Chapter 2
Power System Modelling and SSR Analysis Methods

Abstract  Numerous well documented tools are available for power system modelling and analysis. This chapter will describe the most dominant and commonly used methods to study subsynchronous resonance phenomenon in power system networks. Since this phenomenon is closely related to the operation of turbogenerators, the fundamental structure, and operation of a synchronous machine is also explained. Within this chapter, models for all of the main power system components, including excitation systems, power system stabilizers, transmission lines, loads, synchronous machine and turbine generator mechanical system are described. The modelling of Line Commutated Converter, Voltage Sourced Converter based HVDC system and thyristor controlled series capacitor is also provided.

2.1 Synchronous Generators

The modelling and analysis of the synchronous machine has been subject of investigations since 1920 [1, 2], several more studies investigated the same subject [3–5]. Many books also covered the operation and performance of synchronous machines [1, 6]. Within this section the basic structure and operation of the synchronous machine is described. The synchronous machine is an AC generator, driven by a turbine to convert mechanical energy into electrical energy. Understanding of synchronous machine operation and accurate modelling of its dynamic performance are extremely important in subsynchronous studies.

The two major parts of synchronous machine are ferromagnetic structures. The stationary part which is basically a hollow cylinder, called the stator or armature shown in Fig. 2.1a. The armature has longitudinal slots in which there are coils of the armature windings. These windings carry the current supplied to an electrical load by a generator. The rotor is the part (shown in Fig. 2.1b) which is mounted on the shaft and rotates inside the hollow stator. The winding on the rotor, called field winding, carries DC current and produces magnetic field which induces alternating
voltages in the armature windings. The very high mmf produced by the DC current in the field winding joins the mmf produced by the currents in the armature windings. The resultant flux across the air gap between the stator and rotor generates voltages in the coils of the armature windings and produces electromagnetic torque.

The DC current is supplied to the field winding by an exciter, which may be a generator placed on the same shaft or separate DC source connected to the field winding through brushes bearing on slip rings. Large AC generators usually have exciters consisting of an AC source with solid state rectifiers. In case of generators, the shaft is driven by a prime mover which is usually steam or hydraulic turbine. The electromagnetic torque developed in the generator when it delivers power opposes the torque of prime mover [1, 6].

Figure 2.2 shows the cross section of a three phase synchronous machine with one pair of poles. It can be observed that the opposite sides of a coil, which is almost rectangular are in slots $a$ and $a'$ 180° apart. Similar coils are in $b$ and $b'$ slots and $c$ and $c'$ slots. Coils sides in $a$, $b$, $c$ are separated by 120° in space so that the uniform rotation of a magnetic field generates voltages displaced by 120° in time domain in the armature windings.

The armature windings usually operate at a voltage that is considerably higher than that of the field voltage, and they are subjected to high transient currents. Therefore, generally armature is mounted on the stator to provide more space for insulation and adequate mechanical strength [6].

The balanced three phase stator currents produce magnetic field in the air gap which rotates at synchronous speed. The field produced by the field windings revolves with the rotor. The rotor field and stator field must rotate at the same speed for the production of a steady torque. Hence, the rotor must run at the synchronous speed.
The field winding indicated by \( f \), gives rise to two poles \( N \) and \( S \) as marked in Fig. 2.2. The axis of field poles is called direct axis or \( d \)-axis while the centreline of the interpolar space is called the quadrature axis or \( q \)-axis. In the actual machine the winding has a large number of turns distributed in slots around the circumference of the rotor. The number of field poles is determined by the mechanical speed of the rotor and the electrical frequency of the stator currents. The synchronous speed is given by (2.1)

\[
Synchronous \ Speed = \frac{120f}{P_f}
\]  

(2.1)

Speed is measured in \textit{rev/min}. \( f \) is the synchronous frequency in \textit{Hz} and \( P_f \) is the number of the field poles.

There are two basic rotor structures, salient (100–1500 rpm) and cylindrical (>1500 rpm) depending on speed. Hydraulic turbines operate at low speeds, therefore, relatively large number of poles are required to generate the rated frequency. A rotor with salient or projecting poles and concentrated winding, is better suited mechanically for this kind of prime movers. The poles mounted on the rotor are made of steel laminations and connected to the rotor shaft by means of dovetail joints. Each pole has a pole shoe around which winding is wound. Salient pole rotors often have damper windings to prevent rotor oscillations during oscillations. Steam and gas turbines operate at higher speeds, they have two or four field poles. Their generators have round rotors, made up of solid forged steel. The slots on which windings are fixed, milled on the rotor since the rotor is cylinder, the windage loss is reduced.
2.2 Modelling Power System Components

This section includes models of all main power system components. These models have been used throughout the studies presented later.

2.2.1 Modelling Synchronous Generators

DIgSILENT PowerFactory software is used in the studies presented within this thesis. The descriptions of the models will be centred around the mathematical models available in DIgSILENT PowerFactory. The software provides sixth order generator model for RMS, and eight order model for EMT studies. The per unit stator voltage equations are given by (2.2) and (2.3).

\[
E_d = \frac{1}{\omega_0} \frac{d}{dt} \Psi_d - \frac{\omega}{\omega_0} \Psi_q - R_a i_d \\
E_q = \frac{1}{\omega_0} \frac{d}{dt} \Psi_q - \frac{\omega}{\omega_0} \Psi_d - R_a i_q
\]

where \(E_d\) and \(E_q\) are the \(d\)-axis and \(q\)-axis stator voltages. \(\Psi_d\) and \(\Psi_q\) are the \(d\)-axis and \(q\)-axis stator flux linkage. \(i_d\) and \(i_q\) are the \(d\)-axis and \(q\)-axis currents. \(R_a\) is the stator resistance per phase, \(\omega\) is the angular frequency and \(\omega_0\) is the rated angular frequency.

