Preface

This book is the English edition of the Japanese book *A Group Theoretic Approach to Quantum Information*, which was originally published by Kyoritsu Shuppan, Tokyo, Japan in January 2014. Hence, it is the English translation from the original Japanese book.

Group symmetry is a fundamental concept of quantum theory and has been applied to various fields in physics, particle physics, quantum field theory, cosmology, nuclear physics, condensed matter physics, and quantum optics. Since quantum information is an area to utilize the properties of quantum theory, group symmetry plays an important role in quantum information as well. In contrast, most of studies of quantum information have concentrated on application of methods in information science and proposals of new protocols. Hence, group symmetry has not taken a central role of quantum information. Mathematical foundation of quantum information mainly has been based on the viewpoint of operator algebra. While several individual subareas of quantum information employed group theoretic methods, many people consider that group symmetry does not play an important role in quantum information.

However, there exist so many symmetries based on group representation *behind* several quantum information processes although the symmetries have not been noticed sufficiently. Due to the unawareness, no text book has systematically summarized the method of group symmetry in quantum information. That is, due to the lack of the awareness of the group symmetric structure, these topics have been treated in a completely different way; besides they have common structures based on the group symmetric structure. To resolve such problems, this book deals with fundamental topics of quantum information that are essentially based on group representation so that the reader can understand how group symmetry works as a basic infrastructure of quantum information. These topics can be more deeply understood via group theoretical viewpoint, which brings an easier generalization of respective topics.

Chapter 1 introduces fundamental concepts of quantum information, measurement, state, composite system, computation basis, entanglement, etc, and prepares mathematical notations for quantum system. Then, Chap. 2 summarizes fundamental knowledge for quantum channel and information quantities, which are basis of quantum information. These contents can be understood only with linear algebra, calculus, and elementary probability. These two chapters are preliminary and do not discuss group representation. After these preparations, we deal with respective topics of quantum information by assuming the knowledge of group representation theory. However, the reader does not need to worry about the lack of the knowledge of group representation theory because all the required knowledge for group representation theory is summarized from the basics of group representation in the book *Group Representations for Quantum Theory* [44]. If the reader reads the book, the reader can understand the contents of this book only with linear algebra, calculus, and elementary probability without any additional knowledge for group representation theory. For simplicity, this book refers only the book [44] for any topics of group representation while they are obtained in different literature first. To adopt the notations in the above book, we describe the (projective) unitary representation by the font f. Also, the generators of these representations are written with the same font with the capital letter, e.g., Q.

In fact, if we do not employ any knowledge of representation theory, the contents of the remaining chapters cannot be discussed. This property shows that group representation-approach plays an important role in quantum information. This book has a unique characteristic to explicate quantum information based on group representation. On the other hand, quantum information can be discussed more rigorously than other topics in physics so that the contents of this book are more interesting from the mathematical viewpoint. However, quantum information cannot attract sufficient attention from mathematicians. The main reason seems in the point that the existing studies mainly focus not on the common mathematical structure but on the individual topics. To resolve this issue, this book aims to clarify the common mathematical structure via group representation theory.

Now, we proceed to the details of remaining chapters that address individual topics in quantum information. Chapter 3 addresses entanglement of quantum system via group representation. While existing books mainly deal with entanglement of the bipartite system, this chapter deals with entanglement of the multi-partite system as well as entanglement of the bipartite system. This topic seems to have no deep relation with group representation. However, this topic is essentially based on group representation in an unexpected way behind. For example, many entanglement measures are proposed. However, it is quite difficult to calculate them. A larger part of resolved cases are essentially based on the group symmetry because many entanglement measures are based on optimization and employing the group symmetry can reduce the freedom of the optimization.

Chapter 4 discusses covariant measurements with respect to group representation and gives the theory for optimal measurement. This approach can be applied to so many topics in quantum information. Although quantum state estimation is a very fundamental task in quantum information, the problem to optimize the estimation procedure cannot be exactly solved with the finite resource setting, in general, and it can be exactly solved only when the problem has group symmetric property. Hence, we can say that group symmetric property is essential for state estimation. This idea can be applied to the estimation of unknown unitary action. This problem is much more deeply related to group representation because this problem can be investigated by using Fourier transform in the sense of group representation theory. Further, we deal with approximate quantum state cloning in the same way because we can say the same thing for approximate quantum state cloning, which is also an important topic rooted in the foundation of in quantum theory. That is, all of solved examples of approximate quantum state cloning are based on the group symmetry.

Chapter 5 deals with quantum error correction. As quantum error correction, a stabilizer code and a CSS (Calderbank–Shor–Smolin) code are well known. Since these are essentially based on the discrete Heisenberg representation, group representation plays a crucial role in quantum error correction. To address this topic, we first summarize classical algebraic error correction. Then, based on the discrete Heisenberg representation, we address a stabilizer code, which is a typical quantum error correction, and a CSS code, which is a special case of a stabilizer code. Further, we consider a Clifford code, which is a general framework of algebraic quantum error correction. In addition, we investigate the security analysis of quantum cryptography based on our analysis on CSS codes.

Chapter 6 addresses universal information processing based on Schur duality, which is the joint representation of the special unitary group and the permutation group and can be regarded as quantum analogue of the method of types by Csiszár and Körner, which is one of typical approaches to classical information theory. The universality is the independence of the protocol from the channel or the information source. This chapter contains estimation of density matrix, hypothesis testing of quantum state, entanglement concentration, quantum data compression, and classical-quantum channel. When we do not care about the universality, we can discuss these topics without use of representation theory. However, to construct protocols to achieve the universality, we need to employ Schur duality theory because they cannot be constructed without use of Schur duality.

In the above way, the topics of this book cannot be discussed without group representation. Using the group representation this book deals with these fundamental topics in quantum information very efficiently. Unfortunately, the method of group representation is not the typical approach to quantum information. However, many leading researchers consider that the demand of group symmetry is increasing in the area of quantum information. This is because the area of quantum information is well matured and needs a more systematic approach. Further, to lead the reader to understand, the book contains 32 figures and 71 exercises with solutions. Finally, we make a remark for the organization of this book. We put the symbol * on the title of a section or an example that is very complicated. A part with * will be used only in parts with * so that the reader can understand major parts without reading parts with *. So, the reader is recommended to omit such parts if he/she is not familiar to group representation.

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