown in the figure.



14. What is the force in magnitude and the location acting on a unit width of the dam wall? The water is 15 m deep and the wall is inclined at 60° .
15. The piston of a vertical piston–cylinder device containing a gas has a radius 20 cm and weighs 50 kg. Determine the pressure inside the cylinder. If some heat is transferred to the gas and its volume is doubled, what will be the change of pressure inside the cylinder?
16. A tank H m depth is filled with saline water whose density varies with depth z from the free surface given by $\rho = \rho_0 \sqrt{1 + \tan^2 \frac{\pi z}{4H}}$. ρ_0 is the density of water at the free surface. If H is 4 m determine the gage pressure at the bottom of the tank.
17. A car sunk in a lake 10 m deep. The front seat door is at a height of 25 cm from the lake bottom and is of width 100 cm and height 40 cm. When submerged there is no leakage of pressure of air from the cabin that remained with atmospheric pressure. Determine the hydrostatic force at the center of pressure of the door. If the driver rolled the glass pane a little to make the pressure inside and outside the car same, what will be the hydrostatic pressure at the new center of pressure of the door.
18. A concrete block $0.4 \times 0.4 \times 5$ m of density 2300 kg/m^3 is lowered into salt water of density 1025 kg/m^3 . Determine the tension in the crane rope when the block is completely immersed in water.
19. What is internal energy and enthalpy of a perfect gas? On what state quantity the specific values of internal energy and enthalpy depend on.
20. Define specific heat. Discuss two different ways of performing the heating process and obtain the relations for specific internal energy and specific enthalpy of a perfect gas.
21. Define specific heat under constant volume and specific heat under constant pressure. Show that specific internal energy and specific enthalpy are functions of temperature alone.
22. For a perfect gas show that the specific heats are related to the gas constant R . Derive also expressions for the specific heats in terms of specific heat ratio and the gas constant.
23. What are different possible polytropic forms for pressure—specific volume relations? For a general polytropic law pv^n constant obtain a relation for temperature of air as a function altitude.
24. The sea level values for pressure, temperature and density of air are $p_0 = 101.325 \text{ kPa}$, $T_0 = 288.5 \text{ K}$ and $\rho_0 = 1.225 \text{ kg/m}^3$. Determine the polytropic law. Assuming that this law is valid for the first kilo meter height, find the decrease in temperature for every 100 m.
25. For a closed system state the first law of thermodynamics. Discuss the special case of adiabatic process. What is the change in internal energy for such a system?
26. Describe Joule's apparatus to find mechanical equivalent of heat and demonstrate the irreversible Process using the first law of thermodynamics.
27. Discuss a reversible process and express the first law for a reversible process.

28. State the II Law of Thermodynamics as a statement on the preferred direction of a given heat transfer process. Discuss Clausius statement “It is not possible that at the end of a cycle of changes, heat has been transferred from a colder to a hotter body without providing some other effect”.
29. Define entropy that ascertains the proper direction of a given process in accordance to II law.
30. Derive a relation for change in entropy of a system in terms of initial and final states of thermodynamic variables pressure and temperature. For an isentropic process derive a relation for pressure ratio in terms of temperature or density ratios using specific heat ratio.



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