The per unit rotor voltage equations are given by (2.4)–(2.7)

\[
E_{fd} = \frac{1}{\omega_0} \frac{d}{dt} \psi_{fd} + R_{fd} i_{fd} \\
0 = \frac{1}{\omega_0} \frac{d}{dt} \psi_{1d} + R_{1d} i_{1d} \\
0 = \frac{1}{\omega_0} \frac{d}{dt} \psi_{1q} + R_{1q} i_{1q} \\
0 = \frac{1}{\omega_0} \frac{d}{dt} \psi_{2q} + R_{2q} i_{2q}
\]

where \(E_{fd}\) is the field voltage, \(\psi_{fd}\) is the field flux, \(R_{fd}\) and \(i_{fd}\) are the field resistance and field current. \(\psi_{1d}\), \(\psi_{1q}\) and \(\psi_{2q}\) are the rotor circuit flux linkage in \(d\)-axis and \(q\)-axis. \(R_{1q}\), \(R_{2q}\) are rotor circuit resistances. \(i_{1d}\), \(i_{1q}\), \(i_{2q}\) are the rotor circuit circuits.
The equations of motion of a generator referred to as the swing equation can be expressed as

\[
\frac{d\omega}{dt} = \frac{1}{2H} \left( T_m - T_e - K_D(\omega - 1) \right) 
\]

\[
\frac{d\delta}{dt} = \omega_0(\omega - 1) 
\]

where \( T_m \) is the mechanical torque, \( T_e \) is the electrical torque, \( K_D \) is the damping factor, \( H \) is the inertia constant, \( \delta \) is the rotor angle.

### 2.2.2 Modelling Turbine Generator Mechanical System

The rotor of a turbine generator is a complex mechanical system made up of several rotors of different sizes. The length of the rotor system can exceed 50 m and weigh several hundred tons. The system also has quite a few smaller components such as turbine blades, rotor coils, retaining rings, blowers and pump. An exact analysis of a rotor system may require a continuum model, however, multimass model is adequate for SSR studies [7, 8].

DlgsILENT PowerFactory provides built model of a turbine generator mechanical system. Figure 2.3 shows the turbine generator rotor model used throughout these studies. It is typical model for SSR studies. It consists of a high pressure turbine (HP), an intermediate-pressure turbine (IP), two low pressure turbines (LPA and LPB), the generator rotor (GEN) and the exciter (EXC). They together constitute a linear six-mass spring system. In this model each major element of the system is modelled (such as generator, different sections of turbine, exciter) as a rigid mass connected to adjacent elements by mass less springs. The natural frequencies of the mechanical system calculated using this model are generally lower than system electrical frequency and reasonably match the lower frequency modes of the unit vibration. Lower frequency modes contribute most to shaft stresses caused by terminal short circuits, therefore, this simple model is adequate to evaluate the integrity of the main shaft section.

![Rotor model for transient studies](image-url)
In Fig. 2.3, \( D_i \) represents the external damping, \( D_{i,j} \) is the internal damping in the shaft material and \( K_{i,j} \) represents the stiffness of the shaft. Damping is measured in \( s^{-1} \) or radians/s, and \( K \) is measured in p.u. or p.u. torque/rad.

The mechanical system consisting of generator rotor, exciter and turbines shafts can be viewed as a mass-spring damper system. The equation for the \( i \)th mass connected by elastic shaft sections to mass \((i - 1)\) and mass \((i + 1)\) is given by

\[
\frac{2H}{\omega_0} \frac{d^2 \delta_i}{dt^2} + D_i \frac{d \delta_i}{dt} + D_{i,i-1} \left( \frac{d \delta_i}{dt} - \frac{d \delta_{i-1}}{dt} \right) + D_{i,i+1} \left( \frac{d \delta_i}{dt} - \frac{d \delta_{i+1}}{dt} \right) + K_{i,i-1} (\delta_i - \delta_{i-1}) + K_{i,i+1} (\delta_i - \delta_{i+1}) = (T_{mi} - T_{ei}) = T_a
\]

where \( H \) is the inertia constant expressed in s, \( K \) is the spring constant or stiffness measured in p.u. torque/electrical rad and \( \delta \) is the angular position of the mass \( i \) in electrical radians.

Note: electrical radians = mechanical radians \( \left( \frac{p_f}{2} \right) \) where \( p_f \) is the number of generator field poles.

The differential equation that describes the motion of the generator

\[
\frac{2H}{\omega_0} \frac{d^2 \delta}{dt^2} + D_G \frac{d \delta}{dt} + D_{GB} \left( \frac{d \delta_B}{dt} - \frac{d \delta}{dt} \right) + D_{EG} \left( \frac{d \delta}{dt} - \frac{d \delta_E}{dt} \right) + K_{GB} (\delta - \delta_B) + K_{EG} (\delta - \delta_E) = (T_m - T_e) = T_a
\]

The differential equation that describes the motion of the low pressure turbine B

\[
\frac{2H_B}{\omega_0} \frac{d^2 \delta_B}{dt^2} + D_B \frac{d \delta_B}{dt} + D_{BA} \left( \frac{d \delta_A}{dt} - \frac{d \delta_B}{dt} \right) + D_{GB} \left( \frac{d \delta_B}{dt} - \frac{d \delta_G}{dt} \right) + K_{BA} (\delta_B - \delta_A) + K_{GB} (\delta_B - \delta_G) = T_{LPB}
\]

The differential equation that describes the motion of the low pressure turbine A

\[
\frac{2H_A}{\omega_0} \frac{d^2 \delta_A}{dt^2} + D_A \frac{d \delta_A}{dt} + D_{AIP} \left( \frac{d \delta_A}{dt} - \frac{d \delta_{IP}}{dt} \right) + D_{BA} \left( \frac{d \delta_A}{dt} - \frac{d \delta_B}{dt} \right) + K_{AIP} (\delta_A - \delta_{IP}) + K_{BA} (\delta_A - \delta_B) = T_{LPA}
\]

The differential equation that describes the motion of the intermediate pressure turbine

\[
\frac{2H_{IP}}{\omega_0} \frac{d^2 \delta_{IP}}{dt^2} + D_{IP} \frac{d \delta_{IP}}{dt} + D_{HIP} \left( \frac{d \delta_{IP}}{dt} - \frac{d \delta_{HIP}}{dt} \right) + D_{MPI} \left( \frac{d \delta_{IP}}{dt} - \frac{d \delta_A}{dt} \right) + K_{HIP} (\delta_{IP} - \delta_{HIP}) + K_{AIP} (\delta_{IP} - \delta_A) = T_{IP}
\]
The differential equation that describes the motion of the high pressure turbine

\[
\frac{2H_{HP}}{\omega_0} \frac{d^2 \delta_{HP}}{dt^2} + D_{HP} \frac{d \delta_{HP}}{dt} + D_{HP \dot{\delta}_{IP}} \left( \frac{d \delta_{HP}}{dt} - \frac{d \delta_{IP}}{dt} \right) + K_{HP}(\delta_{HP} - \delta_{IP}) = T_{HP}
\]

Equations (2.11)–(2.15) describe the torque developed in each turbine section.

Mechanical power \( P_m \) and mechanical torque \( T_m \) of the \( i \)th mass are related by

\[
\Delta P_{mi} = T_{mi} \frac{\omega_i}{\omega_0}
\]

where \( \omega_0 \) is the rated angular speed of the rotor and \( \omega_i \) is the angular speed of the \( i \)th shaft section. After initialization (2.16) can be rewritten as (2.17)

\[
\Delta P_{mi} = T_{mi0} \frac{\Delta \omega_i}{\omega_0} + T_{mi}
\]

where \( T_{mi0} \) is the initial torque developed.

The state equation for the turbine generator mechanical system can be expressed as

\[
X_m = [A_m] \dot{X}_m + [B_{m1}] \Delta P_m + [B_{me}] \Delta T_e
\]

where

\[
X_m = [\delta_E \ \delta \ \delta_B \ \delta_A \ \delta_I \ \delta_H \ \Delta \omega_E \ \Delta \omega_G \ \Delta \omega_B \ \Delta \omega_A \ \Delta \omega_I \ \Delta \omega_H]^T
\]

\[
[A_m] = \begin{bmatrix}
0_{6 \times 6} & I_{6 \times 6} \\
A_{21} & A_{22}
\end{bmatrix}
\]

\[
[A_{m21}] = \begin{bmatrix}
-K_{E_E} \frac{2H_E}{\omega_0} & K_{E_E} \frac{2H_E}{\omega_0} & 0 & 0 & 0 & 0 \\
K_{E_E} \frac{2H_E}{\omega_0} & -K_{E_E} + K_{EB} \frac{2H_B}{H_E} & K_{EB} \frac{2H_B}{H_E} & 0 & 0 & 0 \\
0 & K_{EB} \frac{2H_B}{H_E} & -K_{EB} \frac{2H_B}{H_E} & 0 & 0 & 0 \\
0 & 0 & K_{EB} \frac{2H_B}{H_E} & -K_{EB} + K_{EB} \frac{2H_B}{H_E} & K_{EB} \frac{2H_B}{H_E} & 0 \\
0 & 0 & 0 & K_{EB} \frac{2H_B}{H_E} & -K_{EB} + K_{EB} \frac{2H_B}{H_E} & K_{EB} \frac{2H_B}{H_E} \\
0 & 0 & 0 & 0 & K_{EB} \frac{2H_B}{H_E} & -K_{EB} \frac{2H_B}{H_E} \\
\end{bmatrix}
\]

(2.21)
Table 2.1 Comparison of SSR analysis methods

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<td>0.22</td>
<td>–</td>
<td>0.22</td>
<td>–</td>
<td>[120]</td>
</tr>
</tbody>
</table>

\[
[A_{m22}] = \begin{bmatrix}
\frac{dL + dL}{dL} & \frac{dL}{dL} & 0 & 0 & 0 & 0 \\
\frac{dL}{dL} & \frac{dL + dL}{dL} & \frac{dL}{dL} & 0 & 0 & 0 \\
0 & 0 & \frac{dL}{dL} & \frac{dL + dL}{dL} & \frac{dL}{dL} & 0 \\
0 & 0 & 0 & \frac{dL}{dL} & \frac{dL + dL}{dL} & \frac{dL}{dL} \\
0 & 0 & 0 & 0 & \frac{dL}{dL} & \frac{dL + dL}{dL} \\
0 & 0 & 0 & 0 & 0 & \frac{dL}{dL} & \frac{dL + dL}{dL}
\end{bmatrix}
\]

(2.22)

\(F_H, F_I, F_A\) and \(F_B\) represent the fraction of mechanical power delivered by each turbine.

The torques generated by the individual turbine sections depend on the dynamics of the steam turbine and its governing system. The typical values found in literature are given in Table 2.1.

Since the mechanical power is considered constant, no governor action is considered, therefore, the perturbation of input torque \(T_m\) is considered zero.

### 2.2.3 Generator Excitation Systems

The primary function of the excitation system is to provide direct current to synchronous machine field winding. In addition, the excitation system can contribute towards maintaining power system stability by controlling the field voltage \(E_{fd}\) and thereby the field current. This is achieved through AVR which manipulates the field voltage in order to reach the generator stator terminal voltage reference set-point, \(E_i\), and to ensure the first swing stability of the machine.

A power system stabilizer may also be incorporated in order to reduce rotor speed variations following disturbances. Figure 2.4 illustrates operational relationship between the synchronous generator, excitation system and PSS.

Various excitation systems are used in practice, comprehensive details can be found in [9]. Excitation systems used within this thesis are described in the following sections.
2.2.3.1 Manual Excitation

Manual excitation is the most basic and simplest excitation scheme. It maintains the field voltage $E_{fd}$ at a constant value determined through the synchronous generator parameter initialization. This scheme does not employ AVR, therefore, the generator terminal voltage may vary from the desired value if operating conditions change.

2.2.3.2 Static Excitation (IEEE Type STIA)

Static excitation systems provide direct current to field winding of the generator through rectifiers which are fed by either transformers or auxiliary machine windings [9]. A simplified version of IEEE Type ST1A static exciter used within this thesis is presented in Fig. 2.5. It consists of voltage transducer delay, exciter, and Transient Gain Reduction (TGR). The signal $E_{pss}$ is a signal from the PSS, if one is used in conjunction with the exciter.

Two versions of this excitation system are used within this thesis, referred to as ST1A_v1 and ST1A_v2.

**ST1A_v1** considers the transducer delay as negligible ($T_R = 0$) (used in Test network 2, will be introduced later in this chapter).

**ST1A_v2** has no time constant in the exciter block ($T_A^{ex} = 0$), and no transient gain reduction block (used in Test network 1, will be introduced later in this chapter).

![Fig. 2.4 Signals between the synchronous generator, excitation system and power system stabilizer](image)

![Fig. 2.5 Simplified block diagram for the IEEE type ST1A static exciter](image)
2.2.3.3 DC Excitation (IEEE Type DC1A)

Excitation systems which use a DC current generator and commutator are referred to as DC exciters. These type of exciter respond slower than static systems [9]. A simplified version of the IEEE Type DC1A DC excitation system used within this thesis, is shown in Fig. 2.6 (used in Test network 2).

2.2.4 Power System Stabilizers

A power system stabilizer acts to provide additional damping to generator rotor oscillations through supplementary control signal sent to the excitation system. The most commonly and logical input signals to the power system stabilizer are rotor speed deviation $\Delta \omega_r$, terminal frequency and power [1].

Figure 2.7 shows the block diagram of the PSS used within this thesis. To compensate the phase lag between the exciter input and the electrical torque to ensure that the introduced electrical damping torque component is in phase with the rotor speed variation, the PSS must include suitable compensation blocks. This phase compensation is introduced by a number of phase lead/lag blocks which are combined with a washout filter so that steady state changes are ignored.

Fig. 2.6 Simplified block diagram for the IEEE type DC1A DC exciter

Fig. 2.7 Block diagram of a PSS
2.2.5 Transmission Lines

Transmission lines are fundamentally distributed parameter components of the power system. It is essential to model them in some detail for the study of fast switching transients. Transmission lines representation by a single $\pi$ circuit, however, is adequate for the power system dynamic studies involving frequencies below the synchronous frequency [4]. Within this thesis transmission lines are modelled using standard $\pi$ equivalents. Figure 2.8 shows a single phase $\pi$ equivalent of a transmission line.

Basic assumptions in three phase representation of transmission lines are that they are symmetric, self impedance of all three phases is equal and mutual impedance between any two phases is the same, line parameters are constant and the network is linear. It can be demonstrated that in steady case, a symmetric three phase linear network connected to synchronous generators has only fundamental frequency voltages and currents [1]. On the other hand, a lack of symmetry produces imbalanced currents (with negative sequence component) which can results in third harmonic voltage generation. The symmetry is disturbed during imbalanced faults such as single line to ground or line to line faults. However, their duration is brief and their presence can be neglected.

DIgSILENT PowerFactory provides both $\pi$ and distributed model for transmission lines. Throughout this thesis, transmission lines are represented by a lumped parameter model, the common $\pi$ representation.

2.2.6 Loads

Representation of loads in a power network can have a significant impact on analysis results [11, 12]. It was shown that dynamic load models can affect the damping of electromechanical modes and participation of a generator in the mode. Further examples of the effects of the loads modelling can be found in [13]. Within this thesis, a constant impedance load model is used, represented as a shunt admittance $Y_{load}^{i}$ connected to the $i$th load bus. This load model is considered adequate for subsynchronous resonance studies.
2.3 HVDC System Modelling

2.3.1 LCC-HVDC Converters

DIgSILENT PowerFactory provides integrated Line commutated inverter and rectifier models. These models are used in this thesis. The model for load flow calculations, RMS and EMT simulations are based on the fundamental frequency approach. In steady state, the converter is modelled as a load with constant active power $P$ and reactive power $Q$. The transmitted DC power across HVDC system can be expressed by (2.23).

$$P_{dc} = V_{con} \cdot I_{dc}$$  \hspace{1cm} (2.23)

where $V_{con}$ is the converter voltage and $I_{dc}$ is the current through DC line.

The DC voltage of the ideal and uncontrolled converter without load is called ideal no load voltage represented by $V_{dio}$. For a six pulse converter, it is given by (2.24) where $V_{LL}$ is the AC voltage supplied to the converter station.

$$V_{dio} = \frac{3}{\pi} \sqrt{2} V_{ac}$$  \hspace{1cm} (2.24)

The gate control of the thyristors is used to delay ignition of the valves. The time delay due to turn on applied signal is given by

$$\omega t = \alpha$$  \hspace{1cm} (2.25)

and (2.24) is modified as follows

$$V_{dio} = V_{dio0} \cdot \cos(\alpha)$$  \hspace{1cm} (2.26)

By considering the current commutation from one valve to next, (2.26) can be rewritten as shown in (2.27)

$$V_{dio} = V_{dio0} \cdot \cos(\alpha) - \Delta V_{con}$$  \hspace{1cm} (2.27)

where $\Delta V_{con}$ is defined as a function of $I_{dc}$ and commutation reactance $X_c$.

$$\Delta V_{con} = \frac{3}{\pi} X_c \cdot I_{dc}$$  \hspace{1cm} (2.28)

In HVDC systems the commutation reactance is assumed to be the leakage reactance of the converter transformer given by (2.29)
\[ X_c = X_{r,sec} = u_{kr} \frac{V^2_{r,sec}}{S_T} \] (2.29)

\( X_c \) is the commutation reactance, \( S_T \) is the rated power of transformer, \( u_{kr} \) is the short circuit voltage and \( V_{r,sec} \) is the transformer secondary side voltage.

By considering the commutation reactance and ignition angle, the DC voltage given by (2.27) can be written as

\[ V_{di} = V_{di0} \cdot \cos(\alpha) - \frac{u_{kr}}{2} \] (2.30)

The power factor can be calculated assuming symmetrical firing angle and using positive sequence voltage as a reference.

Valves are triggered using the built in trigger circuit which converts the firing angle supplied by the converter controller to six correct firing signals of the discrete thyristors.

2.3.2 Converter Transformer Model

Converter transformer is modelled by three phase three winding transformer, with grounded Wye-Wye and delta connection. The model uses tap setting arrangement.

2.3.3 LCC Converter Controls

DIgSILENT PowerFactory offers the flexibility to use built in converter controls. In this work, the built in controls for LCC HVDC system are used.

The LCC controls mainly consist of generation of firing signals, firing angle and extinction angle measurement. In the studies that will be presented later rectifier is operated at constant current and firing angle control while inverter have constant extinction angle and voltage control.

2.3.4 VSC-HVDC Converters

DIgSILENT PowerFactory provides PWM converter model that represents a self commutated, voltage sourced AC/DC converter.

The circuit in Fig. 2.9 is built from valves with turnoff capability which are usually realized by GTOs or IGBTs. Fundamental frequency models provided in DIgSILENT PowerFactory for load flow, and stability are valid for three level
PWM designs as well. The VSC converter supports sinusoidal and rectangular modulation.

The model of all steady state functions including RMS simulations and EMT simulations are based on a fundamental frequency approach. At fundamental frequency, the ideal loss-less converter can be represented by a DC-voltage controlled AC-voltage source conserving active power between AC and DC side. The Pulse width modulation index \( Pm \) is a control variable of PWM converter.

For \(|Pm| < 1\), the following equations can be applied:

\[
V_{acr} = K_0 P_{mr} V_{dc}
\]

\[
V_{aci} = K_0 P_{mi} V_{dc}
\]

The active power conversion between AC and DC side can be written as

\[
P_{ac} = \text{Re}(V_{-ac} I_{-ac}^*) = V_{dc} I_{dc} = P_{dc}
\]

where \( V_{acr} \) is the real part, \( V_{aci} \) is the imaginary part of AC voltage (rms) value, \( K_0 \) is the constant depending on the modulation method. \( P_{mr} \) and \( P_{mi} \) are real and imaginary part of modulation index respectively. \( V_{-ac} \) is the AC voltage phasor, \( I_{-ac}^* \) is the conjugate complex value of the current phasor.

### 2.3.5 VSC-HVDC Controls

The common feature of all VSC-HVDC systems is the generation of a fundamental frequency AC voltage from a DC voltage; the control of this voltage magnitude and phase is the basic function of the VSC. The phase angle \( \delta \) and thus active power transfer is controlled by shifting the fundamental frequency voltage produced by the converter. The power transfer can be from AC system to converter or vice versa depending on the sign of the phase angle difference. Present HVDC schemes are designed to maintain the nominal DC voltage, and control the converter AC voltage by means of PWM. In VSC PWM conversion, the AC voltage output is varied by means of a modulation index defined as the ratio of the required AC voltage...
magnitude to the maximum AC voltage that can be generated for a given DC size capacitor. When the magnitude of this modulation index is close to one, converter voltage is greater than the AC system, and reactive power is transferred to the AC system. When the index is less than one, converter voltage is lower than the system voltage, and the converter absorbs reactive power [14].

The control system for a voltage source converter has a hierarchy structure, with each inner loop to be faster than its outer loop. Vector control also known as \(dq\) current control forms the most inner loop of VSC-HVDC system within this thesis. In this control strategy, the three phase currents are transformed to \(d\) and \(q\) axes, which are then synchronized with the AC system three phase voltage via a phase locked loop (PLL). The \(d\) and \(q\) voltages generated by vector control are transformed to three phase quantities and converted into line voltages by the VSC.

Figure 2.10 shows the current controller used in this work. The input currents to the controller are the converter’s AC currents expressed in a reference frame and output signals are \(P_{md}\) and \(P_{mq}\).

**2.3.5.1 Outer Control Loops**

Vector current control offers the flexibility of independent control of real and reactive power by means of \(dq\) transformation. Based on this the most inner control loop, different controls strategies can be applied namely, active power control, reactive power control, DC voltage control, AC voltage control, and frequency control. The outer control loops used in the studies within this thesis are described below.

**DC Voltage Control**

Large variations in a DC system voltage are not acceptable in normal operation of a VSC-HVDC system as this might lead to power imbalance or equipment failure.
Therefore, one converter in the DC grid is responsible to maintain a constant DC voltage. This is achieved by adding an outer loop control that modifies the reference $d$-axis current input of the inner current loop. The voltage controller is significantly slower than the inner current loop.

Figure 2.11 shows the structure of the outer voltage loop. It is feedback control which requires the measurement of DC link voltage. During a severe disturbance, large variations in DC link may lead to an unacceptable value of current reference. Therefore, the output current must be limited.

**AC Voltage Control**

VSC-HVDC link can also regulate AC side voltage directly with vector current control loop as the inner loop. The basic operation of this control is similar to the reactive power control which maintains the grid side AC voltage.

AC voltage control is also a feedback control, as shown in Fig. 2.12 and requires the AC voltage at the point of control to be measured. VSC-HVDC link with AC voltage control can provide support to improve the AC network dynamic performance.

**Active and Reactive Power Control**

The control of the active power transferred through HVDC link, and the reactive power generated or absorbed by the VSC can be obtained by means of $d$ and $q$ current references of the $dq$ current controller [15].

\[
P = v_d i_d \quad (2.34)
\]

\[
Q = -v_d i_q \quad (2.35)
\]
2.3.6 VSC Control Structure

Figure 2.14 shows the VSC control system including dq current controller, the most inner control loop, and DC voltage control and reactive power control making the outer loops [15].

The rated DC voltage is a reference signal for DC voltage control, it is compared with the measured DC line voltage. The output of the DC control is the $d$-axis component of the grid current $i_{d\text{-ref}}$. The actual grid current $i_d$ is compared with the reference and difference is fed to the $d$-axis current control. A compensation term $L_g \omega_g i_q$ (where $L_g$ is the grid side filter inductance and $\omega_g = \theta_g$) is added for...
decoupling the $d$ and $q$ axis. Grid side $d$-axis grid voltage is also added as feed-forward control to current controller output to produce $d$-axis converter voltage $V_{dc}$.

The $q$-axis current is used to regulate reactive power in the AC grid. The error signal ($Q_{\text{ref}} - Q$) is fed to the reactive power control to obtain $i_{q\text{-ref}}$. The error signal produced by comparing $i_{q\text{-ref}}$ and $i_q$ is sent to $q$-axis current controller. A compensation term $-L_g \omega_g i_d$ is added for decoupling the control between $d$ axis and $q$ axis to the output of $d$ axis current controller to produce $q$ axis component $V_q$.

The two voltage components $V_d$ and $V_q$ are transformed to three phase voltages for PWM control.

For the other converter, the DC voltage control is replaced by the active power control, and the reactive power control is replaced by the AC voltage control.

### 2.4 Thyristor Controlled Series Capacitors (TCSCs)

The TCSC is modelled as a fixed capacitor in parallel with variable inductive reactance (TCR) as shown in Fig. 2.15.

The TCR and effective TCSC reactance is controlled by firing angle $\alpha$ and is given by the following equations.

\[
X_L(\alpha) = X_L \frac{\pi}{\pi - 2\alpha - \sin 2\alpha}
\]

\[
X_{TCSC}(\alpha) = \frac{X_c^2}{(X_c - X_L)} \left( \frac{2(\pi - \alpha) + \sin(2(\pi - \alpha))}{\pi} \right)
\]

\[
+ \frac{4X_c^2}{(X_c - X_L)} \frac{\cos^2(\pi - \alpha)}{(\kappa^2 - 1)} \frac{\kappa \tan \beta - \tan \beta}{\pi} - X_c
\]

\[
\text{where } \kappa = \sqrt{\frac{X_c}{X_L}}
\]

For practical TCSC implementations, $\kappa$ is typically between 2 and 4 [4, 126]. TCSCs are usually operated such that $\frac{X_{TCSC}}{X_c}$ is between 2 and 3 [10, 16].

![Fig. 2.15 TCSC block diagram](attachment:image.png)
In these studies, TCSC is operated in constant impedance control mode. Thyristors valves are triggered using synchronous voltage reversal approach (SVR) since it provides better damping characteristics in subsynchronous frequency range [17, 18]. Synchronous voltage reversal scheme exploits the fact that capacitor voltage reversal occurs during the thyristor conduction interval. When a thyristor is triggered, a current pulse passes through the thyristor and adds to the line current. Thus, an extra charge is pushed into the capacitor from the thyristor branch. This is, in addition, to the charge due to line current such adding an extra voltage across the capacitor. With no losses, the thyristor valve stops conducting when the capacitor voltage is equal in magnitude but opposite in the direction as it was at the turn on instant. The maximum reactance boost depends on the actual line current and the duration of the boosting action. Figure 2.16 shows the boost in capacitor voltage due to conduction of thyristor valves. Required TCSC impedance can be obtained by an equivalent, instantaneous voltage reversal in the middle of the thyristor conduction interval.

Figure 2.17a illustrates TCSC control structure used in this work. The firing angle is calculated by the impedance control, and start pulse is given to the SVR unit at \( t_{\text{start}} \) as shown Fig. 2.17a, it fires thyristor at \( t_f \). Time \( t_f \) is selected such that the thyristor current reaches at its peak with a fixed delay, \( t_0 \). The SVR block calculates the firing instant, based on the measured instantaneous values of capacitor voltage and line current using (2.45)–(2.41) [17].

\[
\begin{align*}
\text{\( u_{\text{CZ}} \) is the reversal voltage (at instantaneous reversal), \( u_{\text{CM}} \) is the measured capacitor voltage, \( X_0 \) is TCSC reactance at resonance, \( i_{\text{LM}} \) is the measured line current, \( t_Z \) is time instant, when it is desired that the capacitor voltage be zero, \( t_M \) is the time instant when \( i_L(t) = 0 \).} 
\end{align*}
\]

\[
\begin{align*}
u_{\text{CZ}} &= u_{\text{CM}} + X_0 i_{\text{LM}} \lambda W_N (t_Z - t_M) \quad (2.39) \\
u_{\text{CZ}} &= X_0 i_{\text{LM}} [\lambda \beta - \tan(\lambda \beta)] \quad (2.40) \\
t_f &= t_Z - \frac{\beta}{\omega} \quad (2.41)
\end{align*}
\]
the time when line current and capacitor voltage measurements are made. $t_f$ is the thyristor triggering time and $\beta$ is the angle of advance.

2.5 SSR Analysis Methods

There are several analytical methods developed to study subsynchronous resonance phenomenon. Three most frequently used techniques both in the industry, and academia are frequency scanning, eigenvalue analysis and electromagnetic transients simulations [10, 19, 20]. Calculations and results using these three methods have been compared with test results, and there is every indication that all three methods give very good results when accurate data is available [21].

2.5.1 Frequency Scanning Method

Frequency scanning technique involves the determination of the driving point impedance as a function of frequency, looking into the network from a point behind the stator winding of a study generator [22]. All the three aspects of SSR namely,
Induction generator effect, Torsional interactions and transient torque amplification can be identified with the help of this technique. Frequency method is best suited for the preliminary studies.

Presence of reactance minima at the frequency which is near to a slip frequency (difference between system frequency and a torsional mode frequency) indicates a potential problem of transient torque amplification. Frequency scanning provides only an approximation for transient torque amplification if such a problem exists [10, 21, 22]. However, it clearly indicates that transient torque amplification problems do not exist if there are no reactance minimum within ±3 Hz of a slip frequency.

Figure 2.18 adapted from [21] shows the frequency scan results for Navajo Project for the normal system configuration and all series capacitors in service. This figure is selected because it clearly demonstrates that all three aspects of SSR can be identified from frequency scanning method. When frequency scanning is used in conjunction with “interaction equation” given by (2.42), negative damping caused by the fixed series compensation can be calculated with reasonable accuracy [23].

\[
\Delta \sigma_n = -\frac{f - f_n}{8f_n H_n} \left( \frac{R_{sub}}{R_{sub}^2 + X_{sub}^2} \right) + \frac{f + f_n}{8f_n H_n} \left( \frac{R_{sup}}{R_{sup}^2 + X_{sup}^2} \right) \tag{2.42}
\]

where \( f_n \) is mechanical modal frequency, \( H_n \) is equivalent p.u. stored energy for a pure modal oscillations, \( R_{sub}, X_{sub} \) are subsynchronous resistance and reactance determined by the frequency scan at frequency \( (f - f_n) \), \( R_{sup}, X_{sup} \) are

![Figure 2.18](image-url)
supersynchronous resistance and reactance determined by the frequency scan at frequency \((f + f_n)\), respectively.

In Fig. 2.18 reactance zero or significant reactance dip are highlighted by red boxes. Figure 2.18 shows that there can be transient torque amplifications problem for Mode 3 (34 Hz) since slip frequency \((60 - 29 = 31)\) is within \(\pm 3\) Hz of the reactance dip. Severity or presence of transient torque amplification can be determined accurately only from EMT simulations, but frequency scanning is extremely useful to develop scenarios for EMT simulations.

Frequency scanning on the other hand is the best technique to establish the existence of induction generator effect. It is indicated when reactance curve passes through zero at a frequency that corresponds to net negative resistance of the power network [21, 22]. Figure 2.18 shows that induction generator effect is not a problem for Navajo generators since resistance is always positive. This is due to damper windings that were applied to the Navajo generators [24, 25].

Torsional interactions or self excitation due to SSR can be evaluated with adequate accuracy using frequency scanning method with the interaction equation developed in [23], given by (2.42).

Formula (2.42) provides the negative damping due to torsional interaction for Mode \(n\) with the system configuration represented in the specific frequency scanning case. It can be observed that (2.42) consists of two parts, subsynchronous and supersynchronous frequency. First part, subsynchronous part will always contribute negative damping, and the second part, supersynchronous part will always add positive damping.

The severity of torsional interactions can be established for Mode \(n\) by comparing the calculated negative damping given by (2.42) with the Mode \(n\) natural mechanical damping of the turbine generator.

\[
\sigma_{net} = \sigma_n - \Delta \sigma_n \tag{2.43}
\]

where \(\sigma_{net}\) is the net damping of a torsional mode, \(\sigma_n\) is the turbine generator damping for Mode \(n\) and \(\Delta \sigma_n\) is negative damping due to torsional interactions.

The validity of the frequency scanning method has been tested in several case studies and now widely used in the industry [22].

### 2.5.2 Eigenvalue Analysis

Eigenvalue technique presents additional information about the performance of the system. This type of analysis is performed with linearized model of the network and the generators using linear set of differential equations. Therefore, this technique is quite straightforward for studying SSR aspects that can be approximated as linear, i.e., induction generator effect and torsional interactions [19, 21, 26].
2.5 SSR Analysis Methods

Eigenvalue analysis provides both the frequencies of oscillations and damping of each mode, therefore, it is used to investigate the effects of different series compensation levels and systems configurations on the damping of torsional oscillations [27–30]. Eigenvalue analysis uses standard linear, state space form of system equations. The eigenvalues of a system matrix $A$ are given by the values of a scalar parameter $\lambda$ for which there are nontrivial solutions to (2.44)

$$A\phi = \lambda \phi$$

(2.44)

$A$ is a $n \times n$ system matrix and $\phi$ is $n \times 1$ vector.

To find the eigenvalues, (2.44) can be written in the form

$$(A - \lambda I)\phi = 0$$

(2.45)

For a non-trivial solution

$$\det [A - \lambda I] = 0$$

(2.46)

Thus for a complex pair of eigenvalues

$$\lambda = \sigma \pm j\omega$$

(2.47)

The imaginary parts of the eigenvalues represent the natural frequencies of the combined system. The corresponding real part of the eigenvalue is a quantitative measure of the stability of the mode, a negative real part signifies the modal stability. The real part of an eigenvalue is a direct measure of the damping of the mode.

Eigenvalue analysis for torsional interactions first requires identifying those eigenvalues that corresponds to natural mechanical modes of the turbine generator unit. The imaginary parts of these eigenvalues represent torsional mode frequencies, and the real parts indicate the damping of the corresponding mode.

Induction generator effect can also be evaluated with eigenvalue analysis. An unstable eigenvalue that is only related to electrical system may be an indication of induction generator effect. By varying the rotor resistance or series compensation, more insight into induction generator effect can be obtained. There is generally reasonable correlation between the frequency scanning and eigenvalue analysis results.

On the other hand, eigenvalues method has several shortcomings.

- The results are only valid for small disturbances, therefore, this technique can’t be used to study torque amplification.
- Physical nonlinearities of the system can’t be easily included in the model, like magnetic saturation of generators.
Similarly it is very difficult to represent switching devices, for example thyristor valves are represented by approximated linear transfer functions that neglect the effect of switching on the system behaviour.

### 2.5.3 Electromagnetic Transients Simulations

The Electromagnetic Transients Program (EMTP) is a programme for numerical integration of the system differential equations. Unlike transient stability programme which generally models positive sequence quantities only, representing a perfectly balanced system, EMTP presents full three phase model of the system with much more detailed models of the transmission lines, cables, machines and special devices such as series capacitors with complex bypass arrangements. The turbine generator unit can be modelled in detail as lumped parameter models and the bilateral coupling of the mechanical and electrical system is included. It allows the nonlinear modelling of complex system components providing great flexibility to model machines and their controllers, transient faults, circuit breaker action and other types of switching events. Due to its flexibility and generality, EMTP can be used to study all three types of SSR [10, 19, 21].

### 2.6 Comparison of SSR Analysis Methods

Table 2.2 provides a direct comparison of these methods. It can be observed that frequency scanning method has capabilities to detect all three aspects of SSR. Eigenvalue analysis provides more accurate information about the steady state SSR (dynamic instability) but cannot detect transient torque amplification problem. EMT simulations can indicate dynamic instability and transient torques amplification problem, but are not suitable for the studies in large networks due to very detailed modelling requirements.

<table>
<thead>
<tr>
<th></th>
<th>Frequency scanning</th>
<th>Eigenvalue analysis</th>
<th>EMT simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can identify dynamic instability?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Indicate transient torques amplification?</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Detailed models required</td>
<td>No</td>
<td>Moderate</td>
<td>Very</td>
</tr>
<tr>
<td>Applicable to large systems</td>
<td>Yes</td>
<td>Selective eigenvalue analysis</td>
<td>Impractical</td>
</tr>
<tr>
<td>Suitable for analysing effects of controllers</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
2.7 Test Networks

Two standard test networks are used within this thesis. The standard AC networks are presented in the following sections. Modifications made for various case studies conducted (such as compensation of line or addition of HVDC line), will be detailed on case by case basis to avoid any ambiguity.

2.7.1 Test Network 1

A large sixteen machine, sixty eight bus is mostly used within this thesis to investigate SSR phenomenon. This network is presented in the Fig. 2.19 and it was introduced in [31] and extensively used in [32] for damping controller design studies. This network represents a reduced order equivalent model of New England Test System and the New York Power System (NYPS). This network consists of five separate areas: NETS includes G1-G9; NYPS consists of generators G10-G13 and three further infeeds from neighbouring areas are represented separately by equivalent generators G14, G15 and G16. With loading details in [32], NYPS area is importing power from the neighbouring areas due to generation shortage of approximately 2.7 GW.

All generators are represented by eighth order models. Generators G1-G8 are equipped with slow DC1A exciter, whilst G9 uses a fast acting ST1A_v2 static exciter and PSS. The remaining generators (G10-G16) are under constant manual excitation. Power system loads are represented by constant impedance.

The generator G16 is a dynamic equivalent of the whole area, in most of studies conducted, it is replaced by the network shown in Fig. 2.20.

\[ G_{16}(\text{Active Power}) = (G_{16-1} + G_{1-1} + G_{8-1} + G_{9-1})_{\text{ActivePower}} \]

G16-1 parameters are same as that of G16 with inertia constant \( H = 10 \) while G1-1, G8-1 and G9-1 have same parameters as G1, G8 and G9 respectively.

Fig. 2.19 16 Machine, 68 bus test system. Separate areas (NETS, NYPS, G14, G15, G16) and inter-areas links highlighted
2.7.2 Test Network 2

A small four machine, two area network presented in Fig. 2.21, is also utilized within this thesis. It is introduced in [1] for use with small disturbance stability studies. This system requires roughly 400 MW power transfer from bus 7 to 9 through a long transmission corridor. All generators are represented by eighth order model neglecting leakage reactance. All four generators are equipped with ST1A_v1 static exciters and PSSs. All power loads are modelled as constant impedance.

2.8 Summary

This chapter has presented the modelling details of the power system components and analysis methods which will be used throughout this thesis.

This chapter began by describing the basic operation and structure of synchronous machine. Insight into synchronous mechanical and electrical mechanism provides the fundamental knowledge to perform, and analyse subsynchronous resonance phenomenon. Then mathematical models of the power system
components used in the studies within these theses are described. The models included, synchronous machine model and its associated controls, transmission lines, and loads. The models for LCC-HVDC system, VSC-HVDC system and TCSC are also presented.

The chapter briefly reviews the most commonly used subsynchronous resonance analysis methods. These three methods are employed throughout this thesis to perform SSR analysis. Finally, the test networks used throughout this research have been introduced.

References


